Article

Lyra's Geometry in a Bianchi Type II String Dust Cosmological Model with an Electromagnetic Field

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Abstract

We have investigated a Bianchi type II string dust cosmological model with an electromagnetic field in Lyra's Geometry. The magnetic field is due to the electric current flowing along the x-axis such that F_{23} is the only non-vanishing component of F_{ij}. We have assumed the condition $R = S^n$ between a metric potential to get a determinate solution. We found that the model starts with big bang and expansion decreases with time. Furthermore, anisotropy is maintained throughout but the model isotropizes when $n = 1$. The physical and geometrical aspects of the model are also discussed.

Keywords: Bianchi type II model, Lyra's geometry, string dust, electromagnetic field.

1. Introduction

Bianchi type II models play an important role in current modern cosmology, for simplification and description of large-scale behavior of the universe. Bianchi type II spacetime has a fundamental role in constructing cosmological models suitable for describing the early stages of evolution of the universe. Asseo and Sol [1] emphasized the importance of the Bianchi type II universe.

The present-day magnitude of the magnetic field is very small compared to the estimated matter density. It might not have been negligible during the early stage of evolution of the universe, however. Asseo and Sol [1] speculated about the existence of a primordial magnetic field of cosmological origin. Vilenkin [2] pointed out that cosmic strings may cause gravitational lensing. Therefore, it is interesting to discuss whether it is possible to construct an analogue of a cosmic string in the presence of a magnetic field in the framework of Lyra's geometry.

After Einstein (1916) proposed his theory of general relativity, which provided a geometrical description of gravitation, many physicists attempted to generalize the idea of geometrizing gravitation to include a geometrical description of electromagnetism. One of the first attempts was made by Weyl [3], who proposed a more general theory by formulating a new kind of gauge theory involving a metric tensor to geometrize gravitation and electromagnetism. Weyl theory was criticized, however, due to the non-integrability of the length of a vector under parallel displacement. Later, Lyra [4] suggested a modification of Riemannian geometry by introducing a gauge function into the structureless manifold that removed the non-

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integrability condition. This modified geometry is known as Lyra's geometry. Subsequently, Sen [5] formulated a new scalar-tensor theory of gravitation and constructed an analogue of Einstein's field equations based on Lyra's geometry. He found that the static model with finite density in a Lyra manifold is similar to the static model in Einstein's general relativity. Halford [6] pointed out that the constant displacement vector field (β) in Lyra's geometry plays a similar role of cosmological constant (ʌ) in the general theory of relativity. He has also shown that scalar-tensor treatment based in Lyra's geometry predicts the same effects, within observational limits, as in Einstein's theory. The main difference between cosmological theories based on Lyra's geometry and Riemannian geometry lies in the fact that the constant displacement vector (□) arises naturally from the concept of gauge in Lyr'sa geometry whereas the cosmological constant (Λ) was introduced by Einstein in an adhoc manner to find the static solution of his field equations.

Soleng [7] investigated cosmological models based on Lyra's geometry and has shown that the constant gauge vector field either includes a creation field and is identical to Hoyle's creation cosmology (Hoyle, [8],Hoyle and Narlikar [9, 10]) or contains a special vacuum field that, together with the gauge vector term, may be considered a cosmological term. In the latter case, solutions are identical to general relativistic cosmologies with a cosmological term.

Cosmological models based on Lyra's geometry with constant and time-dependent displacement vector fields have been investigated by a number of authors, namely, Beesham [11], Chakraborty and Ghosh [12], Rahaman and Bera [13], Rahaman et al. [14, 15], Pradhan and Vishwakarma [16], Ram and Singh [17], Pradhan et al. [18], Ram et al. [19], Mohanty et al. [20], Bali and Chandnani [21], etc. Singh [22–25] and Singh and Desikan [26] have studied Bianchi type I, III models, Kantowaski-Sachs models and a new class of cosmological models with a time-dependent displacement field. They have also made a comparative study of Robertson-Walker models with a constant deceleration parameter in Einstein's theory with a cosmological term and in cosmological theory based on Lyra's geometry. Bali et. al. [27] have investigated LRS Bianchi type II massive string cosmological models with a magnetic field in Lyra's geometry. An anisotropic Bianchi type III bulk viscous fluid universe in Lyra's geometry has been studied by Kumari et al. [28]. Bali and Chandnani [29] have investigated a Bianchi type III bulk viscous dust-filled universe in Lyra's geometry under certain physical assumptions. Recently, V. K. Yadav and L. Yadav [30] have presented Bianchi type III bulk viscous and barotropic perfect fluid cosmological models in Lyra's geometry with the assumption that the coefficient of viscosity of dissipative fluid is a power function of the energy density. Bali et al. [31] also investigated a Bianchi type I string dust magnetized cosmological model in the framework of Lyra's geometry.

