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A Note on Inextensible Flows of Partially & Pseudo Null Curves in E_1^4

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Abstract

In this paper, we study inextensible flows of partially null and pseudo null curves in E_1^4.We give some necessary and sufficient conditions for inextensible flows of partially null and pseudo null curves in E_1^4

Keywords: Inextensible flows, partially null curves, pseudo null curves, Minkowski space-time.

1 Introduction

Recently, the study of the motion of inextensible curves has been arisen in a number of diverse engineering applications. The flow of a curve is said to be inextensible if the arc length is preserved. Physically, inextensible curve flows give rise to motions in which no strain energy is induced. The swinging motion of a cord of fixed length, for example, a piece of paper carried by the wind, can be described by inextensible curve and surface flows. Such motions arise quite naturally in a wide range of a physical applications. For example, both Chirikjian and Burdick (6) and Mochiyama et al. (17) studied the shape control of hyper-redundant, or snake-like robots. Inextensible curve and surface flows also arise in the context of many problems in computer vision (11), (16) and computer animation (7), and even structural mechanics (19) .

Firstly, Kwon and Park studied inextensible flows of curves and developable surfaces in Euclidean 3-space (14). Inextensible flows of curves have been studied in many different spaces. Gürbüz examined inextensible flows of spacelike, timelike and null curves in (9). After this work Ögrenmiş et al. studied inextensible curves in Galilean space (18) and Yıldız et al. studied inextensible flows of curves according to Darboux frame in Euclidean 3-space (20). Moreover Latifi et al. (2008) studied inextensible flows of curves in Minkowski 3-space (15).

In (4) and (5) the authors focused on timelike and space like curves in E_1^3, respectively. In the recent works (21), (22), Yıldız et al. gave necessary and sufficient conditions for inextensible flows of non-null curves in E^n and E_1^n .

More generally, from the differential geometric point of view, the study of null curves has its own geometric interest. Many of the classical results from Riemannian geometry have Lorentz counterparts. In fact, spacelike curves or timelike curves can be studied by a similar approach to that in positive definite Riemannian geometry. However, null curves have very different properties from spacelike or timelike curves. In other words, the null curve theory has many results which have no Riemannian analogues. The presence of null curves often causes important and interesting differences, as will be the case in the present study (2).

Nowadays, many important and intensive studies are seen about null curves in Minkowski space. Papers in (1), (3), (8),(12) and (13) are obtained some new characterizations of the null curves in Minkowski space.

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In this paper, we define inextensible flows of partially null and pseudo null curves in E_1^4. Then, we give some necessary and sufficient conditions for inextensible flows of them in E_1^4.

2 Preliminaries

Let E_1^4 denote the 4-dimensional Minkowski space -time with the standard flat metric given by

$$< \; , \; > - dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2,$$

where (x_1, x_2, x_3, x_4) is a rectangular coordinate system of E_1^4 .Since g is an indefinite metric, recall that a vector v in E_1^4 can have one of three casual characters: it can be spacelike if $< v, v > > 0$ or $v = 0$, timelike if $< v, v >< 0$ and null (lightlike) if $< v, v > = 0$ and $v \neq 0$. The norm of a vector v is given by$\| v \| = \sqrt{|< v, v >|}$ .Therefore, v is unit vector if $< v, v >= \mp 1$.Next, the vectors v and w are said to be orthogonal if $< v, w >= 0$. Similarly an arbitrary curve $\gamma(s)$ can be locally spacelike, timelike or null (lightlike), if all of its velocity $\gamma^{'}(s)$ are spacelike, timelike or null (lightlike). Next $\gamma(s)$ is a unit speed curve if $< \gamma^{'}(s), \; \gamma^{'}(s) >= \mp 1$.

Recall that a spacelike curve in E_1^4 is called pseudo null curve or partially null curve, if its principal normal or its first binormal vector field is null, respectively [10] . A null curve γ is parametrized by arclength function s, if $< \gamma^{''}(s), \; \gamma^{''}(s) >= 1$. In particular, pseudo null curve or partially null curve $\gamma(s)$ has unit speed, if $< \gamma^{'}(s), \; \gamma^{'}(s) >= 1$.

