Lorentz-Invariant Gravitation Theory

Chapter 9. The equivalence principle and metric tensor of LIGT

1.0. Equivalence of inertial and gravitational masses and its consequences

Interpretation of the equivalence of inertial and gravitational masses by Einstein led him to assertion that the theory of gravity can not be a Lorentz-invariant theory, but it should be a general relativistic theory in a Riemannian – non-flat - space-time.

At the same time a characteristic feature of the Lorentz-invariant theory is a flat space-time. Can we solve this contradiction between our approach and the approach of general relativity?

1.1. Is GTR an L-invariant theory?

It is known that general relativity is considered a relativistic theory, but it is not a L-invariant theory (see "Chapter 1", or (Katanaev, 2013, pp. 777-778))

The general relativity principle, according to Einstein's hypothesis, should be a generalization of the Lorentz-invariance of the special relativity theory. As such principle, Einstein proclaimed the requirement of general covariance. As is known, most physicists - see, e.g., Hilbert, Fock, Logunov (Polak, 1959; Fock, 1964; Logunov, 2002) - do not consider the general covariance to be equivalent with some type of relativity, which generalizes the Lorentz-invariance. This follows from the fact that any Lorentz-invariant theory can always be written in covariant form.

Thus, the absence of such a generalization makes the Lorentz- invariance a basic requirement for any relativistic theory. The real space of such theories is Euclidian (or, conditionally, taking into account time, it is pseudo-Euclidian). Obviously, this is also valid for the gravitation theory. Hence, the Riemannian space is not a real space, but a mathematical model. Indeed, the assertion that the real space is Riemannian is not supported by theory or experiment.

As Jacobi have theoretically shown (see about Jacobi's geometrization of Newton's theory of gravitation in chapter 8 or in (Polak, 1959; Encyclopaedia of matematics, 2011), any conservative potential fields of Newtonian mechanics can be written in the form of Riemann geometry. Thus, in this case the Riemannian space is not a geometric space, but, rather, a physical field in a geometric shape.

As is known, there are numerous representations of the physical characteristics in the geometric form, and they are often very useful and productive. For example, the use of the n-dimensional configuration space provides a new mathematical apparatus. In the modern theory space of different measurements is widely used (including fractional measurements). But hardly anyone would agree that these spaces exist in nature.

Attempts to prove experimentally, that the space of the universe is Riemannian space, gave negative results. Recent measurements (Plank collaboration, 2013a, b) show that the space of the universe is flat with an accuracy to one tenth of a percent. This means that in fact the space-time of universe is Euclidean, or if we operate with 4-forms, pseudo-Euclidean. Thus, we must recognize that in terms of a physical theory the space is always Euclidean space, but physical problems can be formulated mathematically in terms of geometric forms, including the Riemannian space.

Let us analyze the possibility of describing the gravitational interaction without the involvement of a Riemannian space.

1.2. Einstein interpretation of the equivalence of inertial and gravitational masses in building a theory of gravitation

Let us see first of all how in modern physics is described the transition of Einstein from Euclidean space to Riemann, the starting point of which was the principle of equivalence. Is it possible to give a different interpretation of the equivalence of gravitational and inertial mass?

In an inertial reference system, the free motion of all bodies is uniform and rectilinear, and if, say, at the initial time their velocities are the same, they will be the same for all times. Clearly, therefore, if we consider this motion in a given non-inertial system, then relative to this system all the bodies will move in the same way.

The fundamental property of gravitational fields that all bodies, independently of mass, move in them in the same way, remains valid also in relativistic mechanics. The properties of the motion in a non-inertial system are the same as those in an inertial system in the presence of a gravitational field. In other words, a non-inertial reference system is equivalent to a certain gravitational field. This is called the principle of equivalence.

A somewhat more general case is a non-uniformly accelerated motion of the reference system: it is clearly equivalent to a uniform but variable gravitational field (Landau and Lifshitz, 1951).

