Exploration

Extending Lehnert's Revised Quantum Electrodynamics to Fractal Media & Cantor Sets

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ABSTRACT

In a series of papers, Bo Lehnert proposed a novel and revised version of quantum electrodynamics (RQED) based on Proca equations. However, as far as I know there is no paper yet for extending RQED to fractal media and Cantor Sets. Drawing similarity between Proca and Maxwell equations, I extend RQED further in the present paper based on a recent paper by Zhao *et al.* derived Maxwell equations on Cantor sets from local fractional vector calculus. It can be shown that Maxwell equations on Cantor sets in a fractal bounded domain give efficiency and accuracy for describing the fractal electric and magnetic fields. I also extend RQED to anisotropic fractal media based on the work of Martin Ostoja-Starzewski. It is hoped that this paper may stimulate further investigations and experiments for finding physics beyond Standard Model in fractal media. It may be expected to have some impact to fractal cosmology modeling too.

Keywords: Cantor sets, fractional vector calculus, Navier-Stokes, Durand, Maxwell equation, Proca equation, fractal cosmology.

1. Introduction

Conventional electromagnetic theory based on Maxwell's equations and quantum mechanics has been successful in its applications in numerous problems in physics, and has sometimes manifested itself in a good agreement with experiments. Nevertheless, as already stated by Feynman, there are unsolved problems leading to difficulties with Maxwell's equations that are not removed by and not directly associated with quantum mechanics [20]. Therefore QED, which is an extension of Maxwell's equations, also becomes subject to the typical shortcomings of electromagnetic in its conventional form. This reasoning makes a way for Revised Quantum Electrodynamics as proposed by Bo Lehnert.

According to the late Benoit Mandelbrot, fractal geometry is a workable geometric middle ground between excessive geometric order of Euclid and the geometric chaos of general mathematics. It is based on a form of symmetry that had previously been underused, namely invariance, under contraction or dilation [1]. Fractal geometry has many applications including in biology, physics, geophysics, engineering, mathematics, cosmology and other fields of science

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and art. A rapidly growing field is to express electromagnetic wave equations in fractal media. An interesting piece in this regard is a paper by Martin Ostoja-Starzewski, where he is able to derive Maxwell equations in anisotropic fractal media [3]. Therefore it is reasonable to consider an extension of RQED to Cantor Sets and fractal media. That is the purpose of this paper.

In the meantime, it is known that Maxwell himself described his theory of electromagnetism based on elastic ether, therefore it seems worth to re-derive his equations from elasticity theory. Therefore, in the next section I will discuss 3 possible methods to link elasticity theory and Maxwell equations. The third section will discuss Lehnert's RQED. Then I will discuss Maxwell equations on Cantor sets. The fifth section will discuss how to extend Proca equations and RQED on Cantor Sets. The sixth section will discuss how to extend Proca equations and RQED on fractal media. The seventh section will discuss some implications for cosmology including dark matter and massive gravitational wave.

2. Review of 3 methods to derive Maxwell equations from elasticity theory

As far as I know there are at least 3 possible methods to connect elasticity theory and Maxwell equations. Here I will briefly discuss the methods proposed by Algirdas Maknickas, Valery Dmitriyev, and David Zareski.

a. Maknickas's approach [5]:

According to Maknickas, the classical electromagnetic field theory is based on similarity to the classic dynamic of solid continuum media. Therefore he thinks that it is required to consider a *micropolar extension* of electromagnetic field equations based on Cosserat media. In essence, besides the well-known four differential equations of Maxwell, Maknickas proposed additional four Maxwell equations for rotational components using micropolar elasticity analogy [5, p. 5-6]:

- Micropolar Gauss's law for electric field: $\nabla \cdot C = \frac{\rho c}{\gamma_0}$. (1)
- Micropolar Gauss's law for magnetic field: $\nabla \cdot G = 0$. (2)
- Micropolar Maxwell-Faraday equation: $\nabla \times C = -\frac{\partial G}{\partial t}$. (3)
- Micropolar Ampere's circuital law: $\nabla \times G = \beta_0 \left(J_G + \gamma_0 \frac{\partial C}{\partial t} \right)$. (4)

b. Dmitriyev's approach [16, p.7]:

