Article

Bianchi Type-VI_0 Cosmological Model with Variable Deceleration Parameter

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Abstract

Bianch type-VI_0 cosmological model have been investigated with variable deceleration parameter(q)(R.K. Tiwari *et at.*, African Review of Physics**10**,395-402,2015)i.e, $q = \alpha + \beta H$, which yields scale factor $a = e^{\frac{1}{\beta}\sqrt{2\beta t+k}}$.The model has non-singular origin and expands exponentially with cosmic time t till late times. We observe that the model is early decelerating phase and late time accelerating phase. The equation of state parameter(ω) found to be time dependent and its range for derived model is in agreement with the recent observations.The jerk parameter (j) is found to be good agreement with the recent observations. We also discussed statefinder parameters $\{r, s\}$ which predicts that the universe in the model originates from Einstein static era $\{r \to \infty, s \to -\infty\}$ to ΛCDM model $\{r \to 1, s \to 0\}$. The physical and geometrical properties of cosmological model is also discussed.

Keywords: Bianchi Type VI_0, EoS parameter, variable deceleration parameter, jerk parameter, statefinder parameters.

1 Introduction

Several observational studies like distance Supernovae (SNeIa) (Perlmutter *et al.*[1-3], Riess *et al.*[4], Granavich *et al.*[5],Schmidt *et al.*[6], Clocchiatti *et al.*[7], Tonry *et al.*[8]),large scale structure (LSS) Spergel *et al.*[9], Tegmark *et al.*[10]), fluctuation of cosmic Microwave Background Radiation (CMBR) (de Bernardis *et al.*[11],Hanany *et al.*[12]), wilkinson microwave anisotropy probe (WMAP) (Bennettt, *et al.*[13]),Sloan digital skey survey (SDSS) (Adelman-Mc Cartny *et al.* [14], Seljak *et al.*[15]) and Chandra x-ray observatory (Allen *et al.*[16]) have prove that our universe is going through accelerated expansion.To explain the cosmic positive acceleration, mysterious dark energy has been proposed. This dark energy has been defined as unknown form of energy could not have been detected directly and it does not cluster like ordinary matter. Even the fact is that the three fourth of energy density of universe has been fathered by this dark energy which has very prominent role in accelerated expansion of the universe. There are many models proposed by immense authors(Amirhashchi *et al.*[17], Pradhan *et al.*[18], Saha *et al.*[19], Saha [20], Yadav and Saha[21]) with an effort to explain the dark energy such as quintessence, Chaplygin gas, modified Chaplygin gas and holographic dark energy etc.

High-precision measurements of expansion of the universe are required to understand that how the expansion rate changes over time. The evolution of the cosmological equation of state, in general relativity.The equation of state parameter (ω),which is consider as an important quantity in describing the dynamics of the universe which is the ratio of pressure (p) and energy density (ρ) is given by $\omega = \frac{p}{\rho}$, which is not necessarily constant.The present data seem to slightly favour an evolving dark energy with EoS $\omega < -1$ around the present epoch and $\omega > -1$ in the near past.Obviously, ω can not cross -1 for quintessence or phantom alone. Recent observational data limits the equation of state parameter (ω)as $-1.67 < \omega < -0.62$ while a combination of data from observations of SNeIa (Knopet *et al.*[22],CNM anisotropy and galaxy clustering statistics(Tegmark *et al.*[23]) gives the limit on ω as $-1.33 < \omega < -0.79$. As a matter of fact, if ω

