Article

Two-Fluid Cosmological Models in Hypersurface-Homogeneous Spacetime

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Abstract

In this paper, we presented a class of solutions of Einstein's field equations describing two-fluid models of the universe in Hypersurface-Homogenous space time. In these models one fluid is the radiation distribution which represents the cosmic microwave background and the other fluid is the perfect fluid representing the matter content of the universe. Also we have discussed physical and kinematical behaviors of the model.

Key words: Two-fluid, hypersurface-homogenous space time, general relativity.

1. Introduction

The cosmic microwave background (CMB) is one of the corner stones of the homogeneous, isotropic model. Anisotropies within the CMB are associated with little perturbation, superimposed on the superbly smooth background that is believed to seed formation of galaxies and large-scale structure within the universe. Exact solutions of Einstein's field equations and the laws of thermodynamics are presented which both a comoving radiative perfect fluid (modelling the cosmic microwave background) and a non-comoving imperfect fluid (modelling the observed material content of the Universe) act as the source of the gravitational field as represented by the flat FRW line element by Coley [1]. Dunn [2] has found two fluid solutions to the field equations for Godel-type space-time.Coley and Dunn [3] presented exact solutions of the spatially, homogenous, anisotropic Binachi type VI_0 field equations in which the source of the gravitational field consists of two comoving perfect fluids. Pant and Oli [4] presented a class of solutions of Einstein's field equations describing two fluid models of the universe in a locally rotationally symmetric Bianchi type II space-time.

Anisotropic, homogeneous two-fluid cosmological models in a Bianchi type I space–time with a variable gravitational constant G and cosmological constant Λ has been investigated by Oli [5]. Oli [6] presented anisotropic, homogeneous two-fluid cosmological models in a Bianchi I spacetime. Anisotropic, homogeneous two-fluid cosmological models using Bianchi type-V

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space-time have been presented by Adhav et. al.[7]. Adhav et. al. [8] has studied anisotropic, homogeneous two-fluid cosmological models in a Bianchi type III space-time. Katore et. al [9] investigated cosmological models with perfect fluis and dark energy .Mete et. al.[10] have used plane symmetric metric to presente anisotropic, homogeneous two-fluid cosmological models. Anisotropic, homogeneous two-fluid cosmological models using Bianchi type-V space-time have been presented by Mete et. al.[11]. Pawar and Dagwal[12] have studied Bianchi Type IX two fluids cosmological models with matter and radiating source. Mete et. al [13] presented Bianchi type-I metric of the Kasner form describing two-fluid source of the universe in general relativity. R. Venkateswarlu [14] has investigated five dimensional Kaluza-Klein space-time describing two-fluid sources in the context of zero-mass scalar field. Samanta and Debata [15] studied a class of solutions of Einstein's field equations describing two-fluid models of the universe in a five dimensional spherical symmetric spacetime.

Evolution of Bianchi type-V cosmological model is studied in the presence of two-fluid distribution with negative constant deceleration parameter have been investigated by Singh et. al.[16]. Mete et. al.[17] considered anisotropic, homogeneous two-fluid plane symmetric cosmological models in higher dimensions. Two-fluid anisotropic Bianchi type-III cosmological model is investigated by Samanta [18] with variable gravitational constant G and cosmological constant Λ in the framework of Einstein's general relativity.

Motivating with this work, we have presented two-fluid Hypersurface-Homogenous cosmological model. The physical behavior of the model has been discussed in detail.

2. Metric & the Field Equations

Stewart and Ellis [19] have obtained general solutions of Einstein's field equations for a perfect fluid distribution satisfying a barotropic equation of state for the Hypersurface-homogeneous space time given by the metric

$$ds^2 = dt^2 - A^2 dx^2 - B^2 dy - \left[dy^2 + f_K^2(y) dz^2\right], \qquad (1)$$

where , $f_K(y) = \sin y, y, \sinh y$ respectively when $K = 1, 0, -1$.

Hajj-Boutros[20] developed a method to build exact solutions of field equations in case of the metric (1) in presence of perfect fluid and obtained exact solutions of the field equations which add to the rare solutions not satisfying the barotropic equation of state.

