Article

Wünsch's Vectors & Lanczos Potential

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Abstract

For arbitrary spacetimes with Petrov types O, N, III, and D (empty), we show that the Newman-Penrose (NP) components of Lanczos potential are the NP projections of the Wünsch's vectors.

Keywords: Lanczos spin-tensor, Wünsch vector, conformal tensor.

1. Introduction

Here we shall employ the quantities and conventions of [1-3]. The Lanczos potential [4] is a generator for the Weyl tensor:

$$S_{\mu\nu\alpha\beta} = f(x^r)\{S_{\mu\nu\alpha;\beta} - S_{\mu\nu\beta;\alpha} + S_{\alpha\beta\mu;\nu} - S_{\alpha\beta\nu;\mu} + \frac{1}{2}[(H_{\mu\beta} + H_{\beta\mu})g_{\nu\alpha} - (H_{\mu\alpha} + H_{\alpha\mu})g_{\nu\beta} + (H_{\nu\alpha} + H_{\alpha\nu})g_{\mu\beta} - (H_{\nu\beta} + H_{\beta\nu})g_{\mu\alpha}]\} \quad (1)$$

where $f(x^r)$ is a complex scalar, and:

$$H_{\mu\nu} \equiv S_{\mu\ \ \nu;\alpha}^{\ \alpha} = K_{\mu\ \ \nu;\alpha}^{\ \alpha} + i\ {}^*K_{\mu\ \ \nu;\alpha}^{\ \alpha} = K_{\mu\nu} + i\ {}^*K_{\mu\nu}\ . \quad (2)$$

On the other hand, we have the Wünsch vectors [5, 6]:

$$L_\alpha = \nu\, l_\alpha + \pi\, n_\alpha - \lambda\, m_\alpha - \mu\, \bar{m}_\alpha\ , \quad N_\beta = -\tau\, l_\beta - \kappa\, n_\beta + \rho\, m_\beta + \sigma\, \bar{m}_\beta\ , \quad (3)$$

in terms of the Newman-Penrose (NP) null tetrad [7, 8].

In Sec. 2 are considered the Petrov types N, III and O [9] in the canonical tetrad [8, 10], and we exhibit that the NP components of the corresponding Lanczos spintensor are the projections of (3) onto the null tetrad. In Sec. 3 occurs a similar situation for type D empty spacetimes.

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2. Petrov Types N, III & O

In [11] was resolved (1) with $f = 1$ for arbitrary 4-spaces with the following Petrov types in the canonical null tetrad [10]:

a). **N**: $\psi_4 \neq 0, \quad \psi_r = 0, \ r \neq 4$

$$2\Omega_0 = \kappa\,, \qquad 2\Omega_3 = -\lambda\,, \qquad 2\Omega_4 = \sigma\,, \qquad 2\Omega_7 = -\nu\,,$$
$$6\Omega_1 = \rho\,, \qquad 6\Omega_2 = -\pi\,, \qquad 6\Omega_5 = \tau\,, \qquad 6\Omega_6 = -\mu\,, \tag{4}$$

b). **III**: $\psi_3 \neq 0, \quad \psi_r = 0, \ r \neq 3$

$$\Omega_0 = \kappa\,, \qquad \Omega_3 = -\lambda\,, \qquad \Omega_4 = \sigma\,, \qquad \Omega_7 = -\nu\,,$$
$$3\Omega_1 = \rho\,, \qquad 3\Omega_2 = -\pi\,, \qquad 3\Omega_5 = \tau\,, \qquad 3\Omega_6 = -\mu\,, \tag{5}$$

or in terms of the Wünsch's vectors:

$$2c\Omega_0 = -N_\alpha l^\alpha\,, \qquad 6c\Omega_5 = -N_\alpha n^\alpha\,, \qquad 2c\Omega_4 = -N_\alpha m^\alpha\,, \qquad 6c\Omega_1 = -N_\alpha \overline{m}^\alpha\,,$$
$$6c\Omega_2 = -L_\beta l^\beta\,, \qquad 2c\Omega_7 = -L_\beta n^\beta\,, \qquad 6c\Omega_6 = -L_\beta m^\beta\,, \qquad 2c\Omega_3 = -L_\beta \overline{m}^\beta\,, \tag{6}$$

where $c = 1$ and $\frac{1}{2}$ for the types N and III, respectively.

Hence the NP components $\Omega_r\,,\ r = 0, \dots, 7$ are the projections of (3) onto the canonical null tetrad $(l^\alpha, n^\alpha, m^\alpha, \overline{m}^\alpha)$. Type O is included as particular case of the types N (with $\psi_4 = 0$) or III (when $\psi_3 = 0$).

3. Type D Vacuum Spacetime

In [12] we have solutions of (1) for $f(x^r) = \psi_2^{2/3}$ when R_4 is empty and type D, in the canonical tetrad such that $\psi_2 \neq 0,\ \psi_r = 0,\ r \neq 2$:

c). $$\Omega_1 = \rho\, \psi_2^{-2/3}\,, \qquad \Omega_5 = \tau\, \psi_2^{-2/3}\,, \qquad \Omega_a = 0, \quad a \neq 1, 5\,, \tag{7}$$

that is:

$$\Omega_1 = -\psi_2^{-2/3} N_\alpha \overline{m}^\alpha\,, \qquad \Omega_5 = -\psi_2^{-2/3} N_\alpha n^\alpha\,. \tag{8}$$

d). $$\Omega_2 = \pi\, \psi_2^{-2/3}\,, \qquad \Omega_6 = \mu\, \psi_2^{-2/3}\,, \qquad \Omega_b = 0, \quad b \neq 2, 6\,, \tag{9}$$

therefore:

$$\Omega_2 = \psi_2^{-2/3} L_\beta l^\beta \,, \quad \Omega_6 = \psi_2^{-2/3} L_\beta m^\beta \,, \tag{10}$$

then, again the expressions (8) and (10) give the Ω_r as the NP projections of the Wünsch's vectors.

To determine Ω_a for arbitrary spacetimes with Petrov types D, II and I, is an open problem, whose solution could be suggested by the NP components of (3).

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