Article

Comparison of the Algorithmic and Axiomatic Approaches to the Construction of Quantum Field Theory

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Abstract

Two possibilities of the quantum theory construction, indicated by Feynman, are examined. The special features of the structure of the Standard Model (SM) are enumerated, which attest to the fact that SM is not an axiomatic, but an algorithmic theory. Deficiencies of SM and possibilities of overcoming these deficiencies are indicated. The structure of the nonlinear quantum field theory (NQFT) as an axiomatic theory, which makes it possible to overcome deficiencies in the Standard Model, is presented.

Key-words: algorithmic, axiomatic, Standard Model, quantum field theory.

Introduction: "Babylonian" and "Greek" Approach to the Construction of the Physical Theory

Historically, there are several aspects of mathematics. Proof-based mathematics is not the only form. Davis and Hersh (Davis and Hersh, 1982) affirm: "*The mathematics of Egypt, of Babylon, and of the ancient Orient was all of the algorithmic type. Dialectical mathematics -- strictly logical, deductive mathematics -- originated with the Greeks. But it did not displace the algorithmic.*"

But the famous physicist Feynman (Feynman, 1964) argued that, "*In physics, we need the Babylonian method, and not the Euclidian or Greek method*." Here R. Feynman expounds what he calls the Babylonian, as opposed to the Euclidean or axiomatic, approach to mathematics and physics. In the latter, the plan is to deduce statements from a set of axioms, whereas the Babylonian approach exploits alternative representations of physical phenomena and the interchangeability which only mathematical reasoning affords.

Different mathematical representations invite different physical interpretations, and the mathematical generalizations implied may lead to theorems whose generality extends beyond the confines of a proof within a given system. The next step is then the guessing of physical equations, which, Feynman argues, facilitates the guessing of new physical laws in a way that common-sense feeling, philosophical principles, or models cannot (Feynman, 1964):

"*...there are two kinds of ways of looking at mathematics, which for the purpose of this lecture I will call the Babylonian tradition and the Euclidean or Greek tradition. In Babylonian schools in mathematics the student would learn by doing a large number of examples until establishing the general rule (i.e., the theorem). Tables of numerical quantities were available so that they could solve elaborate equations.*

Under the Babylonian system, everything was prepared for calculating things out. But Euclid (under the Greek mathematical system) discovered that there was a way in which all of the theorems of geometry could be ordered from a set of axioms that were simple. The Babylonian mathematics is that you know all of the various theorems and many of the connections in between..."

Historically in the estimation of Babylonian mathematics R. Feynman is not entirely precise. In the Babylonian documents no formulations were found, which can pretend to the role of "*the various theorems*". The proof of the Pythagorean Theorem, for example, does not depend on the specific sizes of the right-angle triangles. Babylonian approach consists in the trivial practical detection and writing out of hundreds of different three numbers for the sides of the right-angle triangles, which have the same relation between the squares of their sides, which is assigned as the Pythagorean Theorem.

Further assertion of R. Feynman cannot also be accepted without criticism:

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"*...The method of always starting from axioms is not efficient in obtaining theorems. In working something out in geometry it is not efficient if each time you have to start back at the axioms. In physics we 'need' Babylonian method and not the Euclidean or Greek method.. . We can deduce often from one part of physics, like the Law of Gravitation, a principle which turns out to be much more valid than the derivation. This does not happen in Mathematics and under Greek methodology; theorems do not come out in places where they're not supposed to be….*"

In our time the mathematicians also frequently begin from axioms. Nevertheless, since there are already thousands of theorems, proven earlier on the basis of initial axioms, most frequently these theorems and lemmas are used for further proof of various facts. But as it is understandable, this is a simple method to use an axiomatic base.

Maybe for this reason not all physicists agreed with Feynman. The book of Leonard Mlodinow (Mlodinow, 2003) offers into R. Feynman's relationship with Murray Gell-Mann. L. Mlodinow note the rivalry between two great physicists, both of whom showed a grudging respect for the other, but who followed different lines of enquiry in their chosen field of Theoretical Physics: Feynman considered himself a "Babylonian" and Murray Gell-Mann a "Greek":

"Feynman used to say that there were two kinds of Physicists, the Babylonians and the Greeks. He was referring to the opposing philosophies of those ancient civilizations. The Babylonians made western civilization's first great strides in understanding numbers and equations, and in geometry. Yet it was the later Greeks – in particular Thales, Pythagoras, and Euclid – whom we credit with inventing Mathematics. This is because the Babylonians cared only whether or not a method of calculation worked – that is, adequately described a real physical situation – and not whether it was exact, or fit (sic) into any greater logical system. Thales and his Greek followers, on the other hand, invented the idea of theorem and proof – and required that for a statement to be considered through, it had to be an exact logical consequence of a system of explicitly stated axioms or assumptions. To put it simply, the Babylonians focused on phenomena, the Greeks on the underlying order."