According to Dmitriyev, classical electrodynamics was found to correspond to incompressible linear elasticity. And this analogy has formal character. By using a new definition:

$$E = \kappa\left[c^2\nabla\times(\nabla\times s) - f\right], \tag{3}$$

And defining the density j of electric current by

$$\varsigma j = -\kappa a v\delta(x - x'), \tag{4}$$

Then he obvtained the Maxwell equations:

$$\partial_t E - c\nabla\times(\nabla\times A) + 4\pi j = 0 \text{, and} \tag{5}$$

$$\nabla\cdot E = 4\pi q\delta(x - x'). \tag{6}$$

c. Zareski's approach [17-19]:

His model is based on the Navier-Stokes-Durand equation of elasticity, as follows:

$$curl\frac{C}{2} + (\sigma+\eta)grad(div\xi) + \eta\nabla^2\xi + f = \partial_t(\rho\partial_t\xi). \tag{7}$$

Then he considers the particular case of conservative elasticity where the elastic medium is governed, by the following equation:

$$(\sigma + 2\eta)grad(div\xi) + f = 0. \tag{8}$$

From these equations, after some changes of variables, he recovers Maxwell equations. The above are 3 methods to connect elasticity theory to Maxwell's equations. Therefore it is appropriate to generalize them further to Proca equations for massive electrodynamics.

3. Lehnert's Revised Quantum Electrodynamics

Conventional electromagnetic theory based on Maxwell's equations and quantum mechanics has been successful in its applications in numerous problems in physics, and has sometimes manifested itself in a good agreement with experiments. Nevertheless, as already stated by Feynman, there are unsolved problems leading to difficulties with Maxwell's equations that are not removed by and not directly associated with quantum mechanics [20]. Therefore QED, which is an extension of Maxwell's equations, also becomes subject to the typical shortcomings of electromagnetic in its conventional form. This reasoning makes a way for Revised Quantum Electrodynamics as proposed by Bo Lehnert.

In a series of papers, Bo Lehnert proposed a novel and revised version of Quantum Electrodynamics, which he calls as RQED. His theory is based on the hypothesis of a nonzero electric charge density in the vacuum, and it is based on Proca-type field equations [20, p. 23]:

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)A_\mu = \mu_0 J_\mu, \mu = 1,2,3,4 \tag{9}$$

where

$$A_\mu = \left(A, \frac{i\phi}{c} \right), \tag{10}$$

With A and ϕ standing for the magnetic vector potential and the electrostatic potential in three-space. In three dimensions equation (9) in the vacuum results in [20, p.23]:

$$\frac{curlB}{\mu_0} = \varepsilon_0 (divE) C + \frac{\varepsilon_0 \partial E}{\partial t}, \tag{11}$$

$$curlE = -\frac{\partial B}{\partial t}, \tag{12}$$

$$B = curlA, divB = 0, \tag{13}$$

$$E = -\nabla\phi - \frac{\partial A}{\partial t}, \tag{14}$$

$$divE = \frac{\bar{\rho}}{\varepsilon_0}. \tag{15}$$

These equations differ from the conventional form, by a nonzero electric field divergence equation (15) and by the additional space-charge current density in addition to displacement current at equation (11). The extended field equations (11)-(15) are easily found also to become invariant to a gauge transformation.[20, p.23]

The main characteristic new features of the present theory can be summarized as follows [20, p.24]:

a. The hypothesis of a nonzero electric field divergence in the vacuum introduces an additional degree of freedom, leading to new physical phenomena. The associated nonzero electric charge density thereby acts somewhat like a hidden variable.
b. This also abolishes the symmetry between the electric and magnetic fields, and then the field equations obtain the character of intrinsic linear symmetry breaking.
c. The theory is both Lorentz and gauge invariant.
d. The velocity of light is no longer a scalar quantity, but is represented by a velocity vector of the modulus c.
e. Additional results: Lehnert is also able to derive the mass of Z boson and Higgs-like boson.[21-23] These would pave an alternative way to new physics beyond Standard Model.

Now let us extend Lehnert's RQED to Cantor sets and fractal media. Such an extension may be found worthwhile, considering there is already a new proposal to extend QED to inhomogenous anisotropic media.[24] Nonetheless, my approaches as presented here are different from [24].

First we will review Maxwell equations on Cantor sets.