In the following we use the notations and concepts from (10), unless otherwise stated.

Let $\{T, N, B_1, B_2\}$ be the moving Frenet frame along a curve γ in E_1^4 , consisting of the tangent, the principal normal, the first binormal and the second binormal vector fields. Depending on the causal character of γ, the Frenet equations have the following forms.

Case (a). If γ is partially null curve, the Frenet formulas read as ((10)):

$$\begin{bmatrix} T^{'} \\ N^{'} \\ B_1^{'} \\ B_2^{'} \end{bmatrix} = \begin{bmatrix} 0 & k_1 & 0 & 0 \\ -k_1 & 0 & k_2 & 0 \\ 0 & 0 & k_3 & 0 \\ 0 & -k_2 & 0 & -k_3 \end{bmatrix} \begin{bmatrix} T \\ N \\ B_1 \\ B_2 \end{bmatrix} \tag{2.1}$$

where the third curvature $k_3(s) = 0$ for each s. Such curve has two curvatures $k_1(s)$ and $k_2(s)$ and lies fully in a lightlike hyperplane of E_1^4 . In particular, the following equations hold

$$< T, T >=< N, N >= 1, < B_1, B_1 >=< B_2, B_2 >= 0, \tag{2.2}$$

$$< T, N >=< T, B_1 >=< T, B_2 >=< N, B_1 >=< N, B_2 >= 0, < B_1, B_2 >= 1. \tag{2.3}$$

Case (b). If γ is pseudo null curve, the Frenet formulas are ((10)):

$$\begin{bmatrix} T^{'} \\ N^{'} \\ B_1^{'} \\ B_2^{'} \end{bmatrix} = \begin{bmatrix} 0 & k_1 & 0 & 0 \\ 0 & 0 & k_2 & 0 \\ 0 & k_3 & 0 & -k_2 \\ -k_1 & 0 & -k_3 & 0 \end{bmatrix} \begin{bmatrix} T \\ N \\ B_1 \\ B_2 \end{bmatrix} \tag{2.4}$$

where the first curvature $k_1(s) = 0$, if γ is straight line, or $k_1(s) = 1$ in all other cases. Such curve has two curvatures $k_2(s)$ and $k_3(s)$ and the following conditions are satisfied:

$$< T, T >=< B_1, B_1 >= 1, < N, N >=< B_2, B_2 >= 0, \tag{2.5}$$

$$< T, N >=< T, B_1 >=< T, B_2 >=< N, B_1 >=< B_1, B_2 >= 0, < N, B_2 >= 1. \tag{2.6}$$

3 Inextensible Flows of partially null curve in E_1^4

Unless otherwise stated, we assume that

$$\gamma : [0, l] \times [0, w) \to E_1^4$$

is a one parameter family of smooth partially or pseudo null curves in E_1^4 , where l is the arclength of the initial curve. Suppose that u is the curve parametrization variable , $0 \le u \le l$. If the speed partially or pseudo null curves γ is given by $v = \left\| \frac{\partial \gamma}{\partial u} \right\|$, then the arclength of γ is given as a function of u by

$$s(u) = \int_0^u \left\| \frac{\partial \gamma}{\partial u} \right\| du = \int_0^u v \, du. \tag{3.1}$$

where

$$\left\| \frac{\partial \gamma}{\partial u} \right\| = \sqrt{\left| < \frac{\partial \gamma}{\partial u}, \frac{\partial \gamma}{\partial u} > \right|}.$$

The operator $\frac{\partial}{\partial s}$ is given by

$$\frac{\partial}{\partial s} = \frac{1}{v} \frac{\partial}{\partial u}. \tag{3.2}$$

where $v = \left\| \frac{\partial \gamma}{\partial u} \right\|$.

In this case; the arclength is as follows $ds = vdu$.