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would be equal -1 (ΛCDM) ,which is mathematically equivalent to the cosmological constant (Λ), upper than -1 (quitessence dark energy)(Steinhardt *et al.*[24]) less than -1 (phantom dark energy)(Caldwell [25]).The recent results in 2009, obtained after a combination of cosmological datasets coming from CMB anisotropies, luminosity distances of high red-shift type I_a supernovae and galaxy clustering , constrain the dark energy EoS to $-1,44 < \omega < -0.92$ at 68% confidence of level (Hinshaw *et al.*[26],Komatsu *et al*[27]). However, it is not at all obligatory to evidence in making a distinction between constant and variable ω,usually the equation of state parameter (ω) is consider as a constant(Kujat *et al.*[28], Bartelmann *et al.*[29]) with phase wise value -1,0,$-\frac{1}{3}$ and 1 for vacuum fluid , dust fluid , radiation and stiff dominated universe , respectively. But in general ,ω is function of time or red-shift (Jimenez[30], Das *et al.*[31]).
We organize the paper as Sec. 2 contains Metric and field equations, Sec. 3 gives Solutions of field equations. Sec. 4 provides Physical and geometrical properties of the model ,sec,5 gives the jurk parameter , Statefinder parameters and finally Sec. 6 supplies Conclusion.

2 The Metric and Field Equations

We consider Bianchi type-VI_0 space time in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{2x}dy^2 + C^2e^{-2x}dz^2 \tag{2.1}$$

where A, B and C are functions of cosmic time t. This ensure that the model is totally anisotropic and spatially homogeneous.
The generalization of EoS parameter of perfect fluid to determine the diagonal form of the energy momentum tensor .

$$T_{ij} = diag[T_{00}, T_{11}, T_{22}, T_{33}] \tag{2.2}$$

Allowing for anisotropy in the pressure of the fluid,and thus in its EoS parameter, gives rise to new possibilities for the evolution of the energy source. To see this, we first parametrize the energy momentum tensor given in (2.2) as follows:

$$T_{ij} = diag[\rho, -p_x, -p_y, -p_z] = diag[1, -\omega_x, -\omega_y, -\omega_z]\rho$$

$$T_{ij} = diag[1, -\omega, -(\omega + \delta), -(\omega + \gamma)]\rho \tag{2.3}$$

where ρ is energy density, p_x, p_y, p_z are the directional pressure, ω_x, ω_y and ω_z are the directional EoS parameters along x,y,z axis respectively, ω is the deviation free EoS parameter,δ and γ are the skewness parameters, and ω,δ and γ are not necessarily constants and might be function of the cosmic time t.
The Einstein's field equations are

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij} \tag{2.4}$$

we choose $8\pi G = 1$, where R_{ij}, R, g_{ij} and T_{ij} are Ricci tensor, Ricci scalar, metric tensor and energy momentum tensor respectively.
The field equations(2.4), with (2.3) for line element(2.1) give rise to

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{1}{A^2} = -\omega\rho \tag{2.5}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} - \frac{1}{A^2} = -(\omega + \delta)\rho \tag{2.6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = -(\omega + \gamma)\rho \tag{2.7}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{C}\dot{A}}{CA}-\frac{1}{A^2}=\rho \tag{2.8}$$

$$\frac{\dot{C}}{C}-\frac{\dot{B}}{B}=0 \tag{2.9}$$

We define average scale factor a for Bianchi type-VI_0 space time as

$$a=(ABC)^{\frac{1}{3}} \tag{2.10}$$

Also, Hubble's parameter(H), expansion scalar(θ), shear scalar (σ), Anisotropic parameter($\overline{A}$), and deceleration parameter(q) as

$$H=\frac{\dot{a}}{a} \tag{2.11}$$

$$\theta=3H \tag{2.12}$$

$$\sigma^2=\frac{1}{2}(\Sigma_{i=1}^3 H_i^2-\frac{1}{3}\theta^2) \tag{2.13}$$

$$\overline{A}=\frac{1}{3}\Sigma_{i=1}^3(\frac{\Delta H_i}{H})^2=\frac{2\sigma^2}{3H^2} \tag{2.14}$$

$$q=-\frac{a\ddot{a}}{\dot{a}^2}=\frac{d}{dt}(\frac{1}{H})-1 \tag{2.15}$$

where $\Delta H_i = H_i - H$ (i=x,y,z) represents the directional Hubble's parameters.