4. Review of Maxwell Equations on Cantor Sets

I will not re-derive Maxwell equations here. For a good reference on Maxwell equations, see for example Julian Schwinger et al.'s book: *Classical Electrodynamics* [9]. Penrose also discusses Maxwell equations shortly in his book: *The Road to Reality*[10].

Zhao et al. were able to write the local fractional differential forms of Maxwell equations on Cantor sets as follows [2, p.4-5]:

- Gauss's law for the fractal electric field: $\nabla^{\alpha} \cdot D = \rho$, (16)

- Ampere's law in the fractal magnetic field: $\nabla^{\alpha} \times H = J_a + \frac{\partial^{\alpha} D}{\partial t^{\alpha}}$, (17)

- Faraday's law in the fractal electric field: $\nabla^{\alpha} \times E = -\frac{\partial^{\alpha} B}{\partial t^{\alpha}}$, (18)

- magnetic Gauss's law in the fractal magnetic field: $\nabla^{\alpha} \cdot B = 0$, (19)

and the continuity equation can be defined as:

$$\nabla^{\alpha} \cdot J = -\frac{\partial^{\alpha} \rho}{\partial t^{\alpha}}, \tag{20}$$

where $\nabla^{\alpha} \cdot r$ and $\nabla^{\alpha} \times r$ are defined as follows:

2.1. In Cantor coordinates [11, p. 2]:

$$\nabla^{\alpha} \cdot u = div^{\alpha} u = \frac{\partial^{\alpha} u_1}{\partial x_1^{\alpha}} + \frac{\partial^{\alpha} u_2}{\partial x_2^{\alpha}} + \frac{\partial^{\alpha} u_3}{\partial x_3^{\alpha}}, \tag{21}$$

$$\nabla^{\alpha} \times u = curl^{\alpha} u = \left(\frac{\partial^{\alpha} u_3}{\partial x_2^{\alpha}} - \frac{\partial^{\alpha} u_2}{\partial x_3^{\alpha}} \right) e_1^{\alpha} + \left(\frac{\partial^{\alpha} u_1}{\partial x_3^{\alpha}} - \frac{\partial^{\alpha} u_3}{\partial x_1^{\alpha}} \right) e_2^{\alpha} + \left(\frac{\partial^{\alpha} u_2}{\partial x_1^{\alpha}} - \frac{\partial^{\alpha} u_1}{\partial x_2^{\alpha}} \right) e_3^{\alpha}. \tag{22}$$

2.2. In Cantor-type cylindrical coordinates [2, p.4]:

$$\nabla^{\alpha} \cdot r = \frac{\partial^{\alpha} r_R}{\partial R^{\alpha}} + \frac{1}{R^{\alpha}} \frac{\partial^{\alpha} r_{\theta}}{\partial \theta^{\alpha}} + \frac{r_R}{R^{\alpha}} + \frac{\partial^{\alpha} r_z}{\partial z^{\alpha}}, \tag{23}$$

$$\nabla^{\alpha} \times r = \left(\frac{1}{R^{\alpha}} \frac{\partial^{\alpha} r_{\theta}}{\partial \theta^{\alpha}} - \frac{\partial^{\alpha} r_{\theta}}{\partial z^{\alpha}} \right) e_{R}^{\alpha} + \left(\frac{\partial^{\alpha} r_{R}}{\partial z^{\alpha}} - \frac{\partial^{\alpha} r_{z}}{\partial R^{\alpha}} \right) e_{\theta}^{\alpha} + \left(\frac{\partial^{\alpha} r_{\theta}}{\partial R^{\alpha}} + \frac{r_{R}}{R^{\alpha}} - \frac{1}{R^{\alpha}} \frac{\partial^{\alpha} r_{R}}{\partial \theta^{\alpha}} \right) e_{z}^{\alpha} . \tag{24}$$

5. Extending Lehnert's Proca Equations on Cantor Sets

Proca equations can be considered as an extension of Maxwell equations, and they have been derived in various ways. It can be shown that Proca equations can be derived from first principles [6], and also that Proca equations may have link with Klein-Gordon equation [7]. However, in this paper I will not attempt to re-derive Proca equations. Instead, I will derive the Proca equations on Cantor Sets, in accordance with Lehnert's approach as outlined above [20].