"*Consequently, their acceleration depends only on the point in space where they happen to be. Can we, therefore, attribute the gravitational characteristics (acceleration) to the points in space, where the bodies are, rather than to the bodies themselves? However, Minkowski's flat space-time doesn't have the properties needed to implement this idea: it is homogeneous, that is, everywhere uniform and isotropic (the same in all directions). This means that the components of the metric (the metric tensor) $\overline{g}_{\alpha\beta}$ are constant '(their individual moduli are either zero or unity). Consequently, we need space-time whose metric tensor has components $g_{\alpha\beta}(x)$ that change from point to point, i.e. the space-time should be curved This enables us to consider geometrical properties of space-time that change at different points*" (Vladimirov et al, 1987, pp. 40-41)

«*Universal gravitation does not fit into the framework of uniform Galilean space. The deepest reason for this fact was given by Einstein. It is that not only the inertial mass, but also the gravitational mass of a body depends on its energy.*

It proved possible to base a theory of universal gravitation on the idea of abandoning the uniformity of space as a wholef and attributing to space only a certain kind of uniformity in the infinitesimal. Mathematically, this meant abandoning Euclidean, or rather pseudo-Euclidean, geometry in favour of the geometry of Biemann » (Fock, 1964) .

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Why is the dependence of the gravitational field on the coordinates and time in Newton's theory $f = f(x,t)$, not suitable to describe the gravity, and the dependence $g_{\alpha\beta} = g_{\alpha\beta}(x,t)$ in general relativity is the only possibility? The use of the heterogeneity of space-time is not tenable. Firstly, it is, in fact, Einstein's postulate who has identified this heterogeneity with Riemannian space-time. Secondly, in a heterogeneous space the conservation laws are not valid, since they are only valid in a homogeneous and isotropic space.

In other words, the authors try to justify a posteriori the introduction of the Riemann geometry by Einstein. This means that we have the right to try to give a different interpretation to the fact of the equivalence of masses.

But above all, it is not clear how is Einstein's equivalence principle related to the Riemann space.

Let us begin with the formulation of the principle of equivalence which Einstein gave himself:

"*A little reflection will show that **the law of the equality of the inertial and gravitational mass is equivalent to the assertion that the acceleration imparted to a body by a gravitational field is independent of the nature of the body**. For Newton's equation of motion in a gravitational field, written out in full, it is:*

(Inertial mass) .(Acceleration) =(Intensity of the gravitational field) .(Gravitational mass).

It is only when there is numerical equality between the inertial and gravitational mass that the acceleration is independent of the nature of the body" (Einstein, 2005).

Let us consider the mathematical basis of the principle of Einstein's equivalence and try to give this mathematics another form.

As we can see, Einstein relied on the Newtonian law of motion of a particle with inertial mass m_{in} in a gravitational field of source with a mass M :

$$m_{in} \frac{d\vec{\upsilon}}{dt} = \gamma_N \frac{m_{gr} M}{r^2} \vec{r}^{\,0} \,, \tag{1.1}$$

where m_{gr} is gravitational mass. Since $m_{in} = m_{gr} = m$, then dividing (1.1) by m we obtain in the case of gravitation the movement equation of the form:

$$\frac{d\vec{\upsilon}}{dt} = \gamma_N \frac{M}{r^2} \vec{r}^{\,0} \,, \tag{1.1'}$$

where acceleration is on the left and the Newton force per unit mass is on the right.

It is easy to see that this equation is the mathematical expression of Einstein's abovementioned principle of equivalence: the power (ie, action) of Newton's gravity exerted on the unit mass (i.e., local point mass), coincides with the acceleration of the moving body in this field (i.e., with the force of inertia acting per unit mass).

Our second question was whether it is possible to give another explanation to this principle.

1.3. Interpretation of the equivalence of inertial and gravitational masses in framework of LIGT

In the GTR we assume that in the (pseudo-) Euclidean reference frame, "*the noninertial frames possessed spatial and temporal inhomogeneities that show up as inertial forces, that depend on the specific characteristics of the reference frame. Obviously, the inertial forces have to have a noticeable effect on the physical processes in these reference frames*" (Vladimirov et al, 1984, p. 38).