The Einstein's field equations for a two fluid source in natural units (gravitational units) are written as

$$R_{ij} - \frac{1}{2} g_{ij} R = -8\pi T_{ij} \, , \tag{2}$$

where R_{ij} is the Ricci tensor, R is the Ricci scalar.

and T_{ij} is the energy momentum tensor for a two fluid source is given by

$$T_{ij} = T_{ij}^{(m)} + T_{ij}^{(r)} \, , \tag{3}$$

where $T_{ij}^{(m)}$ is the energy momentum tensor for matter field and $T_{ij}^{(r)}$ is the energy momentum tensor for radiation field [3]which are given by $T_{ij}^{(m)} = (p_m + \rho_m) u_i^m \, u_j^m - p_m g_{ij}$, (4)

$$T_{ij}^{(r)} = \frac{4}{3} \rho_r u_i^r \, u_j^r - \frac{1}{3} \rho_r g_{ij} \tag{5}$$

together with $g^{ij} u_i^m \, u_j^m = 1, \qquad g^{ij} u_i^r \, u_j^r = 1$. (6)

The off diagonal equations of (2) together with energy conditions imply that the matter and radiation are both co-moving, we get,

$$u_i^m = (0,0,0,1) \, , \qquad u_i^r = (0,0,0,1) \quad . \tag{7}$$

Using the co-moving coordinate system, the non-vanishing components of ${}^mT_i^j$ and ${}^rT_i^j$ can be obtaind as

$${}^mT_4^4 = \rho_m \, , \quad {}^mT_1^1 = {}^mT_2^2 = {}^mT_3^3 = -p_m \, , \quad {}^mT_i^j = 0 \text{ for } i \neq j \tag{8}$$

and

$${}^rT_4^4 = \rho_r \, , \quad {}^rT_1^1 = {}^rT_2^2 = {}^rT_3^3 = -\frac{\rho_r}{3} \, , \quad {}^rT_i^j = 0 \text{ for } i \neq j \, . \tag{9}$$

Now, the field equations (2) for metric (1) with the help of equations (8) and (9) yield set of equations:

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{K}{B^2} = -8\pi \left(p_m + \frac{\rho_r}{3} \right), \tag{10}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -8\pi \left(p_m + \frac{\rho_r}{3} \right), \tag{11}$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} + \frac{K}{B^2} = 8\pi (\rho_m + \rho_r), \tag{12}$$

where overhead dot denotes differentiation with respect to *t*.

3. Solution of the Fueld Equations

Equations (10) to (12) are three independent equations in five unknowns $A, B, \rho_m, \rho_r, p_m$.Shri Ram and Verma [21] have investigated the Hypersurface homogeneous cosmological models with time varying G and Λ term in the presence of bulk viscous fluid. Katore et. al. [22] have obtained hypersurface-homogeneous cosmological model in presence of perfect fluid within the frame work of Barber's second self-creation theory.Chandel et. al.[23] have investigated hypersurface-homogeneous bulk viscous fluid cosmological models with time-dependent cosmological term.They showed that the field equations are solvable for any arbitrary cosmic scale function. We follow the same approach to find exact solutions of the field equations.

Using equations (10) and (11), we obtain

$$\frac{\ddot{B}}{B} - \frac{\ddot{A}}{A} + \frac{\dot{B}^2}{B} - \frac{\dot{A}\dot{B}}{AB} + \frac{K}{B^2} = 0, \tag{13}$$

which on integration, yields

$$-B^2\dot{A} + AB\dot{B} = -K\int A dt + c_1, \tag{14}$$

where c_1 is an integration constant.

We can write equation (14) in the form

$$\frac{d}{dt}\left(B^2\right) - 2\frac{\dot{A}}{A}B^2 = F(t), \tag{15}$$

where $F(t) = -2\frac{k}{A}\int A dt + c_1$. (16)

The linear differential equation (16) has the general solution given by

$$B^2 = A^2\left[\int \frac{F(t)}{A^2} dt + c_2\right], \tag{17}$$

where c_2 is an integration constant.

It is clear that the solution of field equations reduces to integration (17) if $A(t)$ is known as an explicit function of time. We now obtain a particular solution of the field equations for a simple choice of the function, $A(t)$.

We choose

$$A = t^n, \tag{18}$$

where n is a real number.