Definition 3.1. Let γ be a partially or pseudo null curves in E_1^4 and $\{T, N, B_1, B_2\}$ be the Frenet frame of γ in Minkowski space-time. Any flow of the partially or pseudo null curves can be expressed as follows

$$\frac{\partial \gamma}{\partial t} = \beta_1 T + \beta_2 N + \beta_3 B_1 + \beta_4 B_2, \tag{3.3}$$

where β_i are the smooth functions.

Let the arclength variation be

$$s(u, t) = \int_0^u v \, du.$$

For the partially or pseudo null curve does not have any elongation or compression with the condition

$$\frac{\partial}{\partial t}s(u,t)=\int_0^u \frac{\partial v}{\partial t}\,du=0.$$

where $u\in[0,l]$.

Definition 3.2. Let γ be a partially or pseudo null curves in E_1^4 . A partially null curve evolution $\gamma(u,t)$ and its flow $\frac{\partial\gamma}{\partial t}$ are said to be inextensible if

$$\frac{\partial}{\partial t}\left\|\frac{\partial\gamma}{\partial u}\right\|=0. \tag{3.4}$$

Before deriving the necessary and sufficient condition for inelastic partially or pseudo null curves flow, we need the following lemma.

Lemma 3.3. Let $\{T,N,B_1,B_2\}$ be the Frenet frame of a partially null curve γ and

$$\frac{\partial\gamma}{\partial t}=\beta_1T+\beta_2N+\beta_3B_1+\beta_4B_2$$

be a smooth flow of a partially null curves γ in E_1^4 .If $\gamma(s)$ is a unit speed partially null curve in E_1^4 , with curvature $k_3(s)=0$,then we have the following equality

$$\frac{\partial v}{\partial t}=\frac{\partial\beta_1}{\partial u}-\beta_2k_1v. \tag{3.5}$$

Proof. Suppose that $\frac{\partial\gamma}{\partial t}$ be a smooth flow of the partially null curve in E_1^4 , with curvature $k_3(s)=0$.Using definition of γ,we have

$$v^2=<\frac{\partial\gamma}{\partial u},\frac{\partial\gamma}{\partial u}>. \tag{3.6}$$

Then, by differentiating (3.6), we get

$$2v\frac{\partial v}{\partial t}=\frac{\partial}{\partial t}<\frac{\partial\gamma}{\partial u},\frac{\partial\gamma}{\partial u}>.$$

On the other hand, as $\frac{\partial}{\partial u}$ and $\frac{\partial}{\partial t}$ commute, we have

$$v\frac{\partial v}{\partial t}=<\frac{\partial\gamma}{\partial u},\frac{\partial}{\partial u}\left(\frac{\partial\gamma}{\partial t}\right)>.$$

From (3.3), we obtain

$$v\frac{\partial v}{\partial t}=<\frac{\partial\gamma}{\partial u},\frac{\partial}{\partial u}\left(\beta_1T+\beta_2N+\beta_3B_1+\beta_4B_2\right)>.$$

By using (2.1), we have

$$\frac{\partial v}{\partial t}=<T,\left(\frac{\partial\beta_1}{\partial u}-k_1\beta_2v\right)T+\left(\frac{\partial\beta_2}{\partial u}+\beta_1k_1v-\beta_4k_2v\right)N+$$

$$\left(\frac{\partial \beta_3}{\partial u}+k_2\beta_2 v\right)B_1 \quad +\frac{\partial \beta_4}{\partial u}B_2> . \tag{3.7}$$

This clearly forces

$$\frac{\partial v}{\partial t}=\left(\frac{\partial \beta_1}{\partial u}-\beta_2 k_1 v\right).$$