In this case, this heterogeneity does not generate the real Riemannian space-time (although, following the example of Jacobi, this heterogeneity can be displayed mathematically as a Riemannian space).

Indeed, this heterogeneity can be considered as inhomogeneity of the field in space and time of the real Euclidean space and time, but not as heterogeneity of spacetime itself. In accordance with this our interpretation of the principle of equivalence is as follows.

The gravitational field, and not the space and/or time, sets the variable speed of body motion. Therefore, if the field depends on space and time, it is not necessary to bind the body velocity with time and space; it is enough to relate this speed with field itself. **Thus, all we need is to describe the action of force on the movement of the body, to find this relation between field and speed.**

It appears, that based on mathematics, we can actually find this connection. As is known, the equation (1.1 ') can be represented in the energy form.

For this let us rewrite the Newton's motion law in the form:

$$d\vec{\upsilon} = \gamma_N \frac{M}{r^2}\vec{r}^{\,0} dt\,, \tag{1.2}$$

Multiplying the left and right hand side of equation (1.2) on the speed $\vec{\upsilon}$, and taking into account that $\vec{\upsilon} = d\vec{r}/dt$ and $d\vec{r}/r^2 = -d(1/r)$, we have from (1.2) after integration:

$$\frac{\upsilon^2}{2} + \gamma_N \frac{M}{r} = const\,, \tag{1.3}$$

where $\frac{\upsilon^2}{2} = \varepsilon_\kappa / m$ is the kinetic energy of the moving particle per unit mass, and $\gamma_N \frac{M}{r} = \varepsilon_{pot}/m$ is the potential energy of a particle per unit mass at a given point of the gravitational field.

Thus, taking into account the postulate of equivalence and the expression for the potential of the gravitational field $\varphi_N = \gamma_N M/r$, we obtain from (1.1), the relationship between the velocity of the particle and potential of the gravitational field at the position of the particle:

$$\upsilon^2 = 2\varphi_N + const\,, \tag{1.4}$$

Obviously, we can assume that constant "*const*" is either the square of the particle velocity or the double potential at the initial point of reference of the particle motion.

If at the initial moment a particle was at rest, and the motion is only carried out via the potential energy outlay, then during the whole period of motion *const* = 0. For example, this occurs when the reference frame, that is related to the observer, falls freely to the center of gravity source along the radius (*radial infall*) from infinity, where it had a zero velocity. In this case, we have:

$$\upsilon^2 = 2\varphi_N = \frac{2\gamma_N M}{r}\,, \tag{1.5}$$

Thus, as a mathematical consequence of Newton's theory of gravity, we have received another interpretation of the fact of the equality of inertial and gravitational mass. Following the example of Einstein's equivalence principle, it can be expressed as follows: the potential of the gravitational field is equivalent to the square of the velocity of the motion of particles in this field.

In addition, (see chapter 4) the electromagnetic basis of gravitational equations allows one to write the vector potential of the gravitational field through the scalar potential.

2.0. Peculiarities of metric tensor of LIGT

As is known, Einstein came to the metric tensor of the pseudo-Riemannian space on the basis of interpretation of the experimental fact of the equality of inertial and gravitational mass as a so-called Einstein equivalence principle.

Above we have given a different interpretation of the equivalence of masses, in order to obtain an expression for the metric tensor of L-invariant theory of gravitation.

As we mentioned (Chapter 8), in differential geometry, the metric tensor elements are equal to the squares of the Lame scale coefficients. The Lame coefficients indicate how many units of length are contained in the unit coordinates in a given point and are used to transform vectors in the transition from one coordinate system to another.

At the same time, in the framework of general relativity, these two coordinate systems represent basically two dissimilar geometric coordinate systems from a number of well-known rectangular, oblique, or any other coordinates.