Nevertheless, the existence of Babylonian method has actually a deep sense: axiomatic approach can be formulated only if satisfactory number of facts, needed for the generalization, is accumulated.

More than 50 years have passed since the above dispute arose between this great physicists. A question arises, of whether the contemporary quantum field theory is already on that stage, when it can be formulated axiomatically? For example, the preface to the book A.V. Smilga (Smilga, 2001) is devoted to this question:

"*In his well-known popular lectures R. Feynman reflects on the way physical theories are built up and distinguishes two such ways or, rather, two stages in the process of their construction:* 1) *the "Babylonian" stage and* 2) *the "Greek" stage….Feynman writes that a modern physicist is a Babylonian rather than a Greek in this respect: he does not care too much about Rigor, and his God and ultimate Judge is Experiment. Strictly speaking, this is not quite correct. Some branches of classical and also of quantum physics have now quite reached the Greek stage*".

Further we will examine the structure of the contemporary theory of elementary particles - Standard Model - and will note its difficulties. Then we will describe briefly the structure of the axiomatic theory of elementary particles: the nonlinear quantum field theory (NQFT) (Kyriakos, 2009). In conclusion, we will compare the structures of both theories.

1. The Structure of the Contemporary Quantum Theory of Elementary Particles (the Characteristic Properties)

(1) Wave function as the basis of the description of elementary particles

Contemporary theory proceeds from the fact that elementary particles and their interactions are described by wave functions. In other words wave functions are the basic mathematical object of any field theory. From this follows that all properties of particles must be assigned by the properties of wave function. Therefore the purpose of contemporary theory is the search for differential equations, solution of which is the wave function of the given particle, which describes it either in the free state or in state of interaction with other particles.

(2) Lagrange structure of the theory

In the Standard Model the Lagrange approach is used in order to find of the equations of field motion. This means that the equations of motion of free particles and their interactions are derived in the form of the Euler-Lagrange equation from certain postulated Lagrangian.

Each Lagrangian consists of the sum of terms. A part of these terms describes free particles, and other terms describe interaction between these particles. The appropriate equation of motion is obtained by the Lagrangian variation. Since a Lagrangian variation with respect to any function does not affect other functions, a Lagrangian can have many terms, which describe different particles and their interactions.

Until the appearance of the quantum field theory the Lagrangians were selected on the basis of some experimental facts and hypotheses. At present the Lagrangians are selected on the basis of the procedures, in which the symmetries related to different kinds of so-called gauge transformations play important role. These SM Lagrangians give the equations, which describe motion and interaction of all fundamental elementary particles with the high accuracy. Therefore SM is considered as the very successful theory.

(3) Gauge theories, symmetries and transformations

In physics, **gauge theory** is a field theory in which the Lagrangian is invariant under a certain continuous group of transformations, named **gauge invariance**. The Standard Model is a non-abelian gauge theory. The first example of these theories is the Yang–Mills theory (Yang and Mills, 1954). The Standard Model unifies the description of electromagnetism, weak interactions and strong interactions in the language of gauge theory.

Gauge invariance is a form of symmetry. Local phase transformations, which depend on any form of charge (not necessarily electric) are for historical reasons called **gauge transformations**, and the invariance of field physics under them is called **gauge symmetry** or gauge invariance. Particles generally have "internal" symmetries (a form of spin), the "rotations" of which form a **gauge group** of transformations. Associated with this gauge group is a **gauge field** or fields. Associated with the gauge field, or fields, is a covariant derivative, D^{μ}, equal to the ordinary derivative, ∂^{μ}, plus a multiple of the field, or of a "summary" of the fields, respectively.

This is basically a consequence of Noether's theorem, which states that every symmetry has an associated conserved current**.** The local (non-global) nature of gauge invariance is of fundamental importance. It was the essential step in the creation of the electroweak theory. The electroweak theory is complicated by the fact that it treats left-hand and right-handed helicities differently, by recognising an additional charge, the hypercharge. (Yang and Mills, 1954): "… In the present paper we wish to explore the possibility of requiring all interactions to be invariant under independent rotations of the isotopic spin at all space-time points …" (for details see, for example, the book L.H. Ryder (Ryder, 1985))

2. A Specific Example of the Construction of Lagrangian of Interaction on the Basis of the Gauge (Phase) Transformations

Based on the following example, taken from the book of Ian J. R. Aitchison (Aitchison, 2007) we will examine all advantages and disadvantages of this method of theory construction in the particular case of the construction of the theory of interaction of photon with the electron (for details see (Aitchison, 2007)):

"3. *Gauge fields and gauge principle*

In this chapter we will show how the requirement of invariance of the matter Lagrangian under local phase transformations leads to the introduction of vector fields (gauge fields) interacting with the matter fields in a definite way.

In the non-Abelian case, the self-interactions of the vector fields are also prescribed.

