Article

Marder's Dark Energy Model in Saez Ballester Scalar Tensor Theory

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Abstract

We have investigated Marder's cosmological model for dark energy. We obtained the exact solution of Einstein Field Equations by using special law of variation of Hubble Parameter that yields the constant deceleration parameter proposed by Berman. The condition that expansion scalar is proportional to the shear scalar which leads to the relation between metric potential $P_1 = P_3^n$. Some physical parameters of dark energy model are disscssed.

Keywords: Dark energy, scalar tensor theory, Marder's Model.

1 Introduction

In now days, the recent trend of development from astronomical observational data in cosmology gives the clear idea of universe and that idea is as universe is expanding and accelerating. The main goal of the cosmologists is to find out the huge scale structure of the universe. Therefore there have generous curiosity of research worker in amend gravitational theories. Astronomical research observations gives clear draft of universe and that is our universe is full and flat and free from clamped form of energy density spread throughout the universe. The energy density free from clamped form is called Dark Energy with negative pressure being caused to about 74 percent of total energy density. The remaining 26 percent of dark energy density consists of matter consisting near about 22 percent of dark energy density and 4 percent of subatomic partial density. Dark energy is normally generalised by parameter EoS which is given by $\Omega(t) = p \div \rho$, where as p is pressure of fluid and ρ is density of energy. In recent era, several research workers Sahni and Shtanov (1) have discussed dark energy for Braneworld models. SNe Ia Riess et al.(2) investigated dark energy evolution with the Hubble space, Bijan and Yadav (3) have stuided dark energy for Bianchi-II type. Katore et al. (4) have investigated perfect fluid and dark energy for multidimensional Kaluza-Klein. Pawar et al. (5)-(7) have studied dark energy model together with EoS parameter in Theory of General Relativity. Sahoo and Mishra (8) explored axially symmetric cosmological model with anisotropic dark energy.

From last few years, there has been many changes in general relativity. Among few of them are remarkable in which Saez and Ballester (9)-(10), Brans and Dicke (11) proposed the scalar tensor theory in gravitation. Saez Ballester Scalar Tensor Theory is considered as Fundamental Theory of Gravity which deals with the study of early universe. Many astrophysical research worker studied cosmological models in Saez-Ballester Scalar Tensor Theory of general relativity. Among few of them are Singh and Agrawal (12), Shri Ram and Tiwari (13), Reddy and Rao (14) , Singh and Shri Ram (15) have discovered many prospects of cosmological Models in Saez Ballester Scalar Tensor Theory . Katore et al. (16) obtained FRW models,Pawar and Dagwal (17) studied Kantowski-Sachs models in Saez Ballester Scalar Tensor Theory. Mishra and Sahoo (18) obtained Bianchi type VI1 cosmological model with wet dark fluid in scale invariant theory of gravitation Mukherjee (19) found out exact solutions of the cylindrically symmetric Marder's Metric in Sen-Dunn theory of gravitation.Recently Pawar and Solanke (20) investigated Marder's cosmological model with two fluid in same Saez Ballester Theory.

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In present paper, we inspired from the current revial curiosity in Saez Ballester theory of gravity, in particular the Marder's cosmological model for dark energy. In Saez Ballester Scalar Tensor Theory, the metric (1) and field equation is paired with scalar field ϕ which is free from dimension. This pairing provides the acceptable description of weak field.

2 The Metric And Field Equations

We assume Marder'scosmological model of cosmic time given by the line element

$$ds^2 = P_1^2(dx^2 - dt^2) + P_2^2 dy^2 + P_3^3 dz^2 \tag{2.1}$$

where P_1 , P_2 and P_3 are functions of cosmic space time t.