Lemma 3.4. Let $\{T,N,B_1,B_2\}$ be the Frenet frame of a partially null curves γ, and $\frac{\partial \gamma}{\partial t}=\beta_1 T+\beta_2 N+\beta_3 B_1+\beta_4 B_2$ be a smooth flow of a partially null curves γ in E_1^4 , with curvature $k_3(s)=0$. If the flow is inextensible ,then we have the following equality

$$\frac{\partial \beta_1}{\partial u}=\beta_2 k_1 v. \tag{3.8}$$

Proof. Let us assume that the partially null curve flow is inextensible.From (3.4), we have

$$\frac{\partial}{\partial t}s(u,t)=\int_0^u \frac{\partial v}{\partial t}\,du=\int_0^u \left(\frac{\partial \beta_1}{\partial u}-\beta_2 k_1 v\right)\,du=0. \tag{3.9}$$

This clearly forces

$$\frac{\partial \beta_1}{\partial u}-\beta_2 k_1 v=0.$$

We now restrict ourselves to arc length parametrized curves. That is, $v=1$ and the local coordinate u corresponds to the curve arc length s. Then, we have the following lemma.

Lemma 3.5 Let $\{T,N,B_1,B_2\}$ be the Frenet frame of a partially null curves γ and $\frac{\partial \gamma}{\partial t}=\beta_1 T+\beta_2 N+\beta_3 B_1+\beta_4 B_2$ be a smooth flow of a partially null curves γ in E_1^4 .The differentiations of $\{T,N,B_1,B_2\}$ with respect to t is

$$\frac{\partial T}{\partial t}=\left(\frac{\partial \beta_2}{\partial s}+\beta_1 k_1-\beta_4 k_2\right)N+\left(\frac{\partial \beta_3}{\partial s}+\beta_2 k_2\right)B_1+\frac{\partial \beta_4}{\partial s}B_2, \tag{3.10}$$

$$\frac{\partial N}{\partial t}=-\left(\frac{\partial \beta_2}{\partial s}+\beta_1 k_1-\beta_4 k_2\right)T+\psi_2 B_1+\psi_1 B_2, \tag{3.11}$$

$$\frac{\partial B_1}{\partial t}=-\frac{\partial \beta_4}{\partial s}T-\psi_1 N+\psi_3 B_1, \tag{3.12}$$

$$\frac{\partial B_2}{\partial t}=-\left(\frac{\partial \beta_3}{\partial s}+k_2\beta_2\right)T-\psi_2 N+\psi_3 B_2, \tag{3.13}$$

where

$$\psi_1=<\frac{\partial N}{\partial t},B_1>,\quad \psi_2=<\frac{\partial N}{\partial t},B_2>,\quad \psi_3=<\frac{\partial B_1}{\partial t},B_2> .$$

Proof. From the assumption, we have

$$\frac{\partial T}{\partial t}=\frac{\partial}{\partial t}\frac{\partial \gamma}{\partial s}=\frac{\partial}{\partial s}\left(\beta_1 T+\beta_2 N+\beta_3 B_1+\beta_4 B_2\right).$$

Thus, it is seen that

$$\frac{\partial T}{\partial t}=\left(\frac{\partial \beta_1}{\partial s}+k_1\beta_2\right)T+\left(\frac{\partial \beta_2}{\partial s}+\beta_1 k_1-\beta_4 k_2\right)N$$
$$+\left(\frac{\partial \beta_3}{\partial s}+k_2\beta_2\right)B_1+\frac{\partial \beta_4}{\partial s}B_2. \quad (3.14)$$

Substituting (3.8) into (3.14), we get (3.10).
Since

$$\begin{aligned}
&< \ T,N >=0 \ \Rightarrow < T,\frac{\partial N}{\partial t}>=-\ \left(\frac{\partial \beta_2}{\partial s}+\beta_1 k_1-\beta_4 k_2\right),\\
&< \ T,B_1 >=0 \ \Rightarrow < T,\frac{\partial B_1}{\partial t}>=-\frac{\partial \beta_4}{\partial s},\\
&< \ T,B_2 >=0 \ \Rightarrow < T,\frac{\partial B_2}{\partial t}>=-\left(\frac{\partial \beta_1}{\partial s}+k_1\beta_2\right),\\
&< \ N,B_1 >=0 \ \Rightarrow < N,\frac{\partial B_1}{\partial t}>=-\psi_1,\\
&< \ N,B_2 >=0 \ \Rightarrow < N,\frac{\partial B_2}{\partial t}>=-\psi_2,\\
&< \ B_1,B_2 >=1 \Rightarrow < B_1,\frac{\partial B_2}{\partial t}>=-\psi_3,
\end{aligned}$$

we have

$$< N,\frac{\partial N}{\partial t}>=< B_1,\frac{\partial B_1}{\partial t}>=< B_2,\frac{\partial B_2}{\partial t}>=0.$$