The inclusion of electromagnetism: gauge-invariance

The conventional way of introducing the electromagnetic interaction in quantum field theory is via the so-called "minimal prescription", whereby the momentum operator $\hat{\vec{p}}^{\mu}$ *is replaced by* $\hat{\vec{p}}^{\mu} - q\vec{A}^{\mu}$*, for a* *particle of charge* q*, where* $\vec{A}^{\mu}$ *is the 4-vector potential. The corresponding classical Hamiltonian then reproduces, via Hamilton's equations, the correct Lorentz force; in quantum mechanics, since* $\hat{\vec{p}}^{\mu}$ *is replaced by* $i\partial^{\mu}$*, the prescription is*

$$\partial^{\mu} \rightarrow \partial^{\mu} + iq\vec{A}^{\mu}, \qquad (2.1.1)$$

The combination $\partial^{\mu} + iq\vec{A}^{\mu}$ *is of fundamental importance, and is called the "covariant derivative", denoted by* D^{μ}*:*

$$D^{\mu} \equiv \partial^{\mu} + iq\vec{A}^{\mu}, \qquad (2.1.2)$$

The significance of D^{μ} *will emerge as we proceed.*

The rule $\partial^{\mu} \rightarrow D^{\mu}$ *may be taken over to quantum field theory. For example, the Lagrangian for a free Dirac particle of mass* m *is*

$$L_0 = \psi^{+}(i\partial - m)\psi, \qquad (2.1.3)$$

which becomes

$$L_1 = L_0 + L_{\text{int}} = \psi^{+}(i\partial - m)\psi - q\psi^{+}\hat{\alpha}_{\mu}\psi A_{\mu}, \qquad (2.1.4)$$

after the replacement (2.1.1), if the field ψ *corresponds to particles of charge* q*.*

To obtain the complete Lagrangian, we must of course add to (2.1.4) *a part which yields the Maxwell equations for the potentials* $\vec{A}^{\mu}$*; we will defer more detailed consideration of this until the following chapter, since our main concern in this chapter is with symmetries, merely noting here that the standard classical Lagrangian for the electromagnetic field would be*

$$L_{em} = -\frac{1}{2}F_{\mu\nu}F^{\mu\nu}, \qquad (2.1.5)$$

Global and local phase invariance: the gauge principle

We now turn the preceding discussion upside down and introduce the idea, which is crucial to the development of gauge theories.

As we have seen in the previous chapter, global phase invariances are common in particle physics, being associated with various symmetries, and hence with conservation laws via Noether's theorem. The possibility of changing the phase of the SU(2) *transformations concerned in Section* 2.3*, a geometrical interpretation of them is often made – indeed, we used it ourselves in talking about "rotations" in the space of the fields. To be definite, let us now call this the SU*(2) *of isospin. Invariance under global SU*(2) *transformations of the type of putting the physical consequence of such invariance is that we may choose the "axes in isospace" as we please. In other words, the definition of the fields to be associated with the proton, say, (*I_3=1/2*) and with the neutron (*I_3=-1/2*) is, in the limit of exact SU*(2)*-invariance, entirely conventional up to a unitary transformation.*

However, a global symmetry of this type implies that once we have decided what the convention ought to be at one space-time points, because the transformation is not allowed to vary from point to point. Yang and Mills (Yang and Mills, 1954) *were the first to question whether this was entirely reasonable. "It seems", they wrote, "that this is nor consistent with the localized field concept that underlies the usual physical theories. In the present paper we wish to explore the possibility of requiring all interactions to be invariant under independent rotations of the isotopic spin at all space-time points." Thus was the study of the remarkable non-Abelian gauge field theories (see Section* 3.3*) initiated.*

The proposal of Yang and Mills amounts to the requirement that the theory be invariant under local phase transformations. Let us see how this would operate in the very simple case of a U(1) *transformation involving a single phase parameter. We start with the Lagrangian*

$$L_0 = \psi^{+}(i\partial - m)\psi, \qquad (2.1.20)$$

say, which is certainly invariant under the global phase transformation

$$\psi(x) \rightarrow \psi'(x) = \exp(-i\alpha)\psi(x), \qquad (2.1.21)$$

Let us "explore the possibility" of invariance under the local phase transformation

$$\psi(x) \to \psi'(x) = \exp(-i\alpha(x))\psi(x), \quad (2.1.22)$$

Further we are going to demand that a full Lagrangian L_1 *exists which is actually invariant under* (2.1.22)*. Clearly,* L_0 *changes by*

$$\delta L_0 = \psi^+ \hat{\alpha}_\mu \psi \partial_\mu \alpha(x), \quad (2.1.23)$$