In this paper, we have investigated a Bianchi type II string dust cosmological model with an electromagnetic field in Lyra's geometry. To get a determinate solution, we have assumed the condition $\lambda = \rho$ and $R = S^n$ between metric potentials where n is a constant. Various physical and geometrical features of the model are also discussed.

2. Metric and Field Equations

We consider an LRS Bianchi type II metric of the form

$$ds^2 = -dt^2 + R^2(dx^2 + dz^2) + S^2(dy - xdz)^2 \tag{1}$$

where the metric potentials R and S are functions of t alone and $\sqrt{-g} = R^2 S$

The energy momentum tensor for string dust in the presence of a magnetic field is given by

$$T_i^j = \rho\, \mathrm{v}_i v^j - \lambda x_i x^j + E_i^j \tag{2}$$

where ρ is energy density, λ is string density and v^i describes the fluid four velocity vector satisfying the condition

$$\mathrm{v}_i v^i = -1 = -x_i x^i \tag{3}$$

The coordinates are considered to be co-moving so that

$$v^i = (0,0,0,1) \tag{4}$$

Einstein's field equation (in gravitational units $c = 1$, $8\pi G = 1$), in normal gauge for a Lyra manifold were obtained by Sen [5] as

$$R_i^j - \frac{1}{2} R g_i^j + \frac{3}{2}\phi_i \phi^j - \frac{3}{4} g_i^j \phi_k \phi^k = T_i^j \tag{5}$$

where ϕ_i is the displacement field vector defined as

$$\phi_i = (0, 0, 0, \beta(t)) \tag{6}$$

where β is the gauge function.

The electromagnetic field E_i^j given by Lichnerowicz is

$$E_i^j = \bar{\mu}\left[|h|^2 \left(v_i v^j + \frac{1}{2} g_i^j \right) - h_i h^j \right] \tag{7}$$

where $\bar{\mu}$ is the magnetic permeability and h_i is the magnetic flux vector defined by

$$h_i = \frac{\sqrt{-g}}{2\bar{\mu}} \varepsilon_{ijkl} F^{kl} v^j \tag{8}$$

where F^{kl} is the electromagnetic field tensor and ε_{ijkl} is the Levi-Cevita tensor density.

We assume that current is flowing along the x-axis so magnetic field is in yz-plane. So,

$$h_1 \neq 0; h_2 = h_3 = h_4 = 0 \tag{9}$$

and F_{23} is the only non-vanishing component of F_{ij}.

Maxwell's equation is

$$F_{[ij;k]} = 0 \tag{10}$$

which leads to

$$F_{23} = constant = H \tag{11}$$

Now, the components of the electromagnetic field with the help of equations (7), (9) and (11) are obtained as

$$E_1^1 = -\frac{H^2}{2\bar{\mu}R^2S^2} = -E_2^2 = -E_3^3 = E_4^4 \tag{12}$$

Einstein's field equation (5) for the metric (1) together with (12) leads to

$$\frac{R_{44}}{R} + \frac{S_{44}}{S} + \frac{R_4S_4}{RS} + \frac{S^2}{4R^4} + \frac{3}{4}\beta^2 = \lambda + \frac{K}{R^2S^2} \tag{13}$$

$$\frac{2R_{44}}{R} + \frac{R_4^2}{R^2} - \frac{3S^2}{4R^4} + \frac{3}{4}\beta^2 = -\frac{K}{R^2S^2} \tag{14}$$

$$\frac{R_4^2}{R^2} + \frac{2R_4S_4}{RS} - \frac{S^2}{4R^4} - \frac{3}{4}\beta^2 = \rho + \frac{K}{R^2S^2} \tag{15}$$

where $K = \frac{H^2}{2\bar{\mu}}$

and the sub indices 4 in R and S denotes differentiation with respect t.