In contrast, in the L-invariant transformation is examined the transition between two identical from geometric point of view, coordinate systems, which are attached to two reference frames moving relative to each other. Moreover, it was found that a simultaneously this transition requires to take into account the transformation of time.

It is clear that this is not about geometric relationships which may not affect the final results of the solution of physical problems. We proved it by showing that the square of the arc element (interval) in this case is a consequence of the well-known relation between the energy, momentum and mass of the moving particle. Thus, these changes are purely physical. They contribute to the correction of physical problems non-relativistic physics.

However, conventionally this interval can be seen as a geometric object that generates a pseudo-Euclidean geometry, which has in addition to three spatial coordinates, one time coordinate (4-Minkowski geometry, in which SRT is often formulated).

From this geometrical point of view, coordinates and time undergo the change of the scales. These changes can be considered, along with changes of coordinates that take place during the transition between two different coordinate systems. But we should not forget that from the physical point of view it is a completely different transformations and changes of scales.

3.0. Calculation of metric tensor of LIGT

The linear arc element in the 3-dimensional mechanics is expressed through Lame's scale factors in the form of linear elements:

$$ds = \sum_{i=1}^{3} h_i dx_i = h_1 dx_1 + h_2 dx_2 + h_3 dx_3 , \qquad (3.1)$$

where $x_i = \vec{r} = (x_1, x_2, x_3)$, $i = 1, 2, 3$. . In a Cartesian coordinate system $x_i = \vec{r} = (x, y, z)$, and all the Lame coefficients equal to one.

In the L-invariant mechanics it is impossible to enter the line element of the arc since the physical equation, which shows the magnitude of the arc, connects the squares of the energy, momentum and mass, and not the first degrees of these values. The exact expression is obtained in the form of the square of length of arc element, which is often referred to simply as an interval. In the 4-geometry it is of the form:

$$(ds)^2 = \sum_{\mu-0}^{3}(h_\mu dx_\mu)^2 = (h_0 dx_0)^2 + (h_1 dx_1)^2 + (h_2 dx_2)^2 + (h_3 dx_3)^2, \qquad (3.2)$$

or, taking into account that $g_{\mu\mu} = h_\mu h_\mu$, (3.2) receives the form:

$$(ds)^2 = \sum_{\mu-0}^{3} g_{\mu\mu}^L (dx_\mu)^2 = g_{00}^L (dx_0)^2 + g_{11}^L (dx_1)^2 + g_{22}^L (dx_2)^2 + g_{33}^L (dx_3)^2, \qquad (3.2')$$

where $x_\mu = (ict, \vec{r}) = (ict, x_i) = (x_0, x_i)$ $\mu = 0,1,2,3$, $g_{\mu\mu}^L$ is metric tensor in LIGT.

3.1. The Lorentz-Fitzgerald length contraction and time dilation as a change of the scales of coordinates of space and time in LIGT

Using the definition of the metric tensor in LITG given above, let us calculate it in the simplest case. Consider (Pauli, 1958) Lorentz transformation in the transition from the coordinate system *K* to *K*', which is currently moving at a speed υ along the axis x. In this case only the coordinates and time t undergo transformations.

The Lorentz effects of length contraction and time dilation are the simplest consequences of the Lorentz transformation formulae, and thus also of the two basic assumptions of SRT.

$$x = \frac{x' - \upsilon t'}{\sqrt{1-\upsilon^2/c^2}}, \ y = y', \ z = z', \ t = \frac{t' - \frac{\upsilon}{c^2}x'}{\sqrt{1-\upsilon^2/c^2}}, \qquad (3.3)$$

The transformation which is the inverse of (1) can be obtained by replacing υ by $-\upsilon$:

$$x = \frac{x' + \upsilon t'}{\sqrt{1-\upsilon^2/c^2}}, \ y = y', \ z = z', \ t = \frac{t' + \frac{\upsilon}{c^2}x'}{\sqrt{1-\upsilon^2/c^2}}, \qquad (3.3a)$$

