Article

Energy Balance Method for Solving $u^{1/3}$ Force Nonlinear Oscillator

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ABSTRACT

This paper applies He's energy balance method (EBM) to a nonlinear oscillator with fractional potential $u^{1/3}$. The results show that the method is effective and convenient without the requirement of any linearization or small perturbation. The energy balance method can lead to adequately accurate solutions for nonlinear oscillators.

Keywords: Energy balance method, oscillators.

1. Introduction

Energy- balance method (EBM) is one of the well- known methods to solve nonlinear equations. This method was first established by He (He, 2002). EBM was applied many nonlinear oscillators, such as periodic solutions for some strongly nonlinear oscillations (Zhang, 2009), a mass attached to a stretched elastic wire (Jamshidi and Ganji, 2009), nonlinear vibrating equations (Mehdipour et al., 2010), nonlinear oscillators with u^n force (Akbarzade et al., 2008), Van der Pol dampet nonlinear oscillators (Ganji et al., 2010), discontinuities for nonlinear oscillators (Afrouzi et al., 2009; Hu and Tang, 2005), etc.

2. Recent progress

Recently, much researchers have been studying on the oscillations of nonlinear, one dimen- sional systems that the linear terms correspond to the harmonic oscillator differential equations

$$u'' + u = 0. \qquad (2.1)$$

However, dynamical systems for this condition doesn't hold. For this reason, Mickens has studied the equation (Mickens, 2001)

$$u'' + u^3 = \varepsilon f(\lambda, \lambda'), \qquad (2.2)$$

where $f(u,t)$ can be a rational function of u, t and ε is a small positive parameter (Mickens and Ronald, 1996). Consider systems that can be modelled by equations of motion where the potential takes the form

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$$V(u) = V_0 u^{(2n+2)/(2n+1)}, \tag{2.3}$$

where $n = 1,2,3, \ldots$ and V_0 is a positive constant. The force derived from equation (2.3) is

$$f(u) = -\left(\frac{2n+2}{2n+1}\right) V_0 u^{1/(2n+1)} \tag{2.4}$$

that $V(u), f(u)$ are even and odd functions of u respectively

$$V(-u) = V(u), f(-u) = -f(u). \tag{2.5}$$

In the following calculations, the value of n will be taken to be $n = 1$. The summary will present the results for arbitrary positive n. The central result is that a system under the influence of a force such as equation (2.4), has only periodic solutions. Further, the method harmonic balance will be used to calculate an analytical approximation to these periodic solutions. A particle of mass$M$, acted on by the force of equation (2.4), has the equation of motion

$$M\frac{d^2u}{dt^2} + \left(\frac{2n+2}{2n+1}\right) V_0 u^{1/2n+1} = 0. \tag{2.6}$$

By a proper change of both the dependent and independent variables, this equation can be transformed to the dimensionless form as

$$\frac{d^2\overline{u}}{d\overline{t}^2} + \overline{u}^{1/(2n+1)} = 0. \tag{2.7}$$

In this, the "bars" will be dropped to give

$$\frac{d^2u}{dt^2} + u^{1/(2n+1)} = 0. \tag{2.8}$$

For $n = 1$, equation (2.8) becomes

$$\frac{d^2u}{dt^2} + u^{1/3} = 0. \tag{2.9}$$

This type equations by fractional force was applied methods such as homotopy perturbation method (Belendez, 2009), Lindstedt- Poincaré method (Özi¸s and Yıldırım, 2007), etc. In this paper, He’s energy- balance method is applied to equation by fractional power defined by (2.9).

3. Energy-balance Method

Consider firstly the following generalized nonlinear oscillations without forced terms

$$u'' + w_0{}^2 u + \varepsilon f(u) = 0, u(0) = A, u'(0) = 0 \tag{3.1}$$

where f is a nonlinear function of u, u' and u'' (He, 2002).

In this place, variational function can be easily obtained as

$$J(u) = \int_0^t \left\{-\frac{1}{2}u'^2 + \frac{1}{2}{w_0}^2u^2 + \varepsilon F(u)\right\} dt \tag{3.2}$$

where F is potential and $\frac{dF}{du} = f$. Therefore its Hamiltonian can be written in the form

$$H = \frac{1}{2}u'^2 + \frac{1}{2}{w_0}^2u^2 + \varepsilon F(u) = \frac{1}{2}A^2 + F(A) \tag{3.3}$$

or

$$\mathrm{R(t)} = \frac{1}{2}u'^2 + \frac{1}{2}{w_0}^2u^2 + \varepsilon F(u) - \frac{1}{2}A^2 - F(A) = 0 \tag{3.4}$$