In a similar manner above, we can obtain (3.11),(3.12) and (3.13).

Theorem 3.6. Let $\{T,N,B_1,B_2\}$ be the Frenet frame of a partially null curves γ and $\frac{\partial \gamma}{\partial t}=\beta_1 T+\beta_2 N+\beta_3 B_1+\beta_4 B_2$ be a smooth flow of a partially null curves γ in E_1^4 . If the curve flow is inextensible, then there exists the following system of partially differential equation.

$$\frac{\partial k_1}{\partial t}=\frac{\partial^2 \beta_2}{\partial s^2}+\frac{\partial(\beta_1 k_1)}{\partial s}-\frac{\partial(\beta_4 k_2)}{\partial s}-\frac{\partial \beta_4}{\partial s}k_2$$

Proof. From Lemma 3.4, we have

$$\begin{aligned}
\frac{\partial}{\partial s}\frac{\partial T}{\partial t} \ &= \ \left(\frac{\partial^2 \beta_2}{\partial s^2}+\frac{\partial(\beta_1 k_1)}{\partial s}-\frac{\partial(\beta_4 k_2)}{\partial s}\right)N\\
&+\left(\frac{\partial \beta_2}{\partial s}+\beta_1 k_1-\beta_4 k_2\right)(-k_1T+k_2B_1) \quad (3.15)\\
&\left(\frac{\partial^2 \beta_3}{\partial s^2}+\frac{\partial(\beta_2 k_2)}{\partial s}\right)B_1+\frac{\partial^2 \beta_4}{\partial s^2}B_2+\frac{\partial \beta_4}{\partial s}(-k_2N). \quad (3.16)
\end{aligned}$$

Then

$$\begin{aligned}\frac{\partial}{\partial s}\frac{\partial T}{\partial t} &= -\left(\frac{\partial \beta_2}{\partial s}k_1+\beta_1 k_1^2-\beta_4 k_1 k_2\right)T \\ &+\left(\frac{\partial^2 \beta_2}{\partial s^2}+\frac{\partial(\beta_1 k_1)}{\partial s}-\frac{\partial(\beta_4 k_2)}{\partial s}-\frac{\partial \beta_4}{\partial s}k_2\right)N \\ &+\left(\frac{\partial^2 \beta_3}{\partial s^2}+\frac{\partial(\beta_2 k_2)}{\partial s}+\frac{\partial \beta_2}{\partial s}k_2+\beta_1 k_1 k_2-\beta_4 k_2^2\right)B_1+\frac{\partial^2 \beta_4}{\partial s^2}B_2. \qquad (3.17)\end{aligned}$$

Note that

$$\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial s}\right)=\frac{\partial k_1}{\partial t}N-\left(\frac{\partial \beta_2}{\partial s}k_1+\beta_1 k_1^2-\beta_4 k_1 k_2\right)T+\psi_2 k_1 B_1+\psi_1 k_1 B_2. \qquad (3.18)$$

Hence from (3.16) and (3.17), we get

$$\frac{\partial k_1}{\partial t}=\frac{\partial^2 \beta_2}{\partial s^2}+\frac{\partial(\beta_1 k_1)}{\partial s}-\frac{\partial(\beta_4 k_2)}{\partial s}-\frac{\partial \beta_4}{\partial s}k_2.$$

This completes the proof.

