Lorentz-Invariant Gravitation Theory

Chapter 5. The connection of electromagnetic theory and gravitation

Introduction

Here, we will show that the adopted by us the Mossotti-Lorentz postulate does not contradict the existing results of physics, in particular, general relativity. To do this, we will show that the basic solutions of general relativity equations can be expressed in electromagnetic form.

1.0. Introduction

"*The nature of time*, *space* and *reality* (Broekaert, 2005) are to large extent dependent on our interpretation of Special (SRT) and General Theory of Relativity (GTR). In STR essentially two distinct interpretations exist; the "geometrical" interpretation by Einstein based on the *Principle of Relativity* and the *Invariance of the velocity of light* and, the "physical" Lorentz-Poincar´e interpretation with underpinning by *rod contractions*, *clock slowing* and *light synchronization*, see e.g. (Bohm, 1965; Bell, 1987). *It can be questioned whether the "Lorentz-Poincar´e"- interpretation of STR can be continued into GTR*" (Broekaert, 2005).

It can be said that the purpose of our Lorentz-invariant theory of gravitation (LIGT) lie namely in distributing the "physical" interpretation of the Lorentz-Poincaré to gravitation.

Recall that we use the absolute system of units of Gauss, in which all physical units are expressed in terms of mechanical units: centimeter (cm), gram (g), second (s). In our case, it is essential that in this system six vectors of EM theory: $\vec{E}$, $\vec{D}$, $\vec{P}$, $\vec{B}$, $\vec{H}$, $\vec{M}$ (where $\vec{E}$ is electric field strength, $\vec{D}$ is electric displacement, $\vec{P}$ is electric polarization, $\vec{B}$ is magnetic induction, $\vec{H}$ is magnetic field strength, $\vec{M}$ is magnetic polarization), have the same dimension ($g^{1/2}\,cm^{-1/2}\,s^{-1}$). It is believed that this is a major cause why the CGS-Gaussian units have more common use among theoretical physicists. Let also M_s is mass of a big body ("source"), in the gravitational field of which we will explore the movement of another body ("particle") with small mass which is equal to m, and $m << M_s$.

Note that the variety of tasks is determined here not only by the usual parameters of the Newtonian theory of gravitation, but also by whether the source and the particle rotate or not.

2.0. The existing solutions of the Einstein-Hilbert equation

Shortly (Berman, 2007) after the appearance of Einstein's General Relativistic field equations, the first static and spherically symmetric solution became available: it was Schwarzschild's-Droste's metric (Schwarzschild, 1916; Droste, 1917; Hilbert, 1917). It described the gravitational field around a point like mass *M*.

Afterwards, the first rotational metric was developed: Lense-Thirring solution (Thirring and Lense, 1918). It described the field around a rotating sphere at the origin. Nevertheless, it was only an approximate solution, that only represented a weak field, in the slow rotation case.

Reissner and Nordstrom's metric (Reissner, 1916), generalized Schwarzschild's by charging the mass source in the origin.

It was only in the sixties of last century, that a rigorous solution representing the rotation of a central mass, was discovered by R. Kerr, in what is now called Kerr's metric (Kerr, 1963). Immediately afterwards, the generalization to a charged rotating central mass was supplied, which is now called Kerr-Newman's metric (Newman, Couch et al. 1965).

Note that the effects associated with enumerated solutions have been tested with different accuracy. Three effects described by the static Schwarzschild-Droste solution, were checked with an accuracy higher than 1%. Some effects associated with the rotation of the source and the particles were checked with less precision. Most of the other solutions can not be verified by the current state of the art, or because there are no corresponding objects of observation.

3.0. GTR, EMG and EMGT

A lot of the solutions of general relativity are obtained in linear approximation, using the method of perturbation. It was found that the results of this linear theory may be presented in the form of Maxwell's equations. Such a representation has been called gravitoelectromagnetism, or, briefly, GEM.

3.1. Gravito-electromagnetism (EMG)

In general relativity (GR) (Overduin, 2008), "space and time are inextricably bound together. In special cases, however, it becomes feasible to perform a "3+1 split" and decompose the metric of four-dimensional spacetime into a scalar time-time component, a vector time-space component and a tensor space-space component.

When gravitational fields are weak and velocities are low compared to *c*, then this decomposition takes on a particularly compelling physical interpretation: if we call the scalar component a "gravito-electric **(ge-)** potential" and the vector one a "gravito-magnetic **(gm-)** potential", then these quantities are found to obey almost exactly the same laws as their counterparts in ordinary electromagnetism.