3 Solutions of the field equations

Integrating equation(2.9), we have

$$C=mB \tag{3.1}$$

where m is constant.
Putting the value of equation (3.1) in equation(2.7) and subtract the result from equation(2.6), we obtain that the skewness parameters along y and z axis are equal i.e,$\delta=\gamma$.
Equations(2.5)-(2.9) are reduced to

$$2\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}+\frac{1}{A^2}=-\omega\rho \tag{3.2}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{1}{A^2}=-(\omega+\gamma)\rho \tag{3.3}$$

$$2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}-\frac{1}{A^2}=\rho \tag{3.4}$$

Equations (3.2)-(3.4) are three equations with five unknown namely, A, B, ρ, ω, γ. Now to get the determinate solutions, we require two more conditions.
First, we assume that the expansion scalar (θ) is proportional to shear scalar (σ), and using (3.1), we get

$$\frac{1}{\sqrt{3}}\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)=\alpha_0\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right) \tag{3.5}$$

where α_0 is constant.,which yields to

$$\frac{\dot{A}}{A}=m_1\frac{\dot{B}}{B} \tag{3.6}$$

where $m_1 = \frac{2\alpha_0\sqrt{3}+1}{1-\alpha_0\sqrt{3}}$ From equation (3.6), we have

$$A = c_1 B^{m_1} \tag{3.7}$$

where c_1 is constant, without loss of generality ,we take $c_1 = 1$, we have

$$A = B^{m_1} \tag{3.8}$$

This condition is explained to Thorne[32], the observations of the velocity red-shift relation for extagalactic source says that Hubble expansion of the universe is isotropic at that time within$\simeq$ 30 present Kristian and Sachs [33], Kantowski and Sachs[34]. To put more precisely, red-shift studies the limit

$$\frac{\sigma}{H} \leq 0.3$$

on the ratio of the shear tensor (σ) and Hubble parameter (H) in the nearest of our galaxy today.
Second, we assume that the deceleration parameter (q) is a linear function of Hubble parameter(H)(R.K. Tiwari,*et al.*[35-36]) i.e,

$$q = \alpha + \beta H \tag{3.9}$$

where α and β are constants.If $\ddot{a}$ is positive and in this case deceleration parameter (q) will be negative which shows that expansion universe is said to accelerating. Also recent observations have suggested that the present universe is accelerating and value of deceleration parameter (q) lies between 0 to -1.

For mathematical simplicity taking $\alpha = -1$ and then from equation (3.9), we obtain

$$a = e^{\frac{1}{\beta}\sqrt{2\beta t+k}} \tag{3.10}$$

where k is constant. This shows that at $t = 0$ the scale factor a tends to constant, hence the model is non-singular origin.
Using (3.1),(3.8) and (3.10), we have

$$A = k_3 e^{\frac{3m_1}{\beta(m_1+2)}\sqrt{2\beta t+k}} \tag{3.11}$$

$$B = k_1 e^{\frac{3}{\beta(m_1+2)}\sqrt{2\beta t+k}} \tag{3.12}$$

$$C = k_2 e^{\frac{3}{\beta(m_1+2)}\sqrt{2\beta t+k}} \tag{3.13}$$

where $k_1 = m_1^{\frac{-3}{m_1+2}}$, $k_2 = m_1 k_1$, $k_3 = k_1^{m_1}$
Hence the metric (2.1) takes the form

$$ds^2 = -dt^2 + k_3^2 e^{\frac{6m_1}{\beta(m_1+2)}\sqrt{2\beta t+k}} dx^2 + k_1^2 e^{\frac{6}{\beta(m_1+2)}\sqrt{2\beta t+k}} e^{2x} dy^2 + k_2^2 e^{\frac{6}{\beta(m_1+2)}\sqrt{2\beta t+k}} e^{-2x} dz^2 \tag{3.14}$$