In (3.3) Eq., the kinetic energy (E) and potential energy (T) can be expressed as $E = \frac{1}{2}u'^2$, $T = \frac{1}{2}{w_0}^2u^2 + \varepsilon F(u)$ and throughout the oscillation, $H = E + T =$constant, respectively. Nonlinear oscillatory systems contain two important physical parameters, the frequency w and the amplitude of oscillation$A$. Suppose that initial approximate of (3.4) equation can be expressed as

$$u = Acoswt \tag{3.5}$$

Substituting equation (3.5) into the (3.4), it becomes

$$R(t) = \frac{1}{2}A^2w^2\sin^2 wt + \frac{1}{2}{w_0}^2A^2\cos^2 wt + \varepsilon F(Acoswt) - \frac{1}{2}A^2 - F(A) = 0 \tag{3.6}$$

4. Application of Method

Consider a nonlinear oscillator with fractional potential

$$u'' + \varepsilon u^{1/3} = 0, u(0) = A, u'(0) = 0. \tag{4.1}$$

In this equation, variational formulation obtained as

$$J(u) = \int_0^t \left\{-\frac{1}{2}u'^2 + \frac{3}{4}\varepsilon u^{4/3}\right\} dt. \tag{4.2}$$

Therefore, its Hamiltonian can be written in the form

$$H = \frac{1}{2}u'^2 + \frac{3}{4}\varepsilon u^{4/3} = \frac{3}{4}\varepsilon A^{4/3} \tag{4.3}$$

or

$$R(u) = \frac{1}{2}u'^2 + \frac{3}{4}\varepsilon u^{\frac{4}{3}} - \frac{3}{4}\varepsilon A^{\frac{4}{3}} = 0. \tag{4.4}$$

Initial condition can be expressed as

$$u = A \cos wt \tag{4.5}$$

Replacing (4.5) into (4.4), we obtain the following residual equation

$$R(t) = A^{2/3} w^2 \sin^2 wt + \frac{3}{2}\varepsilon \cos^{4/3} wt - \frac{3}{4}\varepsilon = 0. \tag{4.6}$$

by change, if the exact solution had been chosen as the trial function, it would be possible to make R zero for all values of t by approximate choice of w. Since initial condition is an only approximation to the exact solution, R cannot be made zero everywhere. Collocation at $wt = \pi/4$ gives and w frequency obtained.

Additionally, its period can be written in as:

$$T = \frac{2\pi}{w} \tag{3.7}$$

In (4.6), collocation at $wt = \pi/4$ gives

$$w = \sqrt{3 - \frac{3}{\sqrt[3]{4}}}\,\varepsilon^{1/2} A^{-1/3} \tag{4.7}$$

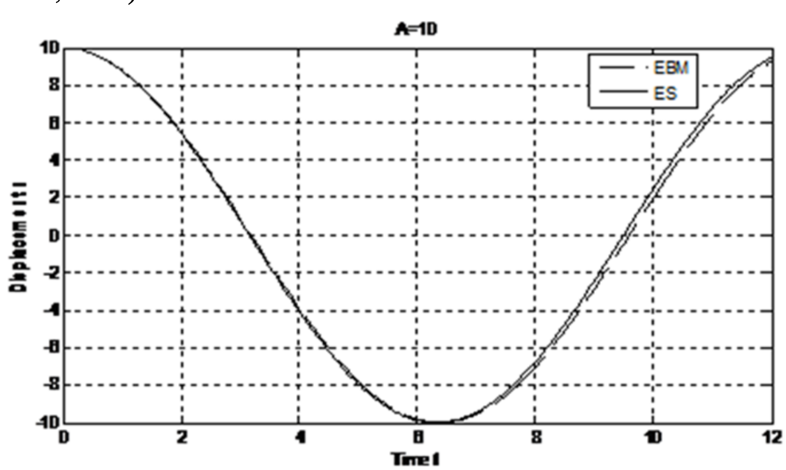
Where $T = 2\pi/w$ is period of the nonlinear oscillator. Its period can be written in the form of:

$$T = \frac{2\pi}{\sqrt{3-\frac{3}{\sqrt[3]{4}}}\,\varepsilon^{1/2} A^{-1/3}} \tag{4.8}$$