In other words, one can construct a "gravito-electric field" $\vec{E}_{ge}$ and a "gravito-magnetic field $\vec{H}_{gm}$, and these fields are obeyed equations that are identical to Maxwell's equations and the Lorentz force law of ordinary electrodynamics.

From symmetry considerations we can infer that the earth's **gravito-electric field** must be radial, and its **gravito-magnetic** one dipolar, as shown in the diagrams 3.1 and 3.2. below:

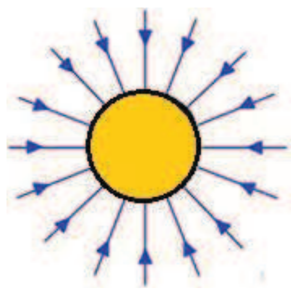

Fig.3.1. Radial gravitation field lines of Earth

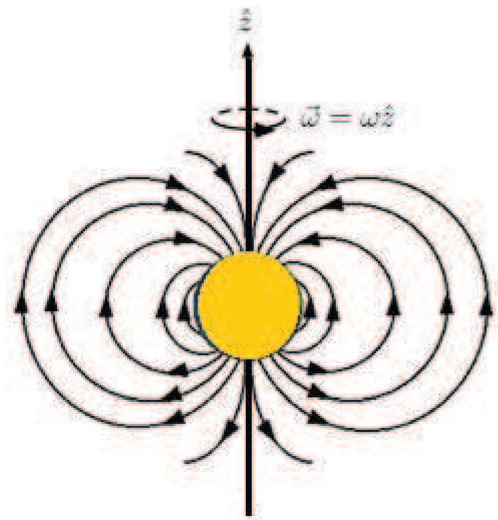

Fig. 3.2. Dipole gravitation field lines of Earth

These facts allow one to derive the main predictions of general relativity, simply by replacing the electric and magnetic fields of ordinary electrodynamics $\vec{E}$ and $\vec{H}$ by $\vec{E}_{ge}$ and $\vec{H}_{gm}$ respectively. However any such identification must be treated with care because the distinction between gravito-magnetism and gravito-electricity is frame-dependent, just like its counterpart in Maxwell's theory".

The mathematical aspect of GEM theory is described in many papers (see, for example, (Mashhoon, 2008))

To avoid misunderstanding, it should be noted that the electromagnetic theory of gravitation (EMGT) and gravitoelectromagnetism (GEM) - are not the same (Mashhoon, 2008). GEM is an auxiliary discipline of general relativity, which allows to physically imagine the phenomena generated by the metric (i.e., geometric) theory: general relativity. In turn, EMGT is an independent theory of gravitation, which arose on the basis of the hypothesis Mossotti and then was developed by number of scientists, including O. Heaviside, H.Lorentz and others (Heaviside, 1912; Lorentz, 1900; Webster, 1912; Wilson, 1921; etc).

4.0. Problem of motion of two interacting bodies in electrodynamics

4.1. Statement of the Problem

For simplicity, we will use the motion law of Newton. In the relativistic case it is of the form:

$$\frac{d\vec{p}}{dt} = \vec{F}\,, \tag{4.1}$$

where $\vec{p} = m\vec{\upsilon}$ is the momentum of the particle, $\vec{F}$ is the force, acting on the particle. In the case of EM field this $\vec{F}$ is Lorentz force:

$$\vec{F} = \vec{F}_e + \vec{F}_m\,, \tag{4.2}$$

where $\vec{F}_e = q\vec{E}$ is the electric part of the Lorentz force, $\vec{F}_m = \frac{q}{c}\vec{\upsilon}\times\vec{H}$ is the magnetic part of the Lorentz force. If we express the vectors of the EM field in terms of potentials $\vec{E} = -grad\varphi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}$, $\vec{H} = rot\vec{A}$, for the Lorentz force we obtain the expression:

$$\vec{F} = q\vec{\nabla}\varphi - \frac{q}{c}\frac{\partial\vec{A}}{\partial t} + \frac{q}{c}\vec{\upsilon}\times\left(\vec{\nabla}\times\vec{A}\right) = q\vec{\nabla}\varphi - \frac{q}{c}\frac{\partial\vec{A}}{\partial t} + \frac{q}{c}\left[\vec{\nabla}\left(\vec{\upsilon}\cdot\vec{A}\right) - \left(\vec{\upsilon}\cdot\vec{\nabla}\right)\vec{A}\right], \qquad (4.3)$$