If we now require that $\delta L_1 = 0$*, then* L_1 *must contain a term in addition to* L_0*, whose change under* (2.1.22) *exactly cancels* $\delta L_0 = 0$*. ch an* L_1 *is, of course,*

$$L_1 = L_0 - q\psi^+ \hat{\alpha}_\mu \psi A_\mu, \quad (2.1.24)$$

provided that, when ψ *undergoes (2.1.22),* A_μ *changes by*

$$A_\mu(x) \to A'_\mu(x) = A_\mu(x) + \frac{1}{q}\partial_\mu \alpha(x), \quad (2.1.25)$$

Writing $\alpha(x)$ *as* $q\chi(x)$ *we recover precisely* (2.1.10) *and* (2.1.11)*, and the original "minimal prescription" Lagrangian. The introduction of a vector field, transforming according to* (2.1.25)*, would seem to be necessary if we are to have local freedom as to phase convention. Furthermore, the vector field has to be one such that transformations of the form* (2.1.25) *can be performed on it without altering the physical results (since* (2.1.25) *is part of the local invariance of* L_1*); such vector fields are called gauge fields. We may say that, at least in this simple case, the requirement of local phase invariance has "generated" the interaction term* $-q\psi^+ \hat{\alpha}_\mu \psi A_\mu$ *between the matter field* ψ *and the gauge field* A_μ*. This is the essence of the gauge principle for generation dynamical theories: suitable gauge fields are introduced, with interactions such that the required local invariance holds.*

We must note at once, however, that if the (gauge) transformation (2.1.25*) on* A_μ *is indeed to be an invariance of the Lagrangian – when combined with* (2.1.22) *– we can not, apparently, allow the vector field* A_μ *to have a mass. This would enter* L *in the form* $\frac{1}{2}m^2A^2$ *(see Chapter* 5*), which is quite clearly not going to be invariant under* (2.1.25)*. Hence, it would appear that this remarkable idea for generating interactions is restricted to massless vector fields only.*

The original paper of Yang and Mills (1954) *generalized invariance under the local one-parameter type of phase transformation* (2.1.22) *to invariance under local isospin-type transformations – that is, under transformations of the type*

$$\psi(x) \to \psi'(x) = \exp(-ig\theta_\alpha(x)t^\alpha)\psi(x), \quad (2.1.26)$$

where the matrices $\hat{t}^\alpha$ *(*$\alpha = 1,2,3$*) represent the algebra of SU*(2) *in the representation appropriate to the multiplet* ψ *. Such invariance will now demand the introduction of three gauge fields* $A^\alpha_\mu(x)$*, which will transform in some way analogous to* (2.1.25)*, when* ψ *undergoes the local phase transformation* (2.1.26)*. The actual form is somewhat more complicated that* (2.1.25)*, and will be given in Section* 3.3*, where we shall also discuss how the local invariance determines the interactions in this case.*

Here too, however, it would seem that the vector fields $A^\alpha_\mu(x)$ *have to be massless…*

The weak interactions of leptons continued to appear to be fundamental, but hopes of describing them in terms of gauge field theory seemed, until 1964*, doomed to disappointment for the same reason as in hadron physics: the mediating vector quanta had to be massive, as we saw in Section*1.1*. The breakthrough came with the realization that the gauge principle could still be at wok, but the associated symmetry could be "hidden" (or "spontaneously broken") – i.e. not unitarily implemented*".

3. Deficiencies in the Structure of the Standard Model Theory

1) It is not difficult to see that at present the theory of Standard Model has namely algorithmic ("Babylonian") structure. It is a collection of rules and procedures, intended for obtaining the correct answer. These procedures are not axiomatics, which by united means could describe all elementary particles. Could it be for this reason that (Gell-Mann, 1981): "*Quantum mechanics … is mysterious, confusing discipline, which none of us really understand*".

2) It is not difficult to understand that in the construction of theory the Lagrange approach is not primary. Moreover, at present the physical sense of Lagrangian is unknown. The differential equations serve as primary expressions for obtaining practical solutions. But with respect to Lagrangian, as the base of the theory, the equations are secondary stage. Thus, obviously, the use of equations of motion as the basis of theory makes the theory simpler.

3) The wave function of photon is introduced in SM in the form of EM potential. At the same time, in EM theory the primary physical characteristic of field is the strength of EM field, and potential is introduced as a mathematical value, determined ambiguously. Obviously, it would be more consecutive to introduce the wave function of the photon directly in the form of the strength of EM field, as this was done in the initial period of development of quantum electrodynamics (QED) (Akhiezer and Berestetskiy, 1965) (let us note that QED can be built on this base without any difficulties).

4) In SM free fundamental fermions (leptons and quarks) are considered as the primary particles. Photon in SM does not appear as independent fundamental field, but as a certain gauge (compensating) field. Moreover this field is postulated here on the basis of the knowledge, borrowed from classical EM theory. At the same time the SM is the Yang–Mills theory, which is the nonlinear generalization of electromagnetic theory. Thus the simplest and consequently fundamental particle in SM is the photon. Obviously, there are no obstacles in considering the photon as fundamental particle within the framework of another theory.