The energy conservation equation $T^i_{i;j} = 0$ leads to

$$\rho_4 + \rho\left(\frac{2R_4}{R} + \frac{S_4}{S}\right) - \lambda\frac{R_4}{R} = 0 \tag{16}$$

and

$$\left(R_i^j - \frac{1}{2}Rg_i^j\right)_{;j} + \frac{3}{2}\left(\phi_i\phi^j\right)_{;j} - \frac{3}{4}\left(g_i^j\phi_k\phi^k\right)_{;j} = 0 \tag{17}$$

which again leads to

$$\frac{3}{2}\phi_i\left[\frac{\partial\phi^j}{\partial x^j}+\phi^l\Gamma^j_{lj}\right]+\frac{3}{2}\phi^j\left[\frac{\partial\phi_i}{\partial x^j}-\phi_l\Gamma^l_{ij}\right]-\frac{3}{4}g^j_i\phi^k\left[\frac{\partial\phi_k}{\partial x^j}+\phi^l\Gamma^j_{lj}\right]-\frac{3}{4}g^j_i\phi_k\left[\frac{\partial\phi^k}{\partial x^j}+\phi_l\Gamma^l_{ij}\right]=0 \tag{18}$$

Equation (18) is identically satisfied for *i= 1, 2, 3*. For *i= j=* 4, equation (18) reduces to

$$\frac{3}{2}\beta\beta_4+\frac{3}{2}\beta^2\left(\frac{2R_4}{R}+\frac{S_4}{S}\right)=0 \tag{19}$$

3. Solution of Field Equations

Field equations (13)-(15) are a system of three equations in four unknowns: R, S, λ, ρ. For complete determination of the model, we assume that shear tensor (σ) is proportional to the expansion (θ) i.e.

$$R=S^n \tag{20}$$

We also assume that string tension density λ is equal to rest density ρ.

i.e. λ = ρ (21)

From equation (19), we have

$$\beta=\frac{\alpha}{R^2S} \tag{22}$$

where α is constant of integration.

From equations (13), (15) and (21), we have

$$\frac{R_{44}}{R}+\frac{S_{44}}{S}-\frac{R_4^2}{R^2}-\frac{R_4S_4}{RS}+\frac{2S^2}{4R^4}+\frac{3}{2}\beta^2=0 \tag{23}$$

Using equations (20) and (22), equation (23) leads to

$$S_{44}-\frac{2n}{(n+1)}\frac{S_4^2}{S}=-\frac{3\alpha^2}{2(n+1)S^{(4n+1)}}-\frac{1}{2(n+1)S^{(4n-3)}} \tag{24}$$

Now on putting $S_4 = f(S)$ in equation (24), we get

$$\frac{df^2}{dS}-\frac{4n}{(n+1)}\frac{f^2}{S}=-\frac{3\alpha^2}{(n+1)S^{(4n+1)}}-\frac{1}{(n+1)S^{(4n-3)}} \tag{25}$$

Equation (25) leads to

$$f^2=\frac{3\alpha^2}{4n(n+2)}S^{-4n}+\frac{1}{4(n^2+n-1)}S^{-4(n-1)} \tag{26}$$

Equation (26) leads to

$$\int\frac{dS}{\sqrt{\frac{3\alpha^2}{4n(n+2)}S^{-4n}+\frac{1}{4(n^2+n-1)}S^{-4(n-1)}}}=t+M \tag{27}$$

where M is constant of integration. The value of S can be determined by equation (27). After a suitable transformation of coordinates, metric (1) reduces to

$$ds^2=\frac{-dT^2}{\left(\frac{3\alpha^2}{4n(n+2)}T^{-4n}+\frac{1}{4(n^2+n-1)}T^{-4(n-1)}\right)}+T^{2n}\left(dX^2+dZ^2\right)+T^2(dY-XdZ)^2 \tag{28}$$

where S = T, x = X, y = Y and z = Z.