Therefore, by using the definitions in equations (21)-(24), we can arrive at Proca equations on Cantor sets from (11) through (15), as follows:

$$\frac{curl^{\alpha} B}{\mu_0} = \varepsilon_0 \left(div^{\alpha} E \right) C + \frac{\varepsilon_0 \partial^{\alpha} E}{\partial t^{\alpha}}, \tag{25}$$

$$curl^{\alpha} E = -\frac{\partial^{\alpha} B}{\partial t^{\alpha}}, \tag{26}$$

$$div^{\alpha} B = 0, \tag{27}$$

$$div^{\alpha} E = \frac{\bar{\rho}}{\varepsilon_0}. \tag{28}$$

where:

$$\nabla^{\alpha} \phi = -\frac{\partial^{\alpha} \vec{A}}{\partial t^{\alpha}} - \vec{E} , \tag{29}$$

$$\vec{B} = \nabla^{\alpha} \times \vec{A}, \tag{30}$$

and Del operator $\nabla^{\alpha} \phi$ can be defined as follows [11, p.2]:

$$\nabla^{\alpha} \phi = \frac{\partial^{\alpha} \phi}{\partial x_1^{\alpha}} e_1^{\alpha} + \frac{\partial^{\alpha} \phi}{\partial x_2^{\alpha}} e_2^{\alpha} + \frac{\partial^{\alpha} \phi}{\partial x_3^{\alpha}} e_3^{\alpha} . \tag{31}$$

To my best knowledge so far, the above extension of Lehnert's RQED on Cantor sets (25)-(30) have not been proposed elsewhere before.

Since according to Blackledge, the Proca equations can be viewed as a *unified wavefield* model of electromagnetic phenomena [7], therefore we can also regard the Proca equations on Cantor sets as a further generalization of Blackledge's *unified wavefield* model.

One persistent question concerning these Proca equations is how to measure the mass of the photon. This question has been discussed in lengthy by Tu, Luo & Gillies [15]. According to their report, there are various methods to estimate the upper bound limits of photon mass. In Table 1 below, some of upper bound limits of photon mass based on dispersion of speed of light are summarized.

Table 1. Upper bound on the dispersion of the speed of light in different ranges of the electromagnetic spectrum, and the corresponding limits on the photon mass. [15, p.94]

Author (year)	Type of measurement	Limits on m_γ (g)
Ross et al. (1937)	Radio waves transmission overland	5.9×10^{-42}
Mandelstam & Papalexi (1944)	Radio waves transmission over sea	5.0×10^{-43}
Al'pert et al. (1941)	Radio waves transmission over sea	2.5×10^{-43}
Florman (1955)	Radio-wave interferometer	5.7×10^{-42}
Lovell et al. (1964)	Pulsar observations on flour flare stars	1.6×10^{-42}
Frome (1958)	Radio-wave interferometer	4.3×10^{-40}
Warner et al. (1969)	Observations on Crab Nebula pulsar	5.2×10^{-41}
Brown et al. (1973)	Short pulses radiation	1.4×10^{-33}
Bay et al. (1972)	Pulsar emission	3.0×10^{-46}
Schaefer (1999)	Gamma ray bursts	4.2×10^{-44}
	Gamma ray bursts	6.1×10^{-39}

From this table and also from other results as reported in [15], it seems that we can expect that someday photon mass can be observed within experimental bound.

6. Extending Revised Quantum Electrodynamics on Fractal Media

It is worth noting here, that Martin Ostoja-Starzewski has derived Maxwell equations in anisotropic fractal media using a different method.[3] Therefore it is interesting to find out how we can extend Lehnert's RQED to fractal media too.

First let us begin with some basic definitions as given by Ostoja-Starzewski [3]:

$$\nabla^D \phi = e_k \nabla_k^D \phi, \tag{32}$$

$$divf = \nabla^D \cdot f, \tag{33}$$

$$curlf = \nabla^D \times f. \tag{34}$$

Based on the above definitions, now I extend Lehnert's RQED to anisotropic fractal media case, as follows:

$$\frac{curl^D B}{\mu_0} = \varepsilon_0 \left(div^D E\right) C + \frac{\varepsilon_0 \partial^D E}{\partial t}, \tag{35}$$

$$curl^D E = -\frac{\partial^D B}{\partial t}, \tag{36}$$

$$div^D B = 0, \tag{37}$$

$$div^D E = \frac{\bar{\rho}}{\varepsilon_0}. \tag{38}$$

To the best of my knowledge, these extensions of Lehnert's RQED to Cantor sets and fractal media have never been proposed elsewhere before.