For combined scalar and tensor fields, the field equations is given by Saez and Ballester are

$$R_{kl} - \frac{1}{2} g_{kl} R - w\phi^m(\phi_{,k}\, \phi_{,l} - \frac{1}{2} g_{kl} \phi_{,n}\, \phi'^n) = -T^k_{kl} \tag{2.2}$$

In same way, scalar field ϕ satisfies equation

$$2\phi^m \phi^l_{;l} + m\phi^{m-1} \phi_{,n}\, \phi'^n = 0 \tag{2.3}$$

And for field equation (2) and (3) we have consequence which is given by

$$T^{kl}_{;l} = 0 \tag{2.4}$$

where w, m are constants terms and $T^{kl}_{;l} = 0$ represents the energy momentum tensor of matter as well as comma and semicolon denotes partial and covariant derivatives respectively.

For five dimension, the energy momentum tensor of dark energy is given by,

$$T^l_k = diag[\rho, -p_x, -p_y, -p_z] \tag{2.5}$$
$$= diag[1, -\Omega_x, -\Omega_y, -\Omega_z]\rho \tag{2.6}$$

where ρ is the energy density dark energy and p_x , p_y , p_z are the pressures on the X , Y and Z axes respectively. Here Ω is EoS parameters of the fluid with no deviation and Ω_x , Ω_y and Ω_z are the EoS parameters in the directions of X , Y and Z axes respectively. We can parameterized energy momentum tensor as follows

$$T^l_k = diag[1, -\Omega, -(\Omega + \alpha), -(\Omega + \beta)]\rho \tag{2.7}$$

For the purpose of simplification, we select $\Omega_x = \Omega$ and introduce skewness parameter α and β which are the deviations from Ω along Y and Z axes respectively.

By Saez-Ballester for metric equation (1) the field equations (2) – (4) using the equation (7) yield the following system of independent field equations (Einstein equations), namely

$$\frac{1}{P_1^2}\left(\frac{\ddot{P}_2}{P_2}+\frac{\ddot{P}_3}{P_3}-\frac{\dot{P}_1\dot{P}_2}{P_1P_2}+\frac{\dot{P}_2\dot{P}_3}{P_2P_3}-\frac{\dot{P}_1\dot{P}_3}{P_1P_3}\right)-\frac{\mu\phi^m\dot{\phi}^2}{2P_1^2}=\Omega\rho \tag{2.8}$$

$$\frac{1}{P_1^2}\left(\frac{\ddot{P}_1}{P_1}+\frac{\ddot{P}_3}{P_3}-\frac{\dot{P}_1^2}{P_1^2}\right)-\frac{\mu\phi^m\dot{\phi}^2}{2P_1^2}=(\Omega+\alpha)\rho \tag{2.9}$$

$$\frac{1}{P_1^2}\left(\frac{\ddot{P}_1}{P_1}+\frac{\ddot{P}_2}{P_2}-\frac{\dot{P}_1^2}{P_1^2}\right)-\frac{\mu\phi^m\dot{\phi}^2}{2P_1^2}=(\Omega+\beta)\rho \tag{2.10}$$

$$\frac{1}{P_1^2}\left(\frac{\dot{P}_1\dot{P}_2}{P_1P_2}+\frac{\dot{P}_2\dot{P}_3}{P_2P_3}+\frac{\dot{P}_1\dot{P}_3}{P_1P_3}\right)+\frac{\mu\phi^m\dot{\phi}^2}{2P_1^2}=-\rho \tag{2.11}$$

$$\ddot{\phi}+\left(\frac{\dot{P}_2}{P_2}+\frac{\dot{P}_3}{P_3}\right)\dot{\phi}+\frac{m\dot{\phi}^2}{2\phi}=0 \tag{2.12}$$

where dot overhead denotes the derivative with respect to cosmic time t.

To seek simplification, we can choose appropriate substitution $P_1 = P_2$. For cosmological model the spatial volume V and the average scale factor R are given by

$$V = P_1^2P_3 \tag{2.13}$$

$$R = (V)^{\frac{1}{3}} = (P_1^2P_3)^{\frac{1}{3}} \tag{2.14}$$

For Marder's Model we have the directional mean Hubble's parameter H as follows

$$H = \frac{1}{3}\left(H_x + H_y + H_z\right) \tag{2.15}$$

where $H_x = H_y = \frac{\dot{P}_1}{P_1}$, $H_z = \frac{\dot{P}_3}{P_3}$ are the directional mean Hubble's parameters along co-ordinate axes respectively.