Take a rod lying along the x-axis, at rest in reference system K'. The position coordinates of its ends, x_1' and x_2' are thus independent of t' and $x_2' - x_1' = l_0$ is the rest lengtl of the rod. On the other hand, we might determine the length of the rod in system K' in the following way. We find x_1 and x_2 as functions of t. Then the distance between the two points which coincide simultaneously with the end points of the rod in system K will be called the length l of the rod in the moving system: $x_2(t) - x_1(t) = l$

Since these positions are not taken up simultaneously in system K', it cannot be expected that l equals l_0. In fact, it follows from (3.3) :

$$x_2' = \frac{x_2(t) - \upsilon t'}{\sqrt{1-\upsilon^2/c^2}}; \quad x_1' = \frac{x_1(t) - \upsilon t'}{\sqrt{1-\upsilon^2/c^2}}$$

for infinitesimal time intervals of length dx has form $dx' = \dfrac{dx}{\sqrt{1-\upsilon^2/c^2}}$.

From here the scaling factor of the Lorentz transformation of coordinates (denote it as k_x^L) will be equal to:

$$k_x^L = \frac{dx'}{dx} = \frac{1}{\sqrt{1-\upsilon^2/c^2}} = \gamma_L, \tag{3.4}$$

where $\gamma_L = 1/\sqrt{1-\upsilon^2/c^2}$ is the Lorentz factor.

The corresponding element λ_{xx} of the metric tensor of the Lorentz transformation will be:

$$\lambda_{xx} = \frac{dx'}{dx}\frac{dx'}{dx} = (\gamma_L)^2, \tag{3.5}$$

The rod is therefore contracted in the ratio $\sqrt{1-\upsilon^2/c^2}:1$, as was already assumed by Lorentz…. It therefore follows that the Lorentz contraction is not a property of a single measuring rod taken by itself, but is a reciprocal relation between two such rods moving relatively to each other, and this relation is in principle observable.

Analogously, the time scale is changed by the motion. Let us again consider a clock which is at rest in K'. The time t' which it indicates in x' is its proper time, τ and we can put its coordinate x' equal to zero. It then follows from (3.3a) that $t = \frac{\tau}{\sqrt{1-\upsilon^2/c^2}}$, which for infinitesimal time intervals dt give: $dt = \frac{dt'}{\sqrt{1-\upsilon^2/c^2}}$.

From here the scaling factor of the Lorentz transformation of time (denote it as k_t^L) will be equal to:

$$k_t^L = \frac{dt'}{dt} = \sqrt{1-\upsilon^2/c^2} = \gamma_L^{-1}, \tag{3.6}$$

The corresponding element λ_{xx} of the metric tensor of the Lorentz transformation will be:

$$\lambda_{tt} = \frac{dt'}{dt}\frac{dt'}{dt} = (\gamma_L)^{-2}, \tag{3.7}$$

Measured in the time scale of K, therefore, a clock moving with velocity υ will lag behind one at rest in K in the ratio $\sqrt{1-\upsilon^2/c^2}:1$. While this consequence' of the Lorentz transformation was already implicitly contained in Lorentz's and Poincare's results, it received its first clear statement only by Einstein.

Then, in framework of LITG the square interval will be as follows:

$$(ds)^2 = \sum_{\mu-0}^{3} \lambda_{\mu\mu}\eta_{\mu\mu}(dx_\mu)^2 = \lambda_{00}\eta_{00}(dx_0)^2 + \lambda_{11}\eta_{11}(dx_1)^2 + \lambda_{22}\eta_{22}(dx_2)^2 + \lambda_{33}\eta_{33}(dx_3)^2, \tag{3.8}$$

where $\eta_{\mu\mu}$ is the geometric metric tensor in LIGT (tensor of pseudo-Euclidian space); $\lambda_{\mu\mu}$ is the physical metric tensor in LIGT. Using the values $\lambda_{00} = \lambda_{tt}$ and $\lambda_{11} = \lambda_{xx}$, according to (3.5) and (3.7), we obtain in the Cartesian system of coordinates:

$$(ds)^2 = -(\gamma_L)^{-2}(dt)^2 + (\gamma_L)^2(dx)^2 + (dy)^2 + (dz)^2, \tag{3.9}$$

Taking into account (1.5) it is easy to see that (3.9) corresponds to the Schwarzschild-Droste solution.