4 Physical and geometrical properties of the model

The spatial volume (V), Hubble's parameter(H), expansion scalar (θ), shear scalar (σ), anisotropic parameter ($\overline{A}$) and deceleration parameter (q) are given as

$$V = e^{\frac{3}{\beta}\sqrt{2\beta t+k}} \tag{4.1}$$

$$H = \frac{1}{\sqrt{2\beta t+k}} \tag{4.2}$$

$$\theta = \frac{3}{\sqrt{2\beta t + k}} \tag{4.3}$$

$$\sigma^2 = 3\left(\frac{m_1 - 1}{m_1 + 2}\right)^2 \frac{1}{(2\beta t + k)} \tag{4.4}$$

$$\overline{A} = 2\left(\frac{m_1 - 1}{m_1 + 2}\right)^2 \tag{4.5}$$

$$q = -1 + \frac{\beta}{\sqrt{2\beta t + k}} \tag{4.6}$$

We observe that at t=0 the scale factor is constant, hence the model has no initial singularity at origin .At $t = \frac{-k}{2\beta} = t_1$ the expansion scalar θ ,Hubble's parameter H and shear scalar σ are infinite,which shows that the universe starts evolving with zero volume at $t = t_1$ with an infinite rate of expansion. The anisotropic parameter is constant hence the model posses anisotropic behavior in whole span of evolution of the universe.
The deceleration parameter q is positive for $t < \frac{\beta^2 - k}{2\beta}$, which indicates the universe is decelerating phase of expansion and q is negative for $t > \frac{\beta^2 - k}{2\beta}$ which indicates the universe is accelerating phase of expansion and is in agreement with recent observations.
From equations (3.11)-(3.13)and (3.4), we have

$$\rho = \frac{9(2m_1 + 1)}{(m_1 + 2)^2(2\beta t + k)} - k_4 e^{\frac{-6m_1}{\beta(m_1+2)}\sqrt{2\beta t + k}} \tag{4.7}$$

where $k_4 = \frac{1}{k_3}$

$$\omega = -\frac{\frac{3}{(m_1+2)}\left\{\frac{-2\beta}{(2\beta t+k)^{\frac{3}{2}}} + \frac{9}{(m_1+2)(2\beta t+k)}\right\} + k_4 e^{\frac{-6m_1}{\beta(m_1+2)}\sqrt{2\beta t+k}}}{\frac{9(2m_1+1)}{(m_1+2)^2(2\beta t+k)} - k_4 e^{\frac{-6m_1}{\beta(m_1+2)}\sqrt{2\beta t+k}}} \tag{4.8}$$

$$\gamma = -\frac{\frac{3\beta(1-m_1)}{(m_1+2)(2\beta t+k)^{\frac{3}{2}}} + \frac{9(m_1^2+m_1-2)}{(m_1+2)^2(2\beta t+k)} - 2k_4 e^{\frac{-6m_1}{\beta(m_1+2)}\sqrt{2\beta t+k}}}{\frac{9(2m_1+1)}{(m_1+2)^2(2\beta t+k)} - k_4 e^{\frac{-6m_1}{\beta(m_1+2)}\sqrt{2\beta t+k}}} \tag{4.9}$$

From equation(4.8), we see that the equation of state parameter (ω) is time dependent. It can be function of re-shift$z$ or scale factor a as well. The equation of state parameter (ω) is linear function of red-shift(z)

$$\omega(z) = \omega_0 + \omega' z \tag{4.10}$$

with $w' = \left(\frac{d\omega}{dz}\right)_{z=0}$ (Huterer and Turner [37], Weller and Albrecht [38]) or non linear as

$$\omega(z) = \omega_0 + \frac{\omega_1 z}{1 + z} \tag{4.11}$$

(Chavalliver and Polarski [39], Linder [40])
The equation of state parameter (ω) is the function of scale factor a

$$\omega(a) = \omega_0 + \omega_a(1 - a) \tag{4.12}$$

where $(\omega_0)$is the present value $a = 1$ and ω_a is the measure of the time variation.