The Mean Anisotropy parameter Δ_m is found to be

$$\Delta_m = \frac{1}{3}\sum_x\left(\frac{\Delta H_x}{H}\right)^2 \tag{2.16}$$

where
$\Delta H_x = H_x - H$; for x, y, z.

Also the expansion scalar Θ and the shear scalar σ^2 are as follows

$$\Theta = u^k_{,k} + u^l\Gamma^k_{lk} = \frac{3\dot{P}_1}{P_1}+\frac{\dot{P}_3}{P_3} \tag{2.17}$$

$$\sigma^2 = \frac{1}{2}\left(\sum_x^z H_x^2 - 3H^2\right) = \frac{1}{3}\left(\frac{\dot{P}_1}{P_1}-\frac{\dot{P}_3}{P_3}\right)^2 \tag{2.18}$$

3 Solution of Field Equation

The field equations (8) to (12) are the system of five independent equations in seven variables, namely $P_1 = P_2$, P_3 , Ω , ϕ , ρ , α and β. Therefore two more constraints relating to these variables are required to evaluate their explicit solution values for system of equations.

We assume the result stated by Bermann (21) which is also known as the special law of variation of mean Hubble's parameter which results in constant of deceleration parameter. The constant of deceleration parameter of universe is denoted by q and is given by

$$q = -\frac{R\ddot{R}}{(\dot{R})^2} \tag{3.1}$$

which leads to the solution

$$R = (c_1 t + c_0)^{\frac{1}{1+q}} \tag{3.2}$$

where $c_1 \neq 0$ and c_0 are integration constants, $q \neq -1$ and index $\frac{1}{1+q}$ is non-negative.

But from the physical condition the shear scalar σ is directly proportional to expansion scalar Θ , using this one we may write

$$P_1 = P_3^n \tag{3.3}$$

where index n is nonzero constant.

On solving the Saez-Ballester field equations from (8) to (12) using (14), (21) and (22) we determined the coefficients $P_1 = P_2$, P_3 in metric equation as follows

$$P_1 = P_2 = (c_1 t + c_0)^{\frac{3n}{(1+q)(2n+1)}} \tag{3.4}$$

$$P_3 = (c_1 t + c_0)^{\frac{3}{(1+q)(2n+1)}} \tag{3.5}$$

Let we take $(c_1 t + c_0) = T$, therefore coefficients P_1, P_2, P_3 become as follow

$$P_1 = P_2 = T^{\frac{3n}{(1+q)(2n+1)}} \tag{3.6}$$

$$P_3 = T^{\frac{3}{(1+q)(2n+1)}} \tag{3.7}$$

With the help of appropriate choice of co-ordinates of system as well as constants in metric equation (1) from above coefficients (P_1, P_2, P_3) values may be written as follows

$$ds^2 = T^{\frac{6n}{(q+1)(2n+1)}} \left(dx^2 + dy^2 - \frac{1}{c_1} dT^2 \right) + T^{\frac{6}{(q+1)(2n+1)}} dz^2 \tag{3.8}$$

4 Some Physical Parameters of the Model

From above calculations at initial cosmic time the equation (24) which represents Marder's Dark Energy Model is nonsingular. The equations given below gives some physical and kinematical parameter of the Marder's Model.

The Spatial Volume V of model (24) is found to be

$$V = T^{\frac{3}{1+q}} \tag{4.1}$$

By the defination of mean Hubble's parameter H is determined as

$$H = \Big((1+q)T \Big)^{-1} \tag{4.2}$$

The Mean Anisotropy Parameter Δ_m of model is calculated as

$$\Delta_m = \frac{2(n-1)^2}{(2n+1)^2} \tag{4.3}$$

The expansion scalar Θ and the shear scalar σ^2 of model are found to be

$$\Theta = \frac{3(3n+1)}{(q+1)(2n+1)T} \tag{4.4}$$

$$\sigma^2 = \frac{3(n-1)^2}{(q+1)^2(2n+1)^2T^2} \tag{4.5}$$

As the shear scalar σ is directly proportional to the expansion scalar Θ, so we get

$$\frac{\sigma^2}{\Theta^2} = \frac{(n-1)^2}{3(3n+1)^2} \tag{4.6}$$

which is non-negative for $n > 1$.