5) The use of potential as the wave function of the photon (and, in fact, of all other particles; see (Aitchison, 2007)) leads to a serious difficulty. The value $m^2 A_\mu A_\mu$ is not a gauge invariant, and thus it is impossible to consider that this term contains mass. This leads to the need to add the mechanism of symmetry braking (which requires the presence of additional particles, which in nature, at least up to today, were not observed) for the generation of the mass of particles. If the strength of EM field is selected as the wave function of photon, a similar term will appear in the form $m\psi^+\psi$. In this form it is invariant in relation to all necessary transformations, and it means, that value m can be interpreted there as mass.

6) The interaction is introduced in SM by means of the gauge transformations of wave functions. In this case the fit procedures, described in the above fragment, are required. Since these transformations are rotations, it is more logical to introduce the rotation transformation of wave functions by postulate. In this case the term of interaction would appear in the equations without the additional assumptions and procedures as a covariant derivative D^μ.

It is possible to assume that to these deficiencies are connected difficulties, which fundamental physics at present is going through and which notes in his last book "*The trouble with physics*" the well-known physicist Lee Smolin (Smolin, 2006):

"*The story I will tell could be read by some as a tragedy. To put it bluntly – and to give away the punch line – we have failed. We inherited a science, physics that had been progressing so fast for so ling that it was often taken as the model for how other kinds of science should be done. For more than two centuries, until the present period, our understanding of the laws of nature expanded rapidly. But today, despite our best effort, what we know for certain about these laws is no more than what we knew back in the* 1970*s.*

How unusual is it for three decades to pass without major progress in fundamental physics? Even if we look back more than two hundred years, to a time when science was the concern mostly of wealthy amateurs, it is unprecedented. Since at least the late eighteenth century, significant progress has been made on crucial questions every quarter century".

4. An Axiomatic Approach

Let us examine now the theory of elementary particles, built on the axiomatic basis, taking into account the above deficiencies in the structure of Standard Model. We will present here very brief results of this theory, referring to the details and proofs in the complete theory (Kyriakos, 2009) (the separate chapter references see in the text).

4.1 Axiomatic Basis of the Theory

The axiomatic basis of the proposed theory is composed by 5 postulates, from which the first 4 are the postulates of contemporary field theory. Postulate 5 expresses the specific nonlinearity of theory, but it does not contradict to the results of contemporary physics.

1) Postulate of fundamentality of the electromagnetic field: Maxwell's equation for the field without sources:

$$\frac{1}{c}\frac{\partial\ \vec{\mathrm{E}}}{\partial\ t} - rot\vec{\mathrm{H}} = 0\,, \tag{4.1.1}$$

$$\frac{1}{c}\frac{\partial\ \vec{\mathrm{H}}}{\partial\ t} + rot\vec{\mathrm{E}} = 0\,, \tag{4.1.2}$$

are fundamental independent equations of motion of fields (Definition: the motions of EM field are called EM waves.)

2) The postulate of the quantization of EM wave fields: electromagnetic wave fields consist of the elementary electromagnetic wave formations (particles) – photons, leptons etc.

3) Postulate of Planck and de Broglie: the relationship between the energy, frequency and wavelength of photon is determined by the following formulas:

$$\varepsilon_{ph} = h\nu = \hbar\omega\,, \tag{4.1.3}$$

$$\lambda = \frac{h}{p_{ph}} = \frac{hc}{\varepsilon}\,, \tag{4.1.4}$$

4) The postulate of superposition of wave fields:
in the general case electromagnetic waves are the superposition of elementary wave fields, the simplest of which are photons.

5) Postulate of the massive particles' generation: for generation of the massive particles the field of photon must undergo the rotation transformation.

Next, we will the theorems of the theory Let us use the above postulates for constructing of the theory of each type of elementary particles.

4.2 Equation of photon (For details see the Chapter 2. "The photon theory")

Using the postulates 1 and 3, we will obtain from Maxwell's equations the wave equation of the photon:

$$\left[\left(\hat{\alpha}_o\hat{\varepsilon}\ \right)^2 - c^2\left(\hat{\vec{\alpha}}\ \hat{\vec{p}}\right)^2\right]\ \Phi = 0\,, \tag{4.2.1}$$

where $\hat{\varepsilon} = i\hbar\,\partial/\partial t$, $\hat{\vec{p}} = -i\hbar\vec{\nabla}$ are the operators of energy and momentum; $\hat{\alpha}_0$; $\hat{\vec{\alpha}}$; $\hat{\beta} \equiv \hat{\alpha}_4$ are Dirac's matrices, while Φ is certain matrix; in this case:

$$\Phi = \begin{pmatrix} E_x \\ E_z \\ iH_x \\ iH_z \end{pmatrix}, \ \Phi^+ = \left(E_x \quad E_z \quad -iH_x \quad -iH_z\right), \tag{4.2.2}$$