5 The Jerk Parameter(j) and Statefinder Parameters$\{r, s\}$

The dimensionless jerk parameter(j) third derivative of the scale factor with respect to cosmic time t(Chiba and Nakamura[41], Blandford *et al.* [42], Visser [43], Sahni[44]) and provides a perfect diagnosis of how much a dark energy model is closed to ΛCDM dynamics. A deceleration to acceleration transition occurs for models with a positive value of j_0 and negative value of q_0. Flat ΛCDM models have a constant jerk j=1. The jerk parameter (j) is defined as

$$j(t) = \frac{\dddot{a}}{aH^3} = \left(\frac{a^2H^2}{2H^2}\right)^{''} \tag{5.1}$$

over dot and primes denote derivatives with respect to cosmic time t and scale factor respectively.
The jerk parameter (j) appears in the fourth term of a Taylor expansion of the scale factor around a_0

$$\frac{a(t)}{a_0} = 1 + H_0(t-t_0) - \frac{1}{2}q_0H_0^2(t-t_0)^2 + \frac{1}{6}j_0H_0^3(t-t_0)^3 + O[(t-t_0)^4] \tag{5.2}$$

Equation (5.1) can be written as

$$j(t) = q + 2q^2 - \frac{\dot{q}}{H} \tag{5.3}$$

From equations (4.2),(4.6) and (5.3), we have

$$j(t) = 1 - \frac{3\beta}{\sqrt{2\beta t + k}} + \frac{3\beta^2}{2\beta t + k} \tag{5.4}$$

For a flat ΛCDM model, jerk parameter (j) has the value j=1.
Sahni *et al.*[45] and Alam *et al.*[46] introduced the statefinder parameters $\{r, s\}$, defined as

$$r = \frac{\dddot{a}}{aH^2} = 1 + 3\frac{\dot{H}}{H} + \frac{\ddot{H}}{H^3} \tag{5.5}$$

and

$$s = \frac{r-1}{3\left(q - \frac{1}{2}\right)} \tag{5.6}$$

For a ΛCDM model, statefinder parameters have the value $\{r, s\} = \{1, 0\}$. For Einstein era we have $\{r, s\} = \{\infty, -\infty\}$.
Now equations (4.2),(4.6) and (5.5),(5.6) result into

$$r = 1 - \frac{3\beta(1-\beta)}{(2\beta t + k)} \tag{5.7}$$

and

$$s = -\frac{3\beta(1-\beta)}{(2\beta t + k)[3\beta - 3\sqrt{2\beta t + k}]} \tag{5.8}$$

We observe that as $t \to t_1$,$\{r, s\} \to \{\infty, -\infty\}$ and $t \to \infty$, $\{r, s\} \to \{1, 0\}$ which depicts that the universe in the model starts from Einstein static era goes to ΛCDM model. This is in agreement with recent observation[47] , which makes our model observationally acceptable.

6 Conclusion

In this paper, we have studied, a new class of anisotropic Bianchi type VI_0 cosmological model with variable equation of state parameter(ω) by using the condition for time dependent deceleration parameter

q. We have obtained the universe starts from non-singular state and expand exponentially with cosmic time t till late times. We show that the deceleration parameter q , initial decelerating phase to present accelerating one.The equation of state parameter (ω) is time dependent and its value in the model lies in the present observational limits. Finally , we notice that the jerk parameter(j) in our model is also found to be in good agreement with recent observations.

Received March 30, 2016; Accepted April 04, 2016

References

[1] S.Perlmutter,*et al.*, Astrophys.J.**483**,565(1997)

[2] S.Perlmutter,*et al.*,Nature 391,51(1998)

[3] S.Perlmutter, G.Aldering,G. Goldhaber, *et al.*,Astrophys.J.**517**,565(1997)

[4] A.G.Riess, *et al.*,Astron. j. **116**,1009(1998)

[5] P.M. Garnavich, *et al.*,Astrophys. J. **493**,L53(1998)

[6] B.P.Schmidt, *et al.*,Astron.J.**507**,46(1998)

[7] A.Clocchiatti, *et al.*,Astron.J.**642**,1(2006)