4.0. Relation between Lorentz factor and characteristics of the Newton gravitational field

The main characteristic of the Lorentz transformation is the Lorentz factor γ_L: $\gamma_L = 1\big/\sqrt{1-\beta^2}$ (where $\vec{\beta} = \vec{\upsilon}/c$), which is determined by the speed of motion of the body $\vec{\upsilon} = \vec{\upsilon}(\vec{r},t)$.

The vector of speed of the particle motion can be considered as its main component, by which its trajectory, acceleration and some other quantities are determined.

On the base of our interpretation of the principle of equivalence of mass, we found relation between field and speed.

In the case of Newton's theory, probably the first, that found this relationship was E.A.Milne (Milne, 1934). Later, independently, and from an other primary bases, this was also done by Arnold Sommerfeld assistant - Wilhelm Lenz. He took advantage of this connection to find a solution to the Kepler problem, which coincides with the results of the Schwarzschild-Droste solution of Einstein-Hilbert equation (Sommerfeld, 1952).

Below we will expand this relationship to the case of the Lorentz-invariant mechanics, to obtain the next approximations in the form of a power series.

Using (1.5) it is easy to find an expression for the Lorentz-factor due to the gravitational field of Newton:

$$\gamma_L = \frac{1}{\sqrt{1-2\varphi_g/c^2}} \text{ или or } \gamma_L = \frac{1}{\sqrt{1-r_S/r}}, \tag{4.1}$$

where $r_S = \dfrac{2\gamma_N M}{c^2}$ is the, so-called, Schwarzschild radius.

5.0. Connection between the characteristics of the gravitational field and the Lorentz factor, taking into account the Lorentz-invariant generalization of mechanics

Within the framework of the Lorentz-invariant theory, the particle mass is a function of velocity and position in the field: $m = m(\vec{r},\vec{\upsilon})$

Let us recall the relativistic relations, binding the energy and momentum in relativistic mechanics.

The law of conservation of the energy-momentum is valid for each of the material particles:

$$\varepsilon_f^2 = c^2 p^2 + m_0^2 c^4, \tag{5.1}$$

where c is the speed of light, m_0 means the particle rest mass, and the full energy ε_f and momentum $\vec{p}$ are defined via relativistic expressions:

$$\varepsilon_f = m_0 c^2 \gamma_L, \quad \vec{p} = m_0 \vec{\upsilon}\gamma_L = \frac{\varepsilon}{c^2}\vec{\upsilon}, \tag{5.2}$$

where, $\gamma_L = 1\big/\sqrt{1-\beta^2}$ called Lorentz-factor, where $\beta = \vec{\upsilon}/c$, $\vec{\upsilon}$ is speed of a particle. Besides, in the relativistic mechanics the kinetic energy ε_κ is entered by the following expression:

$$\varepsilon_\kappa = \varepsilon_f - m_0 c^2 = m_0 c^2 \gamma_L - m_0 c^2 = m_0 c^2(\gamma_L - 1), \tag{5.3}$$

Since $\upsilon < c$, the expressions, containing δ, can be expanded to Maclaurin series (we take here into account only 4 terms):

$$\gamma_L = 1 + \left\{ \frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \frac{5}{16}\beta^6 + \frac{35}{128}\beta^8 + ... \right\}, \tag{5.4}$$

$$\gamma_L^{-1} = 1 - \frac{1}{2}\beta^2 - \frac{1}{8}\beta^4 - \frac{1}{16}\beta^6 - \frac{5}{128}\beta^8 + ..., \tag{5.5}$$

$$\beta\gamma_L = 0 + \beta + 0 + \frac{1}{2}\beta^3 + ... \tag{5.6}$$

Thus we can obtain for energy and momentum the following expressions:

$$\varepsilon_f = m_0 c^2 + m_0 c^2 \left\{ \frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \frac{5}{16}\beta^6 + \frac{35}{128}\beta^8 + ... \right\}, \tag{5.7}$$

$$\begin{aligned} \varepsilon_\kappa = \varepsilon_f - m_0 c^2 = m_0 c^2 \left\{ \frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \frac{5}{16}\beta^6 + \frac{35}{128}\beta^8 + ... \right\} = \\ = \frac{1}{2} m_0 \upsilon^2 + \frac{3}{8} m_0 \left(\frac{\upsilon^4}{c^2} \right) + \frac{5}{16} m_0 \left(\frac{\upsilon^6}{c^4} \right) + \frac{35}{128} m_0 \left(\frac{\upsilon^8}{c^6} \right) + ... \end{aligned}, \tag{5.8}$$

$$p = m_0 \upsilon + \frac{1}{2} m_0 \frac{\upsilon^3}{c^2} + ..., \tag{5.9}$$

At $\beta << 1$ we obtain from (5.7)-(5.9) as first approximation the non-relativistic expressions:

$$\varepsilon_f \approx m_0 c^2 + \frac{1}{2} m_0 \upsilon^2; \;\; p \approx m_0 \upsilon; \; \varepsilon_k \approx \frac{1}{2} m_0 \upsilon^2; \tag{5.10}$$

As we can see, the energy and momentum of a particle can be represented as an expansion in powers of the parameter υ^2/c^2.

Let us consider the problem of motion of a particle of mass m_0 in a gravitational field of a source mass $M >> m_0$. It is obvious that under the influence of gravitational field the particle changes its position and speed. Due to a change in position, the corresponding potential of the gravitational field, changes also and hence the potential energy of the particle and the force of gravity acting on the particle.

According to Newton's theory of gravity, the potential energy of a particle is equal to $\varepsilon_g = m_0 \varphi_N = \gamma_N m_0 M/r$. A change of the speed of the particle is accompanied by a change in its kinetic energy (5.3):

$$\begin{aligned} \frac{\gamma_N m_0 M}{r} = m_0 \varphi_N = \varepsilon_k = \frac{1}{2} m_0 \upsilon^2 + \frac{3}{8} m_0 \left(\frac{\upsilon^4}{c^2} \right) + \frac{5}{16} m_0 \left(\frac{\upsilon^6}{c^4} \right) + \frac{35}{128} m_0 \left(\frac{\upsilon^8}{c^6} \right) + ... = \\ = m_0 c^2 \left\{ \frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \frac{5}{16}\beta^6 + \frac{35}{128}\beta^8 + ... \right\} \end{aligned}, \tag{5.11}$$

In the case of sufficiently small velocities, we obtain a first approximation, $\gamma_N M/r \approx \varphi'_N = \upsilon^2/2$ or; $2\gamma_N M/r \approx 2\varphi'_N = \upsilon^2$, from where:

$$\frac{2\gamma_N M}{c^2 r} = \frac{r_S}{r} \approx \frac{2\varphi'_N}{c^2} = \frac{\upsilon^2}{c^2} = \beta^2, \tag{5.12}$$

where $r_S = 2\gamma_N M/c^2$ is called the gravitational radius of the body of mass M (Schwarzschild radius).

We can use the expressions (5.4) and (5.6), which determine the energy (kinetic and full) and the momentum to give more precisely the motion of a particle in a gravitational field. For this it is possible to clarify the relation between φ_N and r_S by using the following term of the expansion in (5.11) to obtain a second approximation:

$$\frac{2\gamma_N M}{c^2 r} \approx \frac{2\varphi''_N}{c^2} = \frac{\upsilon^2}{c^2} + \frac{3}{4}\frac{\upsilon^4}{c^4} = \frac{2\varphi'_g}{c^2} + \frac{3}{4}\left(\frac{2\varphi'_g}{c^2}\right)^2 = \frac{r_S}{r} + \frac{3}{4}\frac{r_S^2}{r^2}, \tag{5.13}$$