The harmonic functions are the solution of this equation:

$$\begin{cases} \vec{E} = \vec{E}_o e^{-i\omega t + ky} + \vec{E}_o^* e^{i\omega t - ky} \\ \vec{H} = \vec{H}_o e^{-i\omega t + ky} + \vec{H}_o^* e^{i\omega t - ky} \end{cases}, \tag{4.2.3}$$

where energy and momentum are quantified according to postulate 3: $\omega = \varepsilon/\hbar$ and $k = p/\hbar$. Factorizing of (4.2.2), we will obtain the system:

$$\begin{cases} \Phi^+\left(\hat{\alpha}_o\hat{\varepsilon} - c\hat{\vec{\alpha}} \ \hat{\vec{p}}\right) = 0 \\ \left(\hat{\alpha}_o\hat{\varepsilon} + c\hat{\vec{\alpha}} \ \hat{\vec{p}}\right) \Phi = 0 \end{cases}, \tag{4.2.4}$$

These equations, taking into account the quantization of energy and momentum, are the known quantum equations of photon, equivalent to one equation (4.2.1).

The physical sense of these equations is revealed with the substitution of expressions (4.2.2). As a result we obtain Maxwell's equations for the advanced and retarded waves:

$$\begin{cases} \frac{1}{c}\frac{\partial E_x}{\partial t} - \frac{\partial H_z}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial H_z}{\partial t} - \frac{\partial E_x}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial E_z}{\partial t} + \frac{\partial H_x}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial H_x}{\partial t} + \frac{\partial E_z}{\partial y} = 0 \end{cases}, \quad (4.2.5') \qquad \begin{cases} \frac{1}{c}\frac{\partial E_x}{\partial t} + \frac{\partial H_z}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial H_z}{\partial t} + \frac{\partial E_x}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial E_z}{\partial t} - \frac{\partial H_x}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial H_x}{\partial t} - \frac{\partial E_z}{\partial y} = 0 \end{cases}, \quad (4.2.5'')$$

which confirms the EM nature of photon.

The quantum theory of photon shows (Akhiezer and Berestetskii, 1965) that a photon is not a point particle. According to the analysis of quantum equation of the photon L. Landau and R. Peierls (Landau and Peierls, 1930), and later R.J. Cook (Cook, 1982a; 1982b) and T. Inagaki (Inagaki, 1994) showed that the *photon wave function is nonlocal*. In other words, the photon wave function is not determined by the field at the some point, but depends on the field distribution in a certain region, *which has the size of the order of photon wavelength*. This means that the localization of a photon in a smaller region is impossible and, therefore, the concept of a probability density distribution that could be used to find the photon at a fixed point of space does not make sense. Further let us show, how the mass of elementary particles is generated.

4.3 Equation of intermediate boson (massive photon) (For details see the Chapter 3. “The intermediate photon theory and particles’ generation”).

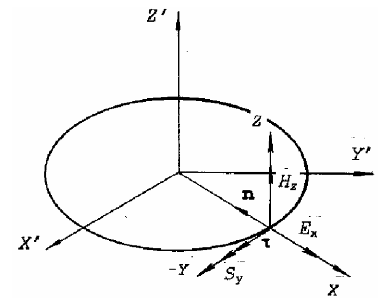
In the framework of NQFT particles acquire mass through an intermediate massive boson. The last is generated with the rotation transformation of EM field. We will use the postulate 5 and produce the rotation transformation $\hat{R}$ of photon fields Φ:

$$\hat{R}\Phi \rightarrow \Psi, \tag{4.3.1}$$

where Ψ is the new wave function, which appears after the transformation of the rotation:

$$\Psi = \begin{pmatrix} \mathrm{E}'_x \\ \mathrm{E}'_z \\ i\mathrm{H}'_x \\ i\mathrm{H}'_z \end{pmatrix} = \begin{pmatrix} \Psi_1 \\ \Psi_2 \\ \Psi_3 \\ \Psi_4 \end{pmatrix}, \qquad (4.3.2)$$

where $(\mathrm{E}'_x \quad \mathrm{E}'_z \quad -i\mathrm{H}'_x \quad -i\mathrm{H}'_z)$ are the vectors of the electromagnetic field, which appear after the rotation transformation and are the wave functions of the new particle within the framework of quantum theory.