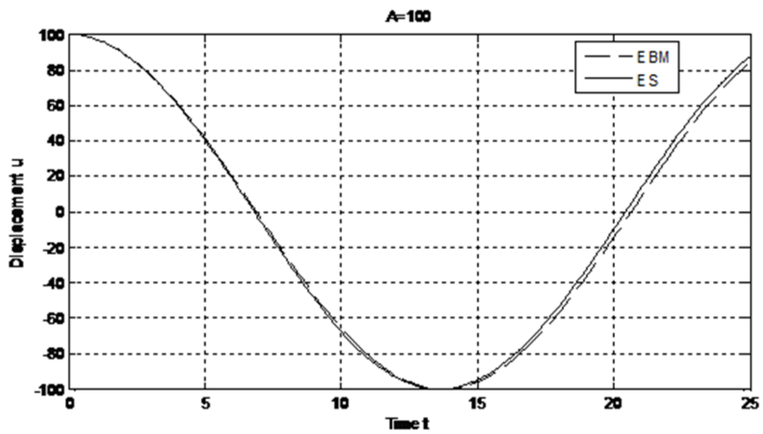
The exact frequency for eq. (4.1)

$$w_{ex} = \frac{2\pi\Gamma(\frac{5}{4})}{\sqrt{6}\Gamma(\frac{3}{4})\Gamma(\frac{1}{2})A^{1/3}} = \frac{1.070451}{A^{1/3}} \tag{4.7}$$

expressed as (Belendez, 2009).



(a)



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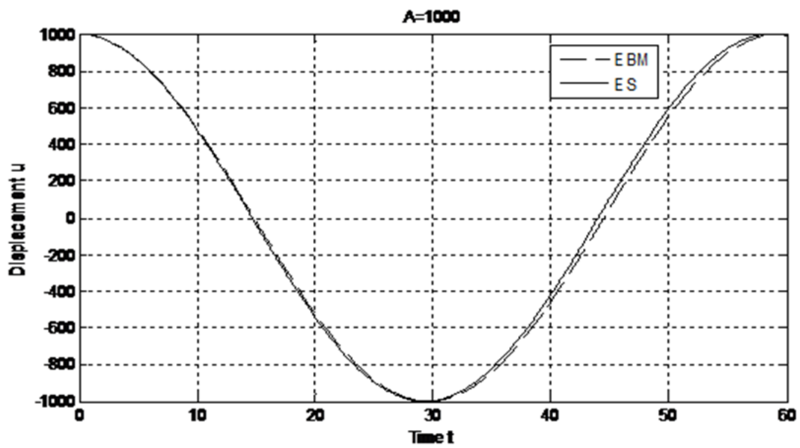

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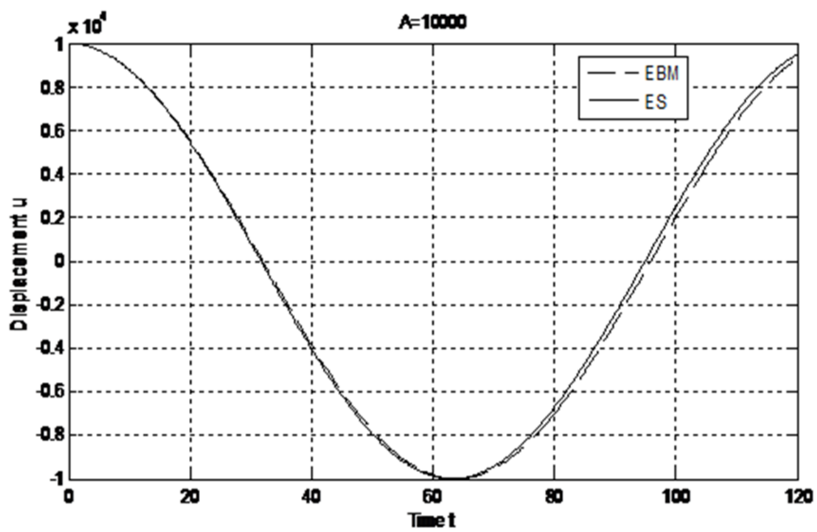

(d)

Fig.1. Comparison of the approximate solutions with the exact solutions to Eq. (4.1) for $\varepsilon = 1$: (a) A = 10; (b) A = 100; (c) A = 1000; (d) A = 10000.