Corollary 3.7 . Let $\{T,N,B_1,B_2\}$ be the Frenet frame of a partially null curves γ and $\frac{\partial \gamma}{\partial t}=\beta_1 T+\beta_2 N+\beta_3 B_1+\beta_4 B_2$ be a smooth flow of a partially null curves γ in E_1^4 .Then,we have the following equalities.

$$k_1=\frac{1}{\psi_1}\left[\frac{\partial^2 \beta_4}{\partial s^2}\right],$$

$$\frac{\partial^2 \beta_3}{\partial s^2}+\frac{\partial(\beta_2 k_2)}{\partial s}+\frac{\partial \beta_2}{\partial s}k_2+\beta_1 k_1 k_2-\beta_4 k_2^2-\psi_2 k_1=0.$$

Theorem 3.8. Let $\{T,N,B_1,B_2\}$ be the Frenet frame of a partially null curves γ and $\frac{\partial \gamma}{\partial t}=\beta_1 T+\beta_2 N+\beta_3 B_1+\beta_4 B_2$ be a smooth flow of a partially null curves γ in E_1^4 .Then, we have

$$k_1=-\left[\frac{\partial \psi_1}{\partial s}\Big/\frac{\partial \beta_4}{\partial s}\right] \qquad (3.19)$$

and

$$k_2=\frac{1}{\psi_1}\left[\frac{\partial \psi_3}{\partial s}\right]. \qquad (3.20)$$

Proof Noting that $\frac{\partial}{\partial s}\left(\frac{\partial B_1}{\partial t}\right)=\frac{\partial}{\partial t}\left(\frac{B_1}{\partial s}\right)$, we have the equations (3.19) and (3.20).

Theorem 3.9. Let $\{T,N,B_1,B_2\}$ be the Frenet frame of a partially null curves γ and $\frac{\partial \gamma}{\partial t}=\beta_1 T+\beta_2 N+\beta_3 B_1+\beta_4 B_2$ be a smooth flow of a partially null curves γ in E_1^4 .Then, there exists the following system of partially differential equation.

$$\frac{\partial k_2}{\partial t}=\frac{\partial \psi_2}{\partial s}+\frac{\partial \beta_3}{\partial s}k_1-\beta_2 k_1 k_2-\psi_3 k_2. \qquad (3.21)$$

Proof By the same way above and considering $\frac{\partial}{\partial s}\left(\frac{\partial B_2}{\partial t}\right)=\frac{\partial}{\partial t}\left(\frac{B_2}{\partial s}\right)$, we reach.

$$\frac{\partial k_2}{\partial t}=\frac{\partial \psi_2}{\partial s}+\frac{\partial \beta_3}{\partial s}k_1-\beta_2 k_1 k_2-\psi_3 k_2.$$

4 Inextensible Flows of pseudo null curve in E_1^4

We omit the proofs of the following theorems became of having close analogy of the theorems given above.

Lemma 4.1. Let $\{T, N, B_1, B_2\}$ be the Frenet frame of a pseudo null curve γ and

$$\frac{\partial \gamma}{\partial t} = \alpha_1 T + \alpha_2 N + \alpha_3 B_1 + \alpha_4 B_2$$

be a smooth flow of a pseudo null curve γ in E_1^4 .If $\gamma(s)$ be unit speed pseudo null curve in E_1^4 , with curvature $k_1(s) = 1$, $k_2(s)$ and $k_3(s) \neq 0$,then we have the following equality

$$\frac{\partial v}{\partial t} = \frac{\partial \alpha_1}{\partial u} - \alpha_4 v.$$