Integrating equation (17), we obtain

$$B^2 = \frac{Kt^2}{n^2-1} + c_1 t^{1+2n} + c_2 t^{2n}. \tag{19}$$

Without loss of generality, we take $c_1 = c_2 = 0$. The solution of equation (19) becomes

$$B^2 = \frac{Kt^2}{n^2-1}, \quad n \neq \pm 1. \tag{20}$$

Hence the geometry of the universe for the hypersurface-homogeneous space-time corresponding to the solution (18) and (20) takes

$$ds^2 = dt^2 - t^{2n}dx^2 - \frac{Kt^2}{n^2-1}\left[dy^2 + f_k^2(y)dz^2\right]. \tag{21}$$

The metric (21) is well defined for $n \neq 1$.

3.1 Model I:

For $K = 1$, the hypersurface-homogeneous cosmological model in (21) reduces to

$$ds^2 = dt^2 - t^{2n}dx^2 - \frac{t^2}{n^2-1}\left[dy^2 + \sin^2 y dz^2\right]. \tag{21a}$$

This model is well defined for $n^2 - 1 > 0$.

3.2 Model II:

For $K = -1$, the hypersurface-homogeneous cosmological model in (21) reduces to

$$ds^2 = dt^2 - t^{2n}dx^2 - \frac{t^2}{1-n^2}\left[dy^2 + \sinh^2 y dz^2\right]. \tag{21b}$$

4. Some Physical & Kenematical Properties

We assume the relation between pressure and energy density of matter field through the "gamma-law" equation of state which is given by

$$p_m = (\gamma - 1)\rho_m, \; 1 \le \gamma \le 2 \quad . \tag{22}$$

We get energy density of matter, energy density of radiation and total energy density as

$$8\pi\rho_m = \left[\frac{2n(2n+1)}{(4-3\gamma)t^2}\right]. \tag{23}$$

$$8\pi\rho_r = \left[\frac{3n(2-2\gamma-n\gamma)}{(4-3\gamma)t^2}\right] \tag{24}$$

$$8\pi\rho = 8\pi(\rho_m + \rho_r) \; ,$$

$$8\pi\rho = \frac{n(n+2)}{t^2}. \tag{25}$$

The density parameter for matter and radiation respectively are given by

$$\Omega_m = \left[\frac{6n(2n+1)}{(4-3\gamma)(n+2)^2}\right]. \tag{26}$$

$$\Omega_r = \left[\frac{9n(2-2\gamma-n\gamma)}{(4-3\gamma)(n+2)^2}\right] \tag{27}$$

and total energy density are given by

$$\Omega = \Omega_m + \Omega_r$$

$$\Omega = \frac{3n}{(n+2)} \; . \tag{28}$$

The anisotropy of the expansion can be parameterized after defining the directional Hubble parameters and the mean Hubble parameter of the expansion.The directional Hubble parameters, which determine the universe expansion rates in the directions of the x, y, z axes, defined as

$$H_x = \frac{\dot{A}}{A}, H_y = H_z = \frac{\dot{B}}{B}, \tag{29}$$

and in terms of the average scale factor $a=(AB^2)^{\frac{1}{3}}$, the Hubble parameter H, which determines the volume expansion rate of the universe , may be generalized to anisotropic cosmological models:

$$H=\frac{\dot{a}}{a}=\frac{1}{3}\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right). \tag{30}$$

The physical quantities of observational interest are the expansion scalar θ, the average anisotropy parameter A_m and the shear scalar σ^2. These are defined as

$$\theta=u^i_{;i}=\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right), \tag{31}$$

$$A_m=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2, \tag{32}$$

$$\sigma^2=\frac{3}{2}A_mH^2. \tag{33}$$

The deceleration parameter yields as

$$q=\frac{d}{dt}\left(\frac{1}{H}\right)-1 \quad . \tag{34}$$

The sign of q indicates whether the model inflates or not. The positive sign corresponds to the standard decelerating model whereas the negative sign indicates inflation.