Using the first approximation (5.4.14), we obtain for the second approximation:

$$\frac{2\varphi''_N}{c^2} = \frac{\upsilon^2}{c^2} + \frac{3}{4}\frac{\upsilon^4}{c^4} = \frac{2\varphi'_g}{c^2} + \frac{3}{4}\left(\frac{2\varphi'_g}{c^2}\right)^2 \approx \frac{r_S}{r} + \frac{3}{4}\frac{r_S^2}{r^2}, \tag{5.14}$$

6.0. Calculation of gravitation field potentials

Thus, according to our axioms, it is sufficient to calculate the gravitational field of the potentials to be able to enter the L-invariant amendments to the gravitational theory of Newton.

In particular, for the calculation of the metric tensor elements of LIGT (see formulae (3.8), (3.9) and (4.1)) we use the potential $\varphi_g = \varphi_N$ of the Newtonian gravitation theory. It is remarkable that this non-relativistic potential gives the relativistic corrections to the solution of the Kepler problem.

Since LITG is based on electromagnetic theory, this calculation is not difficult. We only briefly recall the results of this approach, adequately set out in the chapter 4.

As we have seen, the Maxwell-Lorentz equations can be written in potentials in the form of equations of the electromagnetic field propagation. Using $\vec{E} = -grad\varphi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$, $\vec{B} = rot\vec{A}$, in the case of the Lorentz condition $\frac{1}{c}\cdot\frac{\partial\varphi}{\partial t} + \nabla\vec{A} = 0$, we have:

$$\frac{1}{c^2}\cdot\frac{\partial^2\varphi}{\partial t^2} - \nabla^2\varphi = 4\pi\rho, \tag{6.1}$$

$$\frac{1}{c^2}\cdot\frac{\partial^2\vec{A}}{\partial t^2} - \nabla^2\vec{A} = \frac{4\pi}{c}\rho\vec{\upsilon}, \tag{6.2}$$

From a mathematical point of view, these equations are the d'Alembert non-homogeneous equation. Its solution is known (see chapter 4). Additionally, it turns out that the vector potential associates with the scalar potential by expression:

$$\vec{A} = \varphi\frac{\vec{\upsilon}}{c}, \tag{6.3}$$

In this case, the main characteristics of the electromagnetic vector field are the scalar and vector potentials φ and $\vec{A}$, respectively, or the 4-potential $A_\mu = \left(\frac{\varphi}{c}, -\vec{A}\right)$. ($A_\mu = \frac{\varphi}{c^2} u_\mu$, where $u_\mu = dx_\mu / d\tau$ is 4-speed , dx_μ is 4-movement , τ is the proper time of the particle).

If the system contains a set of particles, each of which generates its own potential, then the potentials φ and $\vec{A}$ of the system of particles depend mainly on the general system parameters – the dimensions of the system, the total charge, etc. It is very important that the calculation of the system potential is defined by the superposition principle, i.e., by summation of the potentials of all the particles. Thus we can determine all the main characteristics of the system's electromagneticfield with the help of the 4-potential.

But before we will find the 4- potential of the system, we need to determine the potentials of a single particle.

As it is known, it is the centrally symmetric potential of Newton that defines the field of a point particle. As we have seen (see chap. 4), the calculation of the potential of a system of point sources - i.e., of a body with known charge density requires the integration of the potential of al l particles over the volume of the body.

Turning to gravitation, it can be expected that in the relativistic theory of gravity, along with the scalar potential φ_g there should be also a vector potential $\vec{A}_g$. There is also no doubt that the relationship (6.3) is valid in the case of the gravitational field. General relativity confirms this, and a great achievement for GR was to demonstrate this characteristic.

Thus, the solutions of equations (6.1) and (6.2) allow us to calculate both Lorentz factor and relativistic corrections to Newton's theory of gravitation.