Lemma 4.2. Let $\{T, N, B_1, B_2\}$ be the Frenet frame of a pseudo null curve γ and $\frac{\partial \gamma}{\partial t} = \alpha_1 T + \alpha_2 N + \alpha_3 B_1 + \alpha_4 B_2$be a smooth flow of a pseudo null curve γ in E_1^4 , with curvature $k_1(s) = 1$, $k_2(s)$ and $k_3(s) \neq 0$.If the flow is inextensible, then we have the following equality

$$\frac{\partial \alpha_1}{\partial u} = \alpha_4 v.$$

Lemma 4.3 Let $\{T, N, B_1, B_2\}$ be the Frenet frame of a pseudo null curve γ and $\frac{\partial \gamma}{\partial t} = \alpha_1 T + \alpha_2 N + \alpha_3 B_1 + \alpha_4 B_2$ be a smooth flow of a pseudo null curve γ in E_1^4 .The differentiations of $\{T, N, B_1, B_2\}$ with respect to t is

$$\frac{\partial T}{\partial t} = \left(\frac{\partial \alpha_2}{\partial s} + \alpha_1 + \alpha_3 k_3\right) N + \left(\frac{\partial \alpha_3}{\partial s} + \alpha_2 k_2 - \alpha_4 k_3\right) B_1 + \left(\frac{\partial \alpha_4}{\partial s} - \alpha_3 k_2\right) B_2,$$

$$\frac{\partial N}{\partial t} = -\left(\frac{\partial \alpha_4}{\partial s} - \alpha_3 k_2\right) T + \psi_2 N + \psi_1 B_1,$$

$$\frac{\partial B_1}{\partial t} = -\left(\frac{\partial \alpha_3}{\partial s} + \alpha_2 k_2 - \alpha_4 k_3\right) T + \psi_3 N - \psi_1 B_2,$$

$$\frac{\partial B_2}{\partial t} = -\left(\frac{\partial \alpha_2}{\partial s} + \alpha_1 + \alpha_3 k_3\right) T - \psi_3 B_1 - \psi_2 B_2,$$

where

$$\psi_1 = < \frac{\partial N}{\partial t}, B_1 >, \quad \psi_2 = < \frac{\partial N}{\partial t}, B_2 >, \quad \psi_3 = < \frac{\partial B_1}{\partial t}, B_2 > .$$

Theorem 4.4. Let $\{T, N, B_1, B_2\}$ be the Frenet frame of a pseudo null curve γ and $\frac{\partial \gamma}{\partial t} = \alpha_1 T + \alpha_2 N + \alpha_3 B_1 + \alpha_4 B_2$ be a smooth flow of a pseudo null curve γ in E_1^4 . If the curve flow is inextensible, then there exists the following system of partially differential equations

$$\frac{\partial^2 \alpha_3}{\partial s^2} + \frac{\partial (\alpha_2 k_2)}{\partial s} - \frac{\partial (\alpha_4 k_3)}{\partial s} - \frac{\partial \alpha_4}{\partial s} k_3 + \frac{\partial \alpha_2}{\partial s} k_2 + \alpha_1 k_2 + 2\alpha_3 k_3 k_2 - \psi_1 = 0,$$

$$\frac{\partial^2\alpha_2}{\partial s^2}+\frac{\partial\alpha_1}{\partial s}+\frac{\partial(\alpha_3k_3)}{\partial s}+\frac{\partial\alpha_3}{\partial s}k_3+\alpha_2k_2k_3-\alpha_4k_3^2-\psi_2=0,$$

$$\frac{\partial^2\alpha_4}{\partial s^2}-\frac{\partial(\alpha_3k_2)}{\partial s}-\frac{\partial\alpha_3}{\partial s}k_2-\alpha_2k_2+\alpha_4k_2k_3=0,$$

$$\frac{\partial k_3}{\partial s}=\frac{\partial\psi_3}{\partial s}-\frac{\partial\alpha_3}{\partial s}-\alpha_2k_2+\alpha_4k_3-\psi_2k_3,$$

$$\frac{\partial k_2}{\partial s}=\frac{\partial\psi_1}{\partial s}+\psi_2k_2,$$

$$\frac{\partial\psi_2}{\partial s}=\frac{\partial\alpha_4}{\partial s}-\alpha_3k_2-\psi_1k_3+\psi_2k_3.$$

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References

[1] H.Balgetir, M.Bektaş, M. Ergüt, On a characterization of null helix, Bull. Inst. Math. Acad. Sinica 29, 71-78, 2001.