Thus, the law (4.1) in the case of motion of a particle in an electromagnetic field takes the form:

$$\frac{d(m\vec{\upsilon})}{dt} = q\vec{E} + \frac{q}{c}\vec{\upsilon}\times\vec{H}\,, \qquad (4.4)$$

As we have noted equations of EM fields can be written in the force form (via field strength, i.e., through the force per unit charge) or in the energy form (via potentials, i.e., through the energy and momentum per unit charge). The connection between them is easy to to be found by multiplying the vector fields at the charge q :

$q\vec{E} = -grad\, q\varphi - \frac{q}{c}\frac{\partial\vec{A}}{\partial t}$, $q\vec{H} = c\cdot rot\frac{q}{c}\vec{A}$. Since the force, energy and momentum are expressed in terms of EM values as $\vec{F}_e = q\vec{E}$, $\varepsilon = q\varphi$, $\vec{p} = \frac{q}{c}\vec{A}$, respectively, and $q\vec{H} = rot\, c\vec{p}$, so for the electric and magnetic forces we obtain correspondingly: $\vec{F}_e = q\vec{E} = -\,grad\,\varepsilon - \frac{\partial\vec{p}}{\partial t}$, $\vec{F}_m = q\left[\vec{\upsilon}\times\vec{H}\right] = \vec{\upsilon}\times rot\, c\vec{p} = \vec{\upsilon}\times\vec{\nabla}\times c\vec{p}$.

Thus, within the framework of EMGT the Lorentz-invariant theory of gravitation must contain not only energy, but also momentum. In other words, it must be characterized not only by gravi-electric field or by corresponding gravi-electric potential, but also by a gravi-magnetic field or appropriate vector potential.

Since we consider the relativistic effects as corrections to the Newtonian mechanics, we will begin from the non-relativistic Newton's law, when the mass of the particle $m = const$. In this case, equation (4.4) can be rewritten as:

$$m\frac{d\vec{\upsilon}}{dt} = \vec{F}_e + \vec{F}_m, \qquad (4.5)$$

Depending on the experimental conditions a few problems for the equation (4.5) can be set, solutions of which are of practical interest and the results of which can be tested in an experiment with sufficient accuracy:

1st task comes from the equation $m\frac{d\vec{\upsilon}}{dt} = \vec{F}_e$ for a spinless or spinning particle, rotating around the stationary source. This task corresponds to the fall of the particle on the source along the radius (the, so-called, radial infall), including the case when the particle rotates in a circular orbit, which is defined by the equilibrium of the force of gravity and centrifugal force.

2nd task comes from the equation $m\frac{d\vec{\upsilon}}{dt}=\vec{F}_m$ for the field, created by the rotation of the source. Since for the usual range of characteristics of the source the field $\vec{H}$ is much smaller than the field $\vec{E}$, this solution can be considered as a small perturbation of the particle motion of the first task. This task is a more difficult one, since $\vec{F}_m$ contains the speed. In addition, as compared with the first problem, the solution is dependent on the particle trajectories around the source; there may be tasks for latitude, azimuth and intermediate, between them, motions of the spinning and spinless particles.

3rd task comes from the equation $m\frac{d\vec{\upsilon}}{dt}=\vec{F}_e+\vec{F}_m$ is a more common type of motion and its solution, of course, is much more difficult relatively to the first two. However, under certain limitations on the parameters of the problem, it is possible to obtain interesting solutions.

Some of these problems in the case of EM theory have already been solved. In this case, it is sufficient only to transfer correctly the solution for the case of the gravitational field. In other cases, there is a need to solve problems from a clean slate.

4.2. Stationary electric field (Coulomb field)

Consider the interaction of two charges q and Q belonging to the bodies with masses m and M, respectively, where $m \ll M$.

The expression for the interaction of two point (or spherical) charges q and Q is given by the Coulomb force:

$$\vec{F}_C=k_e\frac{qQ}{r^2}\vec{r}^{\,0}, \tag{4.6}$$

wherein $k_e=1$ in Gauss system of units, r is distance to particles, $\vec{r}^{\,0}=\vec{r}/r$ is a unit vector.