7. Some Implications for Astrophysics & Cosmology

Beside RQED's implications to Standard Model of Particles, it seems possible to consider that connecting Maxwell equations and elasticity theory can lead to far-reaching implications, such that new explanation of dark matter [17], possible massive gravitational wave [25], and also new explanation of dark energy as elastic strain fluid [26]. It may be expected to have some impact to cosmology modeling on fractal media too. Another possible direction of further research is micropolar fluid cosmology. It should be clear that applications of elasticity theory to cosmology will be fruitful. [27][28][29]

8. Conclusion

In a series of papers, Bo Lehnert proposed a novel and revised version of Quantum Electrodynamics (RQED) based on Proca equations. However, as far as I know there is no paper yet for extending his RQED to fractal media and Cantor Sets. Drawing similarity between Proca and Maxwell equations, in the present paper I extend RQED further based on a recent paper published at *Advances in High Energy Physics* (AHEP) journal, where Yang Zhao et al. derived

Maxwell equations on Cantor sets from the local fractional vector calculus. It can be shown that Maxwell equations on Cantor sets in a fractal bounded domain give efficiency and accuracy for describing the fractal electric and magnetic fields. I also extend RQED to anisotropic fractal media based on the work of Martin Ostoja-Starzewski. It is hoped that this paper may stimulate further investigations and experiments in particular for finding physics beyond Standard Model in fractal media. It may be expected to have some impact to fractal cosmology modeling too.

It shall be noted that the present paper is not intended to be a complete description of physics beyond Standard Model on Cantor sets. Instead, this paper is intended to stimulate further investigations and experiments, and their implications to fractal cosmology modeling.

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References

[1] Mandelbrot, B.B. 1989. Fractal geometry: what is it, and what does it do? *Proc. R. Soc. Lond*. **A**. 423, 3-16, URL: http://users.math.yale.edu/~bbm3/web_pdfs/fractalGeometryWhatIsIt.pdf; [1a] Mandelbrot, B.B. Fractals and the Geometry of Nature. *Encyclopedia Britannica*, URL: http://users.math.yale.edu/~bbm3/web_pdfs/encyclopediaBritannica.pdf

[2] Zhao, Y., Baleanu, D., Cattani, C., Cheng, D-F., & Yang, X-J. 2013. Maxwell's equations on Cantor Sets: A Local Fractional Approach. *Advances in High Energy Physics* Vol. 2013 Article ID 686371, http://dx.doi.org/10.1155/2013/686371, http://downloads.hindawi.com/journals/ahep/2013/686371.pdf

[3] Ostoja-Starzewski, Martin. 2012. Electromagnetism in anisotropic fractal media. *Z. Angew. Math. Phys.*(ZAMP)DOI 10.1007/s00033-012-0230-z . URL: http://mechanical.illinois.edu/media/uploads/web_sites/82/files/zamp_2012.20121002.506b39d642e987.85850489.pdf;[3a] Also in arXiv: 1106.1491 [math-ph], URL: http://arxiv.org/pdf/1106.1491.pdf.

[4] Tajmar, M. 2008. Electrodynamics in Superconductor explained by Proca equations. arXiv:cond-mat/0803.3080

[5] Maknickas, Algirdas. 'Spooky action at a distance' in the Micropolar Electromagnetic Theory. Dec. 5, 2014. http://vixra.org/pdf/1411.0549v4.pdf

[6] Gondran, Michel. 2009. Proca equations derived from first principles. arXiv:0901.3300 [quant-ph], URL: http://arxiv.org/pdf/0901.3300.pdf

[7] Blackledge, Jonathan M. 2007. An Approach to Unification using a Linear Systems Model for the Propagation of Broad-band Signals.*ISAST Transaction on Electronics and Signal Processing*, Vol. 1, No. 1, 2007. URL: http://eleceng.dit.ie/papers/100.pdf

[8] Demir, Suleyman. 2013. Space-time algebra for the generalization of gravitational field equations. *Pramana* Vol. 80 No. 5 (Indian Academy of Sciences), May 2013, 811-823. URL: http://www.ias.ac.in/pramana/v80/p811/fulltext.pdf

[9] Schwinger, Julian, DeRaad, Jr., Lester L., Milton, Kimball A., & Tsai, Wu-yang. 1998. *Classical Electrodynamics*. Reading, Massachusetts: Perseus Books. 591 p.