[2] H.Balgetir, M. Bektaş and J.Inoguchi, Null Bertrand curves in Minkowski 3-space and their characterizations, Note Mat. 23, no. 1, 7-13, .2004.

[3] M. Barros, General helices and a theorem of Lancret, Proc. Amer. Math. Soc. 125,1503-1509, 1997.

[4] S. Bas and T. Körpınar, Inextensible Flows of Spacelike Curves on Spacelike Surfaces according to Darboux Frame in M_1^3 , Bol. Soc. Paran. Mat. 31 (2) , 9–17, 2013.

[5] T.Körpınar and E.Turhan, New Inextensible Flows of Timelike Curves on the Oriented Timelike Surfaces According to Darboux Frame in M_1^3, AMO, Volume 14, Number 2, 2012.

[6] G. Chirikjian, J. Burdick, A modal approach to hyper-redundant manipulator kinematics,IEEE Trans. Robot. Autom. 10, 343–354,1994.

[7] M. Desbrun, M.P. Cani-Gascuel, Active implicit surface for animation, in: Proc. Graphics Interface Canadian Inf. Process. Soc., 143–150, 1998.

[8] A.Ferrandez, A. Gimenez and P. Lucas, Null helices in Lorentzian space forms, International Journal of Modern Physics A 16, 4845–4863, 2001.

[9] N. Gürbüz, Inextensible flows of spacelike, timelike and null curves, Int. J. Contemp. Math. Sciences, Vol. 4, no. 32, 1599-1604, 2009.

[10] K.Ilarslan and E.Nesovic, Some Characterizations of Null, Pseudo Null and Partially Null Rectifiying Curves in Minkowski Space-Time, Taiwanese Journal of Mathematics,Vol 12, No:5,pp 1035-1044, 2008.

[11] M. Kass, A. Witkin, D. Terzopoulos, Snakes: active contour models, in: Proc. 1st Int. Conference on Computer Vision, 259–268,1987.

[12] M.Külahcı, M.Bektaş, M.Ergüt, Curves of AW(k)-type in 3-dimensional Null Cone, Physics Letters A. 371, 275-277, 2007.

[13] M.Külahcı, M.Bektaş,M.Ergüt, On Harmonic Curvatures of Null Curves of the AW(k)-Type in Lorentzian Space Z. Naturforsch. 63a, 248 – 252, 2008.

[14] D. Y. Kwon, F.C. Park, D.P. Chi, Inextensible flows of curves and developable surfaces, Appl. Math. Lett. 18, 1156-1162, 2005.

[15] D.Latifi, A.Razavi A , Inextensible Flows of Curves in Minkowskian Space, Adv. Studies Theor. Phys. 2(16): 761-768, 2008.

[16] H.Q. Lu, J.S. Todhunter, T.W. Sze, Congruence conditions for nonplanar developable surfaces and their application to surface recognition, CVGIP, Image Underst. 56, 265–285, 1993.

[17] H. Mochiyama, E. Shimemura, H. Kobayashi, Shape control of manipulators with hyper degrees of freedom, Int. J. Robot.Res., 18, 584–600, 1999.

[18] A.O. Ogrenmis, M. Yeneroğlu, Inextensible curves in the Galilean Space, International Journal of the Physical Sciences, 5(9),1424-1427, 2010.

[19] D.J. Unger, Developable surfaces in elastoplastic fracture mechanics, Int. J. Fract. 50, 33–38, 1991.

[20] Ö . G. Yıldız, S. Ersoy, M. Masal, A note on inextensible flows of curves on oriented surface, CUBO A Mathematical Journal Vol.16,No: 03, (11-19). October 2014.

[21] Ö . G. Yıldız, M. Tosun, S. O. Karakuş, A note on inextensible flows of curves in E^n, International Electronic Journal of Geometry Volume 6 No. 2 pp. 118-124 (2013).

[22] Ö.G. Yıldız and M.Tosun, A Note on Inextensible Flows of Curves in E_1^n, arXiv:1302.6082v1 [math.DG] 25 Feb 2013