Expression for the field strength of the point source of the electric field is by definition equal:

$$\vec{E}=k_e\frac{\vec{F}_C}{q}=k_e\frac{Q}{r^2}\vec{r}^{\,0}, \tag{4.7}$$

Using the expression of the electric field through the potential $\vec{E}=\vec{\nabla}\varphi\equiv grad\varphi\equiv\partial\varphi/\partial\vec{r}$, it is easy to find:

$$\varphi=k_e\frac{Q}{r}, \tag{4.8}$$

4.3 Magnetic field of a charged particle

A charged particle, as some body, produces a magnetic field, firstly, due to its movement along a certain trajectory, $\vec{H}_l$, and, secondly, due to its own rotation, $\vec{H}_s$.

If the particle is involved in both movements simultaneously, the total field is a superposition of both fields: $\vec{H}=\vec{H}_l+\vec{H}_s$.

Consider how these fields are defined in electrodynamics.

1) If the particle moves along a path with a speed $\vec{\upsilon}$, in the laboratory frame a magnetic field appears, which is equal to:

$$\vec{H}_l = \frac{1}{c}\left[\vec{\upsilon} \times \vec{E}_l\right]. \tag{4.9}$$

The expression for the magnetic field (4.9) can be obtained from the Biot-Savart law when it is applied to a single particle. This magnetic field has the form of circular rings around the axis of the particle motion. It has maximum in a plane, which is perpendicular to the particle motion, and in this cross-section is equal to: $H_l = \frac{\upsilon}{c}\frac{q}{r^2}$. From (4.9) it also follows that this field is υ/c times smaller than the electric.

2) If a charged particle has a rotation (spin), it has a magnetic dipole moment $\vec{\mu}$. The magnetic field of a magnetic dipole is:

$$\vec{H}_s = \frac{3(\vec{r}^{\,0}\vec{\mu})\vec{r}^{\,0} - \vec{\mu}}{r^3}, \tag{4.10}$$

The force lines of the magnetic field are shown in Fig. 3.2. Along the axis of the dipole the field strength is $H_{sII} = \frac{2\mu}{r^3}$, where μ is the absolute value of the magnetic moment of the particle. In the direction, perpendicular to the dipole $H_{s\perp} = \frac{\mu}{r^3}$.

In our case, the task is analogous with the motion of the electron around the proton in a hydrogen atom. In this case, (Davydov, 1965) the atomic nucleus can be considered as a point magnetic dipole with moment $\vec{\mu}$. This dipole creates the potentials:

$$\varphi = 0\,,\ \vec{A} = \frac{[\vec{\mu}\times\vec{r}]}{4\pi r^3} = \left[\vec{\nabla}\times\frac{\vec{\mu}}{4\pi r}\right], \tag{4.11}$$

which correspond to the magnetic field:

$$\vec{H} = rot\vec{A} = \left[\vec{\nabla}\times\vec{A}\right] = \vec{\nabla}\left(\vec{\nabla}\frac{\vec{\mu}}{4\pi r}\right) - \vec{\nabla}^2\left(\frac{\vec{\mu}}{4\pi r}\right), \tag{4.12}$$

Operator

$$W = -\frac{e\hbar}{2mc}\vec{\sigma}\vec{H} = -\frac{e\hbar}{2mc}\sigma_z H\,, \tag{4.13}$$

characterizes the interaction of the magnetic moment of the electron with the magnetic field of the nucleus (here $\vec{\sigma}$ is particle spin).

The expressions for the torque $\vec{M}$ exerted by means of the magnetic field on the magnetic dipole, and the potential energy U of a constant magnetic dipole in a magnetic field, are similar to the corresponding formulas for the electric dipole interaction with the electric field:

$$\vec{M} = \vec{\mu}\times\vec{H}\,,\quad U = -\vec{\mu}\cdot\vec{H}\,, \tag{4.14}$$

It is important to note that the relationship between q and m EMGT is not the same as in classical electrodynamics. In electrodynamics, these are independent quantities. In EMGT there is a close relationship between them due to the electromagnetic origin of matter and interactions, which means that any massive body consists of concentrated electromagnetic field and charges.

Within the framework of the nonlinear theory of elementary particles (NTEP) the concentrated EM field is a combination of different types of the self-interacting electromagnetic waves, which are elementary particles. The mass of these particles is equal to the energy of the EM waves, divided by the square of the speed of light. Since the mass m is determined via the EM field, it appears that mass may be defined through gravitational charge q_g ($m = q_g / \sqrt{\gamma_N}$) so that the dimension of q_g coincides with the dimension of electric charge q. This does not mean that q_g is q. The g-charge q_g is much smaller than the electric charge q and its action is manifested only when the bulk of the charge q is neutralized, as in the case for neutral bodies.