4. Some Physical and Geometrical Features

The energy density (ρ) and string density (λ) for the model (28) are given by

$$\rho=-\frac{K}{S^{2n+2}}+\frac{(n+1)}{4(n^2+n-1)S^{4n-2}} \tag{29}$$

and

$$\lambda=-\frac{K}{S^{2n+2}}+\frac{(n+1)}{4(n^2+n-1)S^{4n-2}} \tag{30}$$

Equation (22) gives

$$\beta=\frac{\alpha}{T^{2n+1}} \tag{31}$$

The scalar expansion θ calculated for flow vector v^i is given by

$$\theta=\frac{2R_4}{R}+\frac{S_4}{S}$$

$$\theta = (2n+1)\sqrt{\left(\frac{3\alpha^2}{4n(n+2)T^{4n+2}} + \frac{1}{4(n^2+n-1)T^{4n-2}}\right)} \tag{32}$$

The shear σ for the model (28) is given by

$$\sigma^2 = \frac{1}{3}(n-1)^2\left(\frac{3\alpha^2}{4n(n+2)T^{4n+2}} + \frac{1}{4(n^2+n-1)T^{4n-2}}\right) \tag{33}$$

From equations (32) and (33) we have

$$\frac{\sigma^2}{\theta^2} = \frac{1}{3}\frac{(n-1)^2}{(2n+1)^2} = \text{constant} \tag{34}$$

The spatial volume (V) is given by

$$V^3 = T^{(2n+1)} \tag{35}$$

The deceleration parameter q is given by

$$q = -\left(\frac{2n-2}{2n+1}\right) + \frac{12}{(2n+1)}\left[\frac{L+MT^4}{2L+NT^4}\right] \tag{36}$$

where $L = 6\alpha^2(n^2+n-1)$, $M = 2(n^2+n-2)$
and $N = 4n(n+2)$

5. Conclusion

We have obtained a Bianchi type II string dust cosmological model with an electromagnetic field in a Lyra manifold. The model starts with a big bang at $T=0$ and expansion in the model decreases as time increases. Expansion in the model stops when $n = -\frac{1}{2}$ and at $T \to \infty$.

Since $T \to \infty, \frac{\sigma}{\theta} \neq 0$ and anisotropy is maintained throughout. However, when $n=1$, the model isotropizes. The displacement vector β is initially large but decreases due to the lapse of time when $2n+1>0$. Model (28) has point-type singularity at $T=0$, where $n>0$.
The deceleration parameter $q=2$ when $n=0$ and $q=+ve$ when $n \geq 1$, which a represents decelerating universe.

