Exploration

General Relativity as Multifractal Analogue of the Standard Model

Ervin Goldfain*

Advanced Technology and Sensor Group, Welch Allyn Inc., Skaneateles Falls, NY 13153

Abstract

We have recently shown that the Standard Model of particle physics (SM) may be configured as a *multifractal set*, with all field components acting as primary generators of this set. The goal of this brief note is to point out that a multifractal description of General Relativity (GR) is also possible, starting from the definition of the metric tensor and relativistic interval. This finding paves the way to an unforeseen homeomorphism between the structure of SM and classical gravity.

Keywords: Standard Model, general relativity, multifractal set, generalized dimension.

1. Multifractals: a concise overview

As it is known, the *box-counting dimension* defines the main scaling property of fractal structures and is a measure of their self-similarity. Multifractals are global mixtures of fractal structures, each characterized by its local box-counting dimension. Self-similarity of multifractals is accordingly defined in terms of a *multifractal spectrum* describing the overall distribution of dimensions. In the language of chaos and complexity theory, multifractal analysis is the study of *invariant sets* and is a powerful tool for the characterization of generic *dynamical systems.*

In the recursive construction of multifractal sets from $i = 1, 2..., N$ local scales r_i with probabilities p_i, the definition of the box-counting dimension leads to [4]

$$\sum_{i=1}^{N} p_i^q r_i^{\tau(q)} = 1 \tag{1a}$$

in which

$$\sum_{i=1}^{N} p_i = 1 \tag{1b}$$

Here, q and $\tau(q)$ are two arbitrary exponents and the latter is typically presented as

$$\tau(q) = (1-q)D_q \tag{2}$$

where D_q plays the role of a *generalized dimension*.

*Correspondence: Ervin Goldfain, Ph.D., Photonics CoE, Welch Allyn Inc., Skaneateles Falls, NY 13153, USA
E-mail: ervingoldfain@gmail.com

2. GR as topological analogue of SM

Consider now the field makeup of the SM, formed by 16 *independent* “flavors”: two massive gauge bosons (W, Z), gluon (g), the Higgs scalar (H), neutrinos, charged leptons and quarks. The SM structure can be conveniently organized in the 4×4 matrix

$$SM = \begin{pmatrix} g & \nu_e & \nu_\mu & \nu_\tau \\ W & e & \mu & \tau \\ Z & u & c & b \\ H & d & s & t \end{pmatrix} \tag{3}$$

The photon (γ) is absent from (3) as it is built from the underlying components of the electroweak sector, whereby $\gamma = \gamma(W_\mu^3, B_\mu)$ and $B_\mu = B_\mu(W_\mu^3, Z)$ [1].

It was shown in [2-3] that, near the electroweak scale M_{EW}, the spectrum of particle masses m_i entering the SM satisfies the “closure” relation

$$\sum_{i=1}^{16} (\frac{m_i}{M_{EW}})^2 = 1 \tag{4}$$

It is apparent that (3) shares the same formal structure with the metric tensor of GR, that is,

$$GR = \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix} \tag{5}$$

where there are only 10 independent entries under the standard assumption $g_{\mu\nu} = g_{\nu\mu}$. Starting from the GR definitions of interval and proper time leads to ($c=1$)

$$\sum_{\mu=0}^{3}\sum_{\nu=0}^{3} g_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = 1 \tag{6}$$

subject to the constraint

$$\sum_{\nu=0}^{3} g^{\mu\nu}g_{\nu\rho} = \delta_\rho^\mu = \begin{cases} 1, & \mu = \rho \\ 0, & \mu \neq \rho \end{cases} \tag{7}$$

Comparing (1), (4) and (6) reveals the following mapping

$$GR: (\, p_i \Rightarrow g^{\mu\nu}g_{\nu\rho},\ q = 1/2\ ,\ D_q = 4,\ \tau(q) = 2\,)$$

$$SM: (\, p_i \Rightarrow 1,\ q = 0,\ D_q = \tau(q) = 2\,) \tag{8}$$

It is interesting to note that $D_0 = 2$ coincides with the fractal dimension of quantum mechanical paths in free space [5-6], while $D_{1/2} = 4$ recovers the four-dimensionality of geodesic paths in classical spacetime.
A couple of conclusions may be drawn from (8), namely:

- GR may be viewed as *topological analogue* of the SM, defined by a half-unitary exponent q and a generalized dimension that is twice its SM counterpart.
- The spectrum of particle *mass scales* (m_i / M_{EW}) and the *four-velocity* ($dx^\mu / d\tau$) form the basis for the multifractal description of SM and GR, respectively.

A follow-up exploration on the multifractal underpinnings of GR is currently under construction [7].

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