Hypothesis of Mossotti – Lorentz suggests (chapter 3) that for the passage to the gravitational field equations it is sufficient to move from electrical quantities to gravity quantities according to certain rules that allow the transition from the electromagnetic field to the residual electromagnetic field of the body. These rules we will try to elucidate below.

5.0. Transition from EM field theory to gravitational field theory

In the works of Lorentz (see, e.g., (Lorentz, 1900)) it was shown in sufficient detail that in the theory of electromagnetic fields the residual electromagnetic field can actually arise. But for the specific purpose of its introduction it is easier and more convenient to use the methods of similarity theory and dimensional analysis (Sedov, 1993).

We will compare the expressions of EM theory with the parallel expressions of gravitational theory and select the correspondences between them. For the control of the conclusions we use dimensional analysis.

The main characteristic of the source field in the one and in the other theory is the expression of the interaction force or the corresponding interaction energy between the two bodies.

5.1. Gravity electrostatic (ge-) field. The transition from the Coulomb's field to the Newton's field

If we assume that gravity is generated by electric field, but quantitatively, by very small part of it (see Addition A1), then Newton's gravitation law:

$$\vec{F}_N = \gamma_N \frac{m \cdot M}{r^2} \vec{r}^{\,0}, \tag{5.1}$$

should take the form of Coulomb's law:

$$\vec{F}_C = k_0 \frac{q \cdot Q}{r^2} \vec{r}^{\,0}, \tag{5.2}$$

where m and q are the mass and electric charge of the particle, M and Q are the mass and electric charge of the source, γ_N is Newton's gravitational constant, and the coefficient k_0 in Gauss's units is $k_0 = 1$. In this case, the definitions of gravitational field strengths of Newton and Coulomb electric field have the form $\vec{E}_N = \frac{\vec{F}_N}{m} = \gamma_N \frac{M}{r^2} \vec{r}^{\,0}$ and $\vec{E} = \frac{\vec{F}_C}{q} = k_0 \frac{Q}{r^2} \vec{r}^{\,0}$, respectively.

We introduce the gravitational charge q_g, corresponding to mass m (Ivanenko and Sokolov, 1949) , by means of the relation:

$$q \rightarrow q_g = \sqrt{\gamma_N}\, m, \tag{5.3}$$

In this case, Newton's law can be rewritten in the form of Coulomb's law:

$$\vec{F}_g = \frac{q_g \cdot Q_g}{r^2}\vec{r}^{\,0} = \gamma_N \frac{m \cdot M}{r^2}\vec{r}^{\,0} = \vec{F}_N, \qquad (5.4)$$

where $Q_g = \sqrt{\gamma_N} M$ is the gravitational charge of source, corresponding to the mass M of the source.

From the comparison of equations (5.2) and (5.4) it follows that the dimensions of the electromagnetic and gravitational charges coincide. At the same time, a gravitational charge (5.3) has electromagnetic origin, and, hence, the corresponding mass is the inertial mass. On the other hand, the law (5.4) comprises the gravitational masses. This implies the equivalence of inertial and gravitational masses.

We introduce the g-field strength within framework of EMGT as:

$$\vec{E} \to \frac{\vec{E}_g}{\sqrt{\gamma_N}}, \qquad (5.5)$$

where the tension of the Coulomb field is equal to: $\vec{E} = \frac{Q}{r^2}\vec{r}^{\,0}$. Substituting the values of gravitation theory here, we get:

$$\vec{E}_g = \sqrt{\gamma_N}\frac{Q_g}{r^2}\vec{r}^{\,0} = \gamma_N \frac{M}{r^2}\vec{r}^{\,0} = \vec{E}_N, \qquad (5.6)$$

where $\vec{E}_N$ is the strength of the Newton gravitational field.

Let us introduce the scalar gravitational potential within the framework of EMTG as:

$$\varphi \to \frac{\varphi_g}{\sqrt{\gamma_N}}, \qquad (5.7)$$

where the potential of the Coulomb field is: $\varphi = \frac{Q}{r}$. Substituting the values of gravitation theory here, we get:

$$\varphi_g = \sqrt{\gamma_N}\frac{Q_g}{r} = \gamma_N \frac{M}{r} = \varphi_N, \qquad (5.8)$$

where φ_N is the potential of the Newton gravitational field.