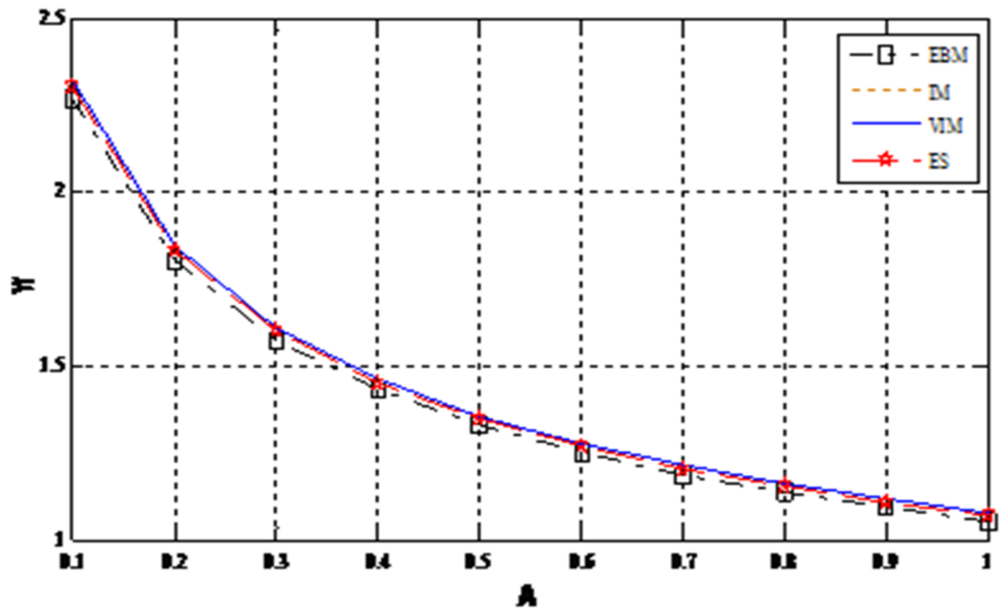

Fig.2. Comparison of the exact and approximate frequencies obtained using different methods (IM= Integral method (Hu and Tang, 2005), VIM= Variational iteration method (Öziş and Yıldırım, 2007) and ES= Exact solution (Belendez, 2009).

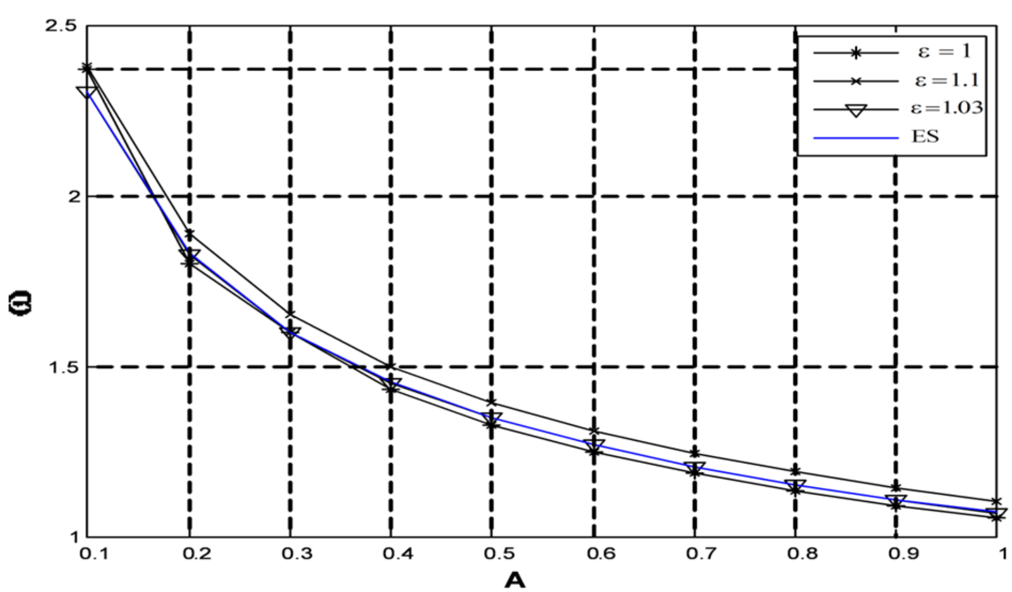

Fig.3. Comparison of the approximate solutions with the exact solution to Eq. (4.1) for $\varepsilon = 1$, $\varepsilon = 1.1$, $\varepsilon = 1.03$.

Table 1. Comparison of the error in between exact frequency and approximate frequency for ε = 1.

A	Exact Sol.	EBM	%Error
0.1	2.30622	2.26996	%1.57227
0.5	1.34868	1.32748	%1.57191
1	1.07045	1.05362	%1.57224
1.5	0.935125	0.920423	%1.5722
5	0.626004	1.57219	%1.57219
10	0.496859	0.489048	%1.57208
20	0.394358	0.388158	%1.57218

In Table 1, we show a comparison of the error in between exact frequency and approximate frequency for (4.1) Eq. In Fig.1, we investigated the comparison of the approximate solutions with the exact solution to (4.1) Eq. for ε = 1 and states of A. In Fig.2, given comparison between approximate frequencies and exact frequency for using different methods. Also in Fig.3, exact solutions compared with approximate solutions for situation of ε.

5. Conclusion

In this article, we applied a technique called energy balance method for nonlinear oscillation applied nonlinear equation with $u^{1/3}$ fractional force. This study compared exact frequency with approximate frequencies obtained for EBM and discussed with table and figures. Method is extremely simple and easy to apply. The method which is proved to be a powerful mathematical tool to study of nonlinear oscillators can be easily extended to any nonlinear equations.

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