[10] Penrose, Roger. 2004. *The Road to Reality: A Complete Guide to the Laws of the Universe*. London: Jonathan Cape. 1123 p. URL: http://staff.washington.edu/freitz/penrose.pdf

[11] Yang, X-J., Baleanu, D., & Tenreiro Machado, J.A. 2013. Systems of Navier-Stokes on Cantor Sets. *Mathematical Problems in Engineering* Vol. 2013, Article ID 769724, http://dx.doi.org/10.1155/2013/769724, URL: http://www.hindawi.com/journals/mpe/2013/769724/, or URL: http://recipp.ipp.pt/bitstream/10400.22/3455/1/ART_TenreiroMachado_2013_DEE.pdf

[12] Christianto, Victor. 2014. A derivation of GravitoElectroMagnetic Proca equations in fractional space. *Prespacetime Journal*, May 2014. URL: www.prespacetime.com.

[13] Christianto, Victor. 2015. London-Proca-Hirsch equations for electrodynamics of superconductors on Cantor Sets. *Prespacetime Journal*, November 2015. http://www.prespacetime.com

[14] Baleanu, D., Tenreiro Machado, J.A., Cattani, C., Baleanu, M.C., & Yang, X-J. 2014. Local Fractional Variational Iteration and Decomposition Methods for Wave Equation on Cantor Sets within Local Fractional Operators. *Abstract and Applied Analysis* Vol. 2014, Article ID 535048, http://dx.doi.org/10.1155/2014/535048; http://downloads.hindawi.com/journals/aaa/2014/535048.pdf

[15] Tu, L-C., Luo, J., & George T. Gillies. 2005. The mass of the photon. *Rep. Prog. Phys*. 68:77-130. (Institute of Physics Publishing), doi:10.1088/0034-4885/68/1/R02

[16] Dmitriyev, Valery P. 2004. *Elasticity and electromagnetism*. URL: http://aether.narod.ru/elast1.pdf

[17] Zareski, David. 2015. Is Dark Matter a Particular State of the Ether?, *Intr. Res. J. Pure and Appl. Physics* Vol. 3 No. 3, pp. 1-7, Dec. 2015.

[18] _______. 2015. *Unification of Physics by the Ether Elasticity Theory*. Saarbrucken, Germany: LAP Lambert Academic Publishing, 2015.

[19] _______. 2014. Unification of Physics Theories by the Ether Elasticity Theory. *Global Journal of Science Frontier Research: A Physics and Space Science* Vol. 14 Issue 6, 2014.

[20] Lehnert, Bo. 2012. A way to Revised Quantum Electrodynamics. *Progress in Physics*, April 2012, Vol. 2, http://www.ptep-online.com

[21] _______. 2013. Higgs-Like Particle due to Revised Quantum Electrodynamics. *Progress in Physics*, July 2013, Vol. 3. http://www.ptep-online.com

[22] _______. 2013. Potentialities of Revised Quantum Electrodynamics. *Progress in Physics*, October 2013, Vol. 4. http://www.ptep-online.com

[23] _______. 2014. Mass-Radius Relations of Z and Higgs-like Bosons. *Progress in Physics*, Vol. 10 Issue 1, January 2014, http://www.ptep-online.com

[24] Lopez, Adrian E. Rubio, & Fernando C. Lombardo. 2015. Quantum Electrodynamics of inhomogeneous anisotropic media. EPJ. Also in arxiv:1411.7758 [hep-th].

[25] Radicella, N., & Tartaglia, A. 2009. Massive gravitational waves from the Cosmic Defect Theory. Arxiv:0911.3365 [gr-qc]

[26] Radicella, N., Sereno, M., & Tartaglia, A. 2012. Dark energy as an elastic strain fluid. MNRAS Advanced Access, Dec. 5, 2012

[27] Levrino, L., & Tartaglia, A. 2012. From the elasticity theory to cosmology and vice versa. arXiv:1205.5577 [gr-qc].

[28] Altenbach, H., & Eremeyev, V.A. 2013. Cosserat Media. http://www.researchgate.net

[29] Maugin, G.A. & Metrikine, A. V. 2010. Mechanics of Generalized Continua: One Hundred Years after the Cosserats. Germany: Springer Science+Business Media, 2010