The Poisson equation for the g-field can serve as test for (5.7). Indeed, for the EM field the Poisson equation can be written as:

$$\Delta\varphi = 4\pi\,\rho_e, \qquad (5.9)$$

where $\rho_e = \frac{dq}{d\tau}$ is the electric charge density, $d\tau$ is the volume element. We introduce the density of gravitational charge ρ_g similarly to the electric density:

$$\rho_e \to \rho_g = \frac{dq_g}{d\tau} = \sqrt{\gamma_N}\rho_m, \qquad (5.10)$$

where $\frac{dm}{d\tau} = \rho_m$ is mass density. Then, replacing the potential and the charge density in (5.9) according to (5.7) and (5.10), we obtain the Poisson equation for the gravitational potential:

$$\Delta\varphi_g = 4\pi \sqrt{\gamma_N}\,\rho_g = 4\pi\,\gamma_N\,\rho_m\,, \qquad (5.11)$$

which corresponds to the Poisson equation for the Newton gravitational field.

5.2. Gravi-magnetic field (gm-field)

In this case, by analogy with electrodynamics, the existence of the variable ge-field and associated with them alternating or direct gm-fields is assumed. The existence of a similar field is confirmed by general relativity and experiments. Unfortunately, since the Newton theory does not contain an analog of magnetic field, the verification of existence of the g-magnetic field within framework of EMGT, can presently be done only by dimensional analysis. Serious confirmation should be obtained by the solution of the corresponding equations of gravitation, which will give equivalent results to the general theory of relativity.

As is known, the magnetic field is generated by the motion of electric charges or movement of an electric field. In this case, we need to obtain an expression for the magnetic field, similar to Coulomb's law for the electric field. This is the Biot-Savart–Laplace law.

For simplicity, we will consider the special case of uniform motion of a source charge Q, which create a current I (current from motion of charge q will be denoted by i). In real tasks, of course, charges and masses are divided into point (differential) values, and field calculated by integrating over a set of point charges.

Magnetic vector $\vec{H}$ that occurs when the charge Q moves at a speed $\vec{\upsilon}$ in circuit element dl, will be:

$$\vec{H} = \frac{Q[\vec{\upsilon}\times\vec{r}]}{|r|^3} = \frac{I\left[d\vec{l}\times\vec{r}\right]}{|r|^3}\,, \qquad (5.12)$$

Using the gravitational charge density ρ_g according to (5.10), similarly to the electric current $i = dq/dt$ and the current density $\vec{j}$, we will define respective g-current (or current of mass) as:

$$i \to i_g = \frac{dq_g}{dt} = \rho_g \upsilon_n dS = \sqrt{\gamma_N}\,\rho_m \upsilon_n dS\,, \qquad (5.13)$$

and the density of g-current of mass, as:

$$\vec{j} \to \vec{j}_g = \left(i_g/dS\right)\vec{n} = \sqrt{\gamma_N}\,\rho_m \upsilon_n\,, \qquad (5.14)$$

where $\vec{\upsilon}$ is the velocity of the charge in a conductor with a cross section dS, and υ_n the projection of the velocity on the normal to dS.

If the e-charge q moves close to the e-current (or permanent magnet field $\vec{H}$), this current (or field $\vec{H}$) acts on the charge via the magnetic part of the Lorentz force F_{Lm}:

$$\vec{F}_{Lm} = q\left[\vec{\upsilon}\times\vec{H}\right] = \frac{q\cdot Q}{|r|^3}\left[\vec{\upsilon}\times\left(\vec{\upsilon}'\times\vec{r}\right)\right] = \frac{i\cdot I}{|r|^3}\left[d\vec{l}\times\left(d\vec{l}'\times\vec{r}\right)\right], \qquad (5.15)$$

Let us introduce the strength of gm-field within framework of EMGT as:

$$\vec{H} \to \frac{\vec{H}_g}{\sqrt{\gamma_N}}, \tag{5.16}$$

where the magnetic field $\vec{H}$ is given by (5.12).

Substituting the corresponding physical quantities according to (5.13) and (5.16) in (5.12), we will obtain the gravi-magnetic (gm-) vector that arises when the charge $Q_g = M\sqrt{\gamma}$ moves at a speed $\vec{\upsilon}$ in an element of a circuit dl:

$$\vec{H}_g = \sqrt{\gamma_N}\frac{Q_g\left[\vec{\upsilon}\times\vec{r}\right]}{|r|^3} = \gamma_N \frac{M\left[d\vec{l}\times\vec{r}\right]}{|r|^3}, \tag{5.17}$$

or

$$\vec{H}_g = \sqrt{\gamma_N}\frac{I_g\left[d\vec{l}\times\vec{r}\right]}{|r|^3} = \gamma_N \frac{\left[d\vec{l}\times\vec{r}\right]}{|r|^3}\rho_m \upsilon_n dS , \tag{5.18}$$

where $\vec{r}$ is the distance between the test particle and the moving charged source or element of current I_g, which generate the gm-vector.