[8] J.L. Tonry, *et al.*,Astron.J.**594**,1(2003)

[9] D.N. Spergel, *et al.*, Astrophys.J.Suppl.Ser.**148**,175(2003)

[10] M. Tengmark, *et al.*, Astrophys.J.**606**,702(2004b)

[11] P. de Bernardis, *et al.*, Nature **404**,955(2000)

[12] S. Hanany, *et al.*, Astron.J.**545**,L53(2000)

[13] C.L.Bennett, *et al.*,Astrophys.J.Suppl.Ser.**148**,1(2003)

[14] J.K. Adelman-McCarthy, *et al.*,Astrophys.J.Suppl.Ser.**162**,38(2006)

[15] U. Seljek, *et al.*, Phys. Rev. D**71**,103515(2005)

[16] S.W.Allen,*et al.*, Mon.Nor.R. Astron. Soc.**353**,457(2004)

[17] H. Amirhashchi, A. Pradhan, B. Saha, Astrophys.Space Sci.**333**,295(2011a)

[18] A. Pradhan, H. Amirhashchi, B. Saha, Int. J. Theor. Phys.**50**,2923(2011a)

[19] B. Saha,H. Amirhashchi, A. Pradhan, Astrophys.Space Sci.**342**,257(2012)

[20] B. Saha, Phys.Part.Nucl. **45**,349(2013b)

[21] A. K. Yadav, B. Saha, Astrophys.Space Sci.**337**,759(2012)

[22] R. K. Knop, *et al.*, Astrophys.J. **598**,102(2003)

[23] M. Tegmark, *et al.*,Astrophys.J.**606**, 702(2004a)

[24] P. J. Steinhardt. L. M. Wang, I. Zlatey, Phys.Rev. D**59**,023504(1999)

[25] R.R. Caldwell, Phys. Lett. B**545**,23(2002)

[26] G.Hinshaw, *et al.*,Astrophys.J.Suppl.Ser.**170**,288(2007)

[27] E. Komatsu, *et al.*,Astrophys. J. Suppl. Ser. **180**,330(2009)

[28] J. Kujut, *et al.*, Astrophys. J. **572**,1,(2002)

[29] M. Bartelmann, *et al.*, New Astron. Rev. **49**,199(2005)

[30] R. Jimenez,New Astron. Rev.**47**,761(2006b)

[31] A. Das, *et al.*, Phys. Rev.D**72**,043528(2005)

[32] K. S. Thorne, Astrophys. J.**148**,51(1967)

[33] J. Kristian, R. K. Sachs.,Astrophys.J. **143**,379(1966)

[34] R. Kantowski, R. K. Sachs., J. Math. Phys.**7**,433,(1966)

[35] R.K. Tiwari, Rameshwar Singh and B. K. Shukla, Afri. Rev.Phys.**10**,395(2015)

[36] R.K. Tiwari and B. K. Shukla:Prespacetime J.**7**,400(2016)

[37] D. Huterer, M. S. Turner, Phys. Rev. D**64**, 123527(2001)

[38] J. Weller, A. Albrecht,Phys. Rev. D**65**,103512(2002)

[39] M. Chevallier, D. Polarski,Int. J. Mod. Phys. D**10**,213(2001)

[40] A. D. Linde, Phys.Lett. B**108**,389(1982)

[41] T. Chiba, T. Nakamura,Prog. Theor. Phys.**100**,1077(1998)

[42] R. D. Blandford, M. Amin, E. A. Baltz, Phys. Lett. B**535**,329(2004)

[43] M. Visser, Class. Quantum Gravity **21**, 2603(2004)

[44] V. Sahni, arXiv:astro-ph/0211084(2002)

[45] V. Sahni *et al.*, JETP Lett. **77**, 201 (2003)

[46] U. Alam *et al.*, Mon. Not. R. Astron. Soc. **344**, 1057 (2003)

[47] Nasr Ahmad, Anirudh Pradhan, arXiv: 1303.3000v1 [Phys. Gen-ph] (2013)