Let us examine the EM wave, which moves along the circular path, so that vectors $\vec{E}, \vec{H}$ and Poynting's vector $\vec{S}$ move as shown in the figure:

Displacement current in equations (4.2.5) is determined by the expression:

$$j_{dis} = \frac{1}{4\pi}\frac{\partial \vec{\mathrm{E}}}{\partial t}, \qquad (4.3.3)$$

The electric field vector of the expression (4.3.3), during the motion along the curvilinear trajectory, can be recorded in the form:

$$\vec{\mathrm{E}} = -\mathrm{E}\cdot\vec{n}, \qquad (4.3.4)$$

where $\mathrm{E} = \left|\vec{\mathrm{E}}\right|$, and $\vec{n}$ is the unit vector of the normal to the curve. After differentiation the displacement current of the plane wave, which moves along the ring, can be recorded in the form:

$$\vec{j}_{dis} = -\frac{1}{4\pi}\frac{\partial \mathrm{E}}{\partial t}\vec{n} + \frac{1}{4\pi}\omega_p \mathrm{E}\cdot\vec{\tau}\,, \qquad (4.3.5)$$

where $\omega_p = \frac{\varepsilon_p}{\hbar} = \frac{m_p c^2}{\hbar} \equiv c\mathrm{K}$, $m_p = \varepsilon_p / c^2$ is a mass, which corresponds to photon energy ε_p; $\vec{j}_n = \frac{1}{4\pi}\frac{\partial \mathrm{E}}{\partial t}\vec{n}$ and $\vec{j}_\tau = \frac{\omega_p}{4\pi}\mathrm{E}\cdot\vec{\tau}$ are the normal and tangential components of displacement current of “nonlinear” EM waves, respectively.

A more general expression can be obtained, describing rotation in the curvilinear geometry of Riemann (Kyriakos, 2009). In this case it occurs that the currents are determined by the connections of field, i.e., by the symbols of Ricci (or, in the most general case, by Christoffel symbols; see also the paper http://redshift.vif.com/JournalFiles/V11NO3PDF/V11N3KYR.pdf).

The physical sense of the generation of mass consists of the following. At the moment of rotation transformation, a self-interaction of own fields occurs in the photon (mass-free boson). Due to this fact the photon fields revolve in the small region of space. In this case its energy does not move from infinity to infinity with the speed of light, but it is locked in a small space region. This concentration of photon energy is a massive particle, one of characteristics of which is the value ε_p / c^2 . This particle is structureless; it does not have the specific size, but it has some characteristics of the energy distribution, which define its existence as separate object. According to the relationship of Einstein the value $m_p = \varepsilon_p / c^2$ is the mass of a particle.

It is understandable that for the generation of masses of particles in NQFT the use of Higgs's mechanism is not required. This frees theory from many known problems and makes theory considerably simpler. It is remarkable that in NQFT the mass does not appear as primary characteristic, but as the ratio of energy to the square of the speed of light. Its property - to be coefficient in the mechanical momentum, which determines the inertia of particle - can be shown by the Ehrenfest theorem.

Because of the rotation, this mass assigns an angular momentum of particle, i.e. spin (in this case, equal to 1). Simultaneously the tangential current appears, which with volume (occupied with particle) integration, determines the particle charge. Since in this case the current is sinusoidal, electrical charge of "massive photon" is equal to zero.

As a result of the transformation of rotation we will obtain the equation of intermediate boson (massive photon):

$$\left(\hat{\alpha}_o\hat{\varepsilon} - c\hat{\vec{\alpha}}\cdot\hat{\vec{p}} - K\right)\left(\hat{\alpha}_o\hat{\varepsilon} + c\hat{\vec{\alpha}}\cdot\hat{\vec{p}} + K\right)\Psi = 0\,, \qquad (4.3.6)$$

Or, taking into account the value K (see above), we will obtain this equation in form:

$$\left(\hat{\varepsilon}^2 - c^2\hat{\vec{p}}^2 - m_p^2c^4\right)\Psi = 0, \qquad (4.3.7)$$

4.4 Equation of leptons (For details see the Chapter 4. "An electron theory (linear approach)")

Now let us produce, conditionally speaking, the breaking of the intermediate boson symmetry (which in nature occurs spontaneously due to electromagnetic repulsion of the parts of the massive boson with each other). In the case of the plane-polarized initial photon this gives the possibility to obtain two oppositely charged particles with half-integral spin of the type of electron and positron.

Multiplying equation (4.3.7) to the left on Ψ^+ and making factorizing, we will obtain the equations of two particles, which are located in the field of each other:

$$\left[\left(\hat{\alpha}_o\hat{\varepsilon} + c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right) + \hat{\beta}\ m_pc^2\right]\psi = 0\,, \qquad (4.4.1')$$

$$\psi^+\left[\left(\hat{\alpha}_o\hat{\varepsilon} - c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right) - \hat{\beta}\ m_pc^2\right] = 0\,, \qquad (4.4.1'')$$

Here $\psi = \begin{pmatrix} E_x \\ E_z \\ H_x \\ H_z \end{pmatrix} \equiv \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}$ is lepton wave function, which corresponds to electromagnetic field after the breakdown of intermediate boson (this ψ-function is not the vector, but a so-called semi-vector, i.e. spinor).