Solution of equation (12) using the equations (22) and (23) is obtained below

$$\phi = \left\{ \frac{c(n+2)}{2} \left(\frac{(q+1)(2n+1)}{(q+1)(2n+1)-3(n+1)} \right) T^{\left(1-\frac{3(n+1)}{(q+1)(2n+1)}\right)} \right\}^{\frac{2}{n+2}} \tag{4.7}$$

The Energy Density ρ of model is calculated from equation (11) using equations (22) and (23) as follows

$$\rho = -\frac{T^{\frac{-6n}{(q+1)(2n+1)}}}{(q+1)^2(2n+1)^2} \left(\frac{[9n(n+2)]}{T^2} + \frac{\omega}{2} \frac{(\frac{c(q+1)(2n+1)(n+1)}{2})^{2(2+m)}}{[(q+1)(2n+1)-3(n+1)]^{2(m+1)}} T^{\frac{2m}{n+m}-\frac{6(n+1)m}{(q+1)(2n+1)(n+2)}} \right) \tag{4.8}$$

From equation (8) E_0S parameter Ω of model using the equations (22), (23) and (31) is found to be

$$\Omega = \frac{1}{\rho} \left[\frac{T^{\frac{-6n}{(q+1)(2n+1)}}}{(q+1)^2(2n+1)^2} \left(\frac{[9-3(q+1)(2n+1)(n+1)]}{T^2} - \frac{\omega}{2} \times \frac{(\frac{c(q+1)(2n+1)(n+1)}{2})^{2(2+m)}}{[(q+1)(2n+1)-3(n+1)]^{2(m+1)}} T^{\frac{2m}{n+m}-\frac{6(n+1)m}{(q+1)(2n+1)(n+2)}} \right) \right] \tag{4.9}$$

For the model, the skewness parameter get vanish that is

$$\alpha = 0 \tag{4.10}$$

From equation (10) the skewness parameter β using the equations (22) , (23) and (32) is determined as

$$\beta = \frac{1}{\rho} \frac{T^{\frac{-6n}{(q+1)(2n+1)}-2}}{(q+1)^2(2n+1)^2} \left[9(n^2-1) - 3(q+1)(2n+1)(3n+1) \right] \tag{4.11}$$

5 Discussion

From above results it can be illustrated as the spatial volume goes on increasing as cosmic time T goes on increasing which leads to the fact that our universe expands from past infinite time for positive value of index $\frac{3}{q+1}$. Hubble's parameter H, the scalar of expansion Θ, Shear scalar σ^2, energy density ρ, EoS parameter Ω and skewness parameter β diverge for initial time $T = 0$, while they vanish for large scale of time T. As the mean anisotropic parameter is constant for model, it is uniform through organic evolution of universe. One thing is resemble between the parameter Ω , ρ , β that they are functions of cosmic time T, while $\alpha = 0$ for the model. For the model the scalar field ϕ vanishes for time $T = 0$ where as it becomes infinitely for large scale values of time T.

6 Conclusion

In this paper, we have studied Marder's Cosmological model for dark energy in Saez Ballester Scalar Tensor theory of general relativity. In obtained model, some physical and kinematical properties are discussed. Among which Hubble's parameter, EoS parameter of dark energy are dependent on cosmic time T. Skewness parameter get vanished for the model. In frame work of Saez Ballester Scalar Tensor Theory, Marder's model is expanding as well as free from initial singularity i.e. at $T = 0$, where as anisotropy parameter remains same in organic evolution of universe and is independent of cosmic time T. In present paper, spaital volume goes on increasing as time increases which conclude that our universe expands finitely from infinite past.

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