Using (5.15), for the gravi-magnetic Lorentz force we obtain:

$$\vec{F}_{Lgm} = \frac{q_g Q_g}{|r|^3}\left[\vec{\upsilon}\times\left(\vec{\upsilon}'\times\vec{r}\right)\right] = \gamma_N \frac{mM}{|r|^3}\left[\vec{\upsilon}\times\left(\vec{\upsilon}'\times\vec{r}\right)\right], \tag{5.19}$$

Since the magnetic field $\vec{H}$ in the electrodynamics can be expressed via a vector potential $\vec{A}$ by the expression:

$$\vec{H} = rot\vec{A}, \tag{5.20}$$

it is useful to define the transition from the EM vector potential $\vec{A}$ to the gravitational $\vec{A}_g$. We assume that:

$$\vec{A} \to \frac{\vec{A}_g}{\sqrt{\gamma_N}}, \tag{5.21}$$

Then, using (5.16), we can rewrite (5.20) in the form:

$$\vec{H}_g = rot\vec{A}_g, \tag{5.22}$$

Expression (5.21) also satisfies the full EM expression for the electric strength vector:

$$\vec{E} = -grad\varphi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}, \tag{5.21}$$

Using (5.5) , (5.7) and (5.21) , we obtain for g-field:

$$\vec{E}_g = -grad\varphi_g - \frac{1}{c}\frac{\partial \vec{A}_g}{\partial t}, \qquad (5.21')$$

Thus, we have shown that the basic EM quantities and equations can be associated with similar quantities and equations for the g-field.

Recall that in framework of EMGT the Hamilton-Jacobi equation serves as the Lorentz-invariant law of motion. In the external source field, characterized by energy ε_{ex} and momentum $\vec{p}_{ex}$, this equation has the form:

$$\frac{1}{c^2}\left(\frac{\partial S}{\partial t} + \varepsilon_{ex}\right)^2 - \left(\frac{\partial S}{\partial x} - p_{x\,ex}\right)^2 - \left(\frac{\partial S}{\partial y} - p_{y\,\mathrm{ex}}\right)^2 - \left(\frac{\partial S}{\partial z} - p_{z\,\mathrm{ex}}\right)^2 = m^2c^2, \qquad (5.4)$$

Then, the Hamilton-Jacobi equation of motion of a particle in an EM field of the source:

$$\frac{1}{c^2}\left(\frac{\partial S}{\partial t} + q\varphi\right)^2 - \left(\frac{\partial S}{\partial x} - \frac{q}{c}A_x\right)^2 - \left(\frac{\partial S}{\partial y} - \frac{q}{c}A_y\right)^2 - \left(\frac{\partial S}{\partial z} - \frac{q}{c}A_z\right)^2 = m^2c^2, \qquad (5.22)$$

can be rewritten for the g-field in the same form:

$$\frac{1}{c^2}\left(\frac{\partial S}{\partial t} + m\varphi_g\right)^2 - \left(\frac{\partial S}{\partial x} - \frac{m}{c}A_{xg}\right)^2 - \left(\frac{\partial S}{\partial y} - \frac{m}{c}A_{yg}\right)^2 - \left(\frac{\partial S}{\partial z} - \frac{m}{c}A_{zg}\right)^2 = m^2c^2, \qquad (5.23)$$

As an illustration of the correctness of the relationship of electromagnetic and gravitoelectromagnetic quantities, we put in Appendix A.2. to this article the table of dimensions of physical quantities, used in the article.

Additions:

A.1. Relationship between electric and gravitational charges

It is easy to show that the gravitational field is a small fraction of the electromagnetic field.

To this corresponds the fact that the gravitational charge of the electron is less than its electric charge $e >> q_g$, where $q_g = m_e\sqrt{\gamma}$ (here $e = 4{,}8\cdot 10^{-10}$ unit. SGSEq is electron charge (1 unit CGSEq = $g^{1/2}sm^{3/2}s^{-1}$), $m_e = 0{,}91\cdot 10^{-27}$ g is electron mass, $\gamma = 6{,}67\cdot 10^{-8}$ $cm^3/g\ sec^2$ is the gravitational constant. It is easy to see that the dimension of the gravitational charge of the electron coincides with the dimension of electric charge and its magnitude in 10^{21} times less. Indeed, $e/m_e\sqrt{\gamma} \approx 2\cdot 10^{21}$.