In the simplest case of the production electron-positron pair $m_p = 2m_e$, and from (4.1) we have:

$$\left[\left(\hat{\alpha}_o\hat{\varepsilon} + c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right) + 2\hat{\beta}\ m_ec^2\right]\psi = 0\,, \qquad (4.4.2')$$

$$\psi^+\left[\left(\hat{\alpha}_o\hat{\varepsilon} - c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right) - 2\hat{\beta}\ m_ec^2\right] = 0\,, \qquad (4.4.2'')$$

It is obvious that in order to become free, the electron and positron must spend energy. It is not difficult to calculate, that during their removing from each other an amount of energy must be spent, equal to the amount, which is necessary for the formation of particle themselves. The external field of particles arises due to this process. Using a linear writing of the energy-momentum conservation law, we will obtain for the external field of the particle:

$$\hat{\beta}\ m_ec^2 = -\varepsilon_{ex} - c\hat{\vec{\alpha}}\ \vec{p}_{ex} = -e\varphi_{ex} - e\hat{\vec{\alpha}}\ \vec{A}_{ex}\,, \qquad (4.4.3)$$

where "ex" indicates "external"; then, substituting (4.4.3) in (4.4.2), we obtain Dirac's equation with the external field:

$$\left[\hat{\alpha}_0\left(\hat{\varepsilon} \mp \varepsilon_{ex}\right) + c\hat{\vec{\alpha}}\cdot\left(\hat{\vec{p}} \mp \vec{p}_{ex}\right) + \hat{\beta}\ m_ec^2\right]\psi = 0\,, \qquad (4.4.4)$$

At a sufficiently great distance between the particles, when these fields are not important, we obtain Dirac's equations for the free particles:

$$\left[\left(\hat{\alpha}_o\hat{\varepsilon} + c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right) + \hat{\beta}\ m_ec^2\right]\psi = 0\,, \qquad (4.4.5')$$

$$\psi^{+}\left[\left(\hat{\alpha}_{o}\hat{\varepsilon}-c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right)-\hat{\beta}\ m_{e}c^{2}\right]=0, \qquad (4.4.5'')$$

4.5 Equation of the massive neutrino (For details see the Chapter 8. "The theory of massive neutrino")

In the book (Kyriakos, 2009) it is shown that from the circularly polarized photon field, neutrino is formed with all its known properties. It is noticeable that in this case the helicities of neutrino and antineutrino are mutually opposite and no transformation can change this property. In other words, the neutrino has always the left spirality, and antineutrino – right spirality (note that in SM this property is not explained and is accepted as a postulate).

4.6. Equation of the hadrons (For details see the Chapter 9. "About hadrons theory")

The formation of different hadrons is also connected with the described characteristics of leptons. According to the fourth postulate, wave fields can form superpositions. It is possible to show that with the superposition of elementary fields, which are equivalent to leptons, different hadrons can be formed, which are described by Yang–Mills equation. Moreover from two lepton-like fields mesons can be formed, and with the superposition of three lepton-like fields - baryons.

Conclusion: Basic Differences of NQFT from the Standard Model

1) As we see, NQFT is an axiomatic theory, not a collection of rules and procedures. All equations and consequences of theory are derived from a limited number of statements, accepted as the axioms. We do not consider the question, if this axiom system is complete. Moreover, judging by the formulation of other axiomatic theories, it is possible to use another system of axioms. We considered the simplest of them, which was necessary and sufficient for describing of all fundamental elementary particles.

2) At the basis of NQFT lies the free electromagnetic field equation, but not a Lagrangian. The equations of motion of fields of elementary particles and their interactions are the consequences of this axiom along with some others. This makes it possible to avoid the complicated mathematical apparatus, connected to the use of a Lagrange approach.

3) At the basis of NQFT lies only one field: the fundamental electromagnetic field. This contradicts neither SM nor everything that we know about the microcosm. All the remaining fields (except gravitational) appear from this fundamental field on the basis of postulates. Thus, the nonlinear theory is the united electromagnetic theory of the elementary particles (unification is not considered a gravitational field, since the equation of GRT, according to Einstein, has geometric origin).

4) Into framework of NQFT the wave functions of particles are not potentials, as in SM, but the strength of the electromagnetic field of different configuration. The potentials are used in NQFT as auxiliary values: as energy and momentum per charge unit of particles.

5) Nevertheless, in framework of NQFT it is shown that all interpretations, accepted in SM, are completely legitimate and reflect the formal mathematical side of peculiarities of elementary particles. Thus, NQFT does not contradict SM, only generalizes and refines its results.

6) NQFT does not require the use of Higgs's mechanism for mass generation. Nevertheless, NQFT works equally well, both in the region of low and high energies, since the NQFT does not violate the unitarity in the region of high energies, as it takes place in SM if we do not use Higgs's mechanism.

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