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References

[1] Asseo, E. and Sol. H.: 'Extragalactic magnetic fields', Physics Reports, 148(6), 307-436 (1987).
[2] Vilenkin, A.:'Cosmic strings,' Physical Review D, vol. 24, no. 8, 2082–2089, (1981).
[3] Weyl,H.,SitzungsberichteDerPreussischenAkademieDer issenschaften,(1918).
[4] Lyra,G.:'Uber eine modifikation der riemannschen geometrie,' Mathematische Zeitschrift, vol. 54, pp. 52–64, (1951).
[5] Sen, D. K.: 'A static cosmological model,' Zeitschrift f¨ur Physik C, vol. 149, pp. 311–323, (1957).
[6] Halford, W. D.:'Scalar-tensor theory of gravitation in a Lyra manifold,' Journal of Mathematical Physics, vol. 13, no. 11, pp. 1699–1703, (1972).
[7] Soleng, H. H.:'Cosmologies based on Lyra's geometry,' General Relativity and Gravitation, vol. 19, no. 12, pp. 1213–1216, (1987).
[8] Hoyle, F.: 'A new model for the expanding universe,' Monthly Notices of the Royal Astronomical Society, vol. 108, pp. 372–382, (1948).
[9] Hoyle, F.and Narlikar, J. V.:'On the avoidance of singularities in C-field cosmology,' Proceedings of the Royal Society A, vol. 278, pp. 465–478, (1964).
[10] Hoyle, F. and Narlikar, J. V.:'Time symmetric electrodynamics and the arrow of time in cosmology,' Proceedings of the Royal Society A, vol. 277, pp. 1–23, (1964).
[11] Beesham, A.:'FLRW cosmological models in Lyra's manifold with time dependent displacement field,' Australian Journal of Physics, vol. 41, no. 6, pp. 833–842, (1988).
[12] Chakraborty, S. and Ghosh, A. :'Generalized scalar tensor theory in four and higher dimension,' International Journal of Modern Physics D, vol. 9, no. 5, pp. 543–549, (2000).
[13] Rahaman, F. and Bera, J.: 'Higher-dimensional cosmological model in Lyra geometry,' International Journal of Modern Physics D, vol. 10, no. 5, pp. 729–733, (2001).
[14] Rahaman, F., Chakraborty, S. and Bera, J. :'Inhomogeneous cosmological model in Lyra geometry,' International Journal of Modern Physics D, vol. 11, no. 9, pp. 1501–1504, (2002).
[15] Rahaman, F., Begum, N., Bag, G. and Bhui, B. C.: 'Cosmological models with negative constant deceleration parameter in Lyra geometry,' Astrophysics and Space Science, vol. 299, no. 3, pp. 211–218, (2005).
[16] Pradhan, A. and Vishwakarma, A. K.:'A new class of LRS Bianchi type-I cosmological models in Lyra geometry,' Journal of Geometry and Physics, vol. 49, no. 3-4, pp. 332–342, (2004).
[17] Ram, S. and Singh, P. : 'Anisotropic cosmological models of Bianchi types III and IV in Lyra's geometry,"' International Journal of Theoretical Physics, vol. 31,no. 12,pp. 2095–2102, (1992).
[18] Pradhan, A., Yadav,V. K. and Chakrabarty, I.: 'Bulk viscous FRW cosmology in Lyra geometry,' International Journal of Modern Physics D, vol. 10, no. 3, pp. 339–349, (2001).
[19] Ram, S., Zeyauddin, M. and Singh, C. P.: 'Anisotropic Bianchi type V perfect fluid cosmological models in Lyra's geometry,' Journal of Geometry and Physics, vol. 60, no. 11, pp. 1671–1680, (2010).
[20] Mohanty,G.,Mahanta,K.L.andSahoo,R. R.:'Non-existence of five dimensional perfect fluid cosmological model in Lyra manifold'Astrophysics and Space Science, vol. 306, no. 4, pp. 269–272, (2006).
[21] Bali, R. and Chandnani,N. K.:'Bianchi type V barotropic perfect fluid cosmological model in Lyra geometry,' International Journal ofTheoretical Physics, vol. 48, no. 5, pp. 1523–1533, (2009).
[22] Singh, T. and Singh, G. P.: 'Bianchi type-I cosmological models in Lyra's geometry,' Journal of Mathematical Physics, vol. 32, no. 9, pp. 2456–2458, (1991).
[23] Singh, T.and Singh, G. P.:'Some cosmological models with constant deceleration parameter,' Il Nuovo Cimento B, vol. 106, no. 6, pp. 617–622, (1991).
[24] Singh,T.and Singh, G. P.: 'Bianchi type III and Kantowski-Sachs cosmological models in Lyra geometry,' International Journal of Theoretical Physics, vol. 31, no. 8, pp. 1433–1446, (1992).
[25] Singh, T. and Singh, G. P.: 'Lyra's geometry and cosmology: a review,' Fortschritte der Physik, vol. 41, no. 8, pp. 737–764, (1993).

[26] Singh, G. P. and Desikan, K. :'A new class of cosmological models in Lyra geometry,' Pramana—Journal of Physics, vol. 49, no. 2, pp. 205–212, (1997).

[27] Bali, R., Yadav, M. K. and Gupta, L.K.:'LRS Bianchi Type II Massive String Cosmological Models with Magnetic Field in Lyra's Geometry', Advances in Mathematical Physics, Volume 2013, Article ID 892361, (2013).

[28] Kumari, P., Singh, M. K., and Ram, S.: 'Anisotropic Bianchi Type-III Bulk Viscous Fluid Universe in Lyra Geometry', Advances in Mathematical Physics, Volume 2013, Article ID 416294, (2013).

[29] Bali, R. and Chandnani, N.K.:'Bianchi type-III bulk viscous dust filled universe in Lyra geometry,' Astrophysics and Space Science, vol. 318, no. 3-4, pp. 225–230, (2008).

[30] Yadav, V. K. and Yadav, L.: 'Bianchi type-III bulk viscous and barotropic perfect fluid cosmological models in Lyra's geometry,' International Journal of Theoretical Physics, vol. 50, no. 5, pp. 1382–1394, (2011).

[31] Bali, R., Gupta, L. K. and Chandnani, N. K.:'Bianchi type I string dust magnetized cosmological models in Lyra geometry,' Communications in Theoretical Physics, vol. 54, pp. 197–202, (2010).