Using equations (18) and (20), the Hubble parameter, Scalar Expansion, Mean anisotropic parameter, Shear Scalar and deceleration parameter are given by:

The Hubble parameter:

$$H=\frac{n+2}{3t}. \tag{35}$$

The scalar expansion:

$$\theta=\frac{n+2}{t} \tag{36}$$

The mean anisotropic parameter:

$$A_m=\frac{2(n-1)^2}{(n+2)^2}. \tag{37}$$

The shear scalar:

$$\sigma^2=\frac{(n-1)^2}{3t^2}. \tag{38}$$

The deceleration parameter:

$$q = \frac{1-n}{n+2} \quad . \tag{39}$$

Spatial Volume:

$$V = \frac{t^{2+n}}{n^2 - 1} . \tag{40}$$

It can be seen that the Hubble parameter , scalar expansion,shear scalar approach zero as t becomes infinitely large while they all diverge at the initial epoch, i.e. at $t = 0$. We observe that the spatial volume is zero at $t = 0$, and it increases with cosmic time. This means that the model starts expanding with a big-bang at $t = 0$.Also, since the average anisotropy parameter is constant, the model remains anisotropic throughout the evolution of the universe which is quite in agreement with the observational evidence of the early universe.The deceleration parameter q is positive for $n < 1$and is negative for $n > 1$. Thus the models represent decelerating universe for $n < 1$and inflationary accelerating universe for $n > 1$.Thus our theortical models are consistent with the results of recent observations.The $\frac{\sigma}{\theta} =$ constant , anisotropy in the models are maintained throughout.At initial epoch i.e.at $t = 0$, ρ_m and ρ_r are all infinite .The physical behaviors of the models (21a) and (21b) are same as of the model (21).

Case I: Dust model

In order to investigate the physical behaviour of the fluid parameters, we consider the particular case of dust which can be reduced from relation (22) when $\gamma = 1$. The universe of this type is termed as dust universe.

The density parameter for matter and radiation respectively are given by

$$\Omega_m = \left[\frac{6n(2n+1)}{(n+2)^2} \right].$$

$$\Omega_r = \left[\frac{-9n^2}{(n+2)^2} \right]$$

and total energy density are given by

$$\Omega = \Omega_m + \Omega_r$$

$$\Omega = \frac{3n}{(n+2)} \ .$$

Case II: Radiation universe $\left(\text{when } \gamma = \frac{4}{3}\right)$

In this case, in relation (22) we take $\gamma = \frac{4}{3}$. The universe of this kind is termed as a radiating universe.

The density parameter for matter and radiation respectively are given by

$$\Omega_m = \infty.$$

$$\Omega_r = \infty$$

Case III: Hard universe $\left[\gamma \in \left(\frac{4}{3}, 2\right), let\, \gamma = \frac{5}{3}\right]$

In this case, in relation (22) we take $\gamma = \frac{5}{3}$.The universe of this kind is termed as a hard universe.

The density parameter for matter and radiation respectively are given by

$$\Omega_m = -\left[\frac{6n(2n+1)}{(n+2)^2}\right].$$

$$\Omega_r = \left[\frac{3n(5n+4)}{(n+2)^2}\right]$$

and total energy density are given by

$$\Omega = \Omega_m + \Omega_r$$

$$\Omega = \frac{3n}{(n+2)} .$$

Case IV: Zeldovich Universe $(\gamma = 2)$

In this case the relation (22) we take $\gamma = 2$.The universe of this kind is termed as a Zeldovich universe.

The density parameter for matter and radiation respectively are given by

$$\Omega_m = \left[\frac{-3n(2n+1)}{(n+2)^2} \right].$$

$$\Omega_r = \left[\frac{9n(1+n)}{(n+2)^2} \right]$$

and total energy density are given by

$$\Omega = \Omega_m + \Omega_r$$

$$\Omega = \frac{3n}{(n+2)} .$$

5. Conclusion

In this paper, we have provided a detailed Homogenous - Hypersurface cosmologies dominating by two relativistic cosmic fluids. Here one fluid represents the matter content of the universe and another fluid is chosen to model the CMB radiation.The cosmological evolution of the model is expansionary, with the scale factors monotonically increasing function of time. The universe start expanding with a big-bang singularity at $t = 0$.The physical and kinematical parameters diverge at this initial singularity. The physical and kinematical parameters are well defined and are decreasing functions for $0 < t < \infty$, and ultimately tend to zero for large time. The $\frac{\sigma}{\theta} =$ constant, anisotropy in the models are maintained throughout.It is important to note that the results obtained resembles with the investigations of Samanta [18] and Singh et. al.[16].

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