For a proton (the only stable heavy particle), this value is of the order $e/m_p\sqrt{\gamma} \approx 2\cdot 10^{18}$. The heaviest known elementary particles are the highly unstable bosons $W^{\pm}$ (mass ≈80 GeV). This is about 100 times more than the mass of the proton, giving a ratio of no less than 10^{16}.

A.2. Dimensions of electromagnetic and gravi-electromagnetic quantities

For the verification of the correctness of correlations in the transition from the EM physical quantities to the gravitation quantities, the accordance of their dimensions plays an important role. The worded below (far from exhaustive) list confirms that electrodynamics can be considered as the basis of mechanics.

Electromagnetic theory

e-charge	$[q] = \mathrm{g}^{1/2}\,\mathrm{cm}^{3/2}\,\mathrm{s}^{-1}$
e-charge density	$[\rho_e] = \mathrm{g}^{1/2}\,\mathrm{cm}^{-3/2}\,\mathrm{s}^{-1}$
e-current	$[i] = \mathrm{g}^{1/2}\,\mathrm{cm}^{3/2}\,\mathrm{s}^{-2}$
e-current density	$[j] = \mathrm{g}^{1/2}\,\mathrm{cm}^{-1/2}\,\mathrm{s}^{-2}$
Coulomb force	$[F_C] = \mathrm{g\,cm\,s}^{-2}$
Strength of e-fields	$[E] = \mathrm{g}^{1/2}\,\mathrm{cm}^{-1/2}\,\mathrm{s}^{-1}$
Strength of m-field	$[H] = \mathrm{g}^{1/2}\,\mathrm{cm}^{-1/2}\,\mathrm{s}^{-1}$
Scalar potential	$[\varphi] = \mathrm{g}^{1/2}\,\mathrm{cm}^{1/2}\,\mathrm{s}^{-1}$
Vector potential	$[A] = \mathrm{g}^{1/2}\,\mathrm{cm}^{1/2}\,\mathrm{s}^{-1}$
Field energy	$[\varepsilon_e] = \mathrm{g\,cm}^2\,\mathrm{s}^{-2} = [q\varphi]$
Field energy density	$[\rho_\varepsilon] = \mathrm{g\,cm}^{-1}\,\mathrm{s}^{-2} = [E^2] = [H^2]$

Gravitation theory of Newton

Newton's force	$[F_N] = \text{г см с}^{-2}$
Newton's gravitational constant	$[\gamma_N] = \text{г}^{-1}\,\text{см}^3\,\text{с}^{-2}$ $([\sqrt{\gamma_N}] = \text{г}^{-1/2}\,\text{см}^{3/2}\,\text{с}^{-1})$
Field strength	$[E_N] = \text{см/с}^2$ (acceleration)
Scalar potential	$[\varphi_N] = \text{см}^2/\text{с}^2 \equiv (\text{см/с})^2$ (velocity square)
Scalar potential	$[\varepsilon_N] = [m\varphi_N] = \text{г см}^2/\text{с}^2$

Electromagnetic gravitation theory (EMGT)

g-charge	$[q_g] = \text{г}^{1/2}\text{см}^{3/2}\text{с}^{-1} = [q_e] = [m\sqrt{\gamma_N}]$
g-charge density	$[\rho_g] = \text{г}^{1/2}\,\text{см}^{-3/2}\,\text{с}^{-1}$
g-current	$[i_g] = \text{г}^{1/2}\,\text{см}^{3/2}\,\text{с}^{-2}$
g-current density	$[j_g] = \text{г}^{1/2}\,\text{см}^{-1/2}\,\text{с}^{-2}$
g-force	$[F_g] = \text{г см с}^{-2} = [F_N]$
Strength of ge-fields	$[E_g] = \text{см/с}^2 = [E_N] = [E\sqrt{\gamma_N}]$
Strength of gm-field	$[H_g] = \text{см/с}^2 = [H\sqrt{\gamma_N}]$
Scalar potential of g-fields	$[\varphi_g] = \text{см}^2/\text{с}^2 = [\varphi\sqrt{\gamma_N}]$
Vector potential of g-fields	$[A_g] = \text{см}^2/\text{с}^2 = [A\sqrt{\gamma_N}]$
g-field energy	$[\varepsilon_g] = \text{г см}^2/\text{с}^2 = [m\varphi_g]$