Exploration

On Sinusoidal Time Variation of the Newtonian Gravitational Constant

G. G. Nyambuya[1]
National University *of* Science *&* Technology,
Faculty *of* Applied Sciences – School *of* Applied Physics,
Fundamental Theoretical and Astrophysics Group,
P. O. Box 939, Ascot, Bulawayo,
Republic *of* Zimbabwe.

Abstract

Recently, Anderson *et al.* (2015) published results showing strong correlation between the measured values of the Newtonian gravitational constant G and the 5.9 year oscillation of the length of day. Following this publication of Anderson *et al.*, (2015), Schlamminger *et al.* (2015) compiled a more complete set of the published measurements of G made in the last 35 years where they performed a least-squares regression to a sinusoid with period 5.90 years and found this fit to yields still a reasonable fit to these data thus somewhat putting credence to this claim of Anderson *et al.* (2015). However, it is yet to be established as to whether or not this signal is gravitational in origin. In this article, we point out that in principle this sinusoidal signal has a place in the gravitomagnetic model that we are currently working on.

Keywords: Time variation, sinusoidal, Newtonian Gravitational Constant, gravitomagnetic.

1 Introduction

Using about a dozen measurements of Earth-based laboratory measurements of Newton's gravitational 'constant' G made since 1962, Anderson et al. (2015) notes that these measurements have yielded values that differ by far more than their reported random plus systematic errors. Upon an inspection of these values by way of plotting them against time, Anderson et al. (2015) unearthed a very surprising, unexpected and interesting behaviour associated with these measurements – they find that these values for G have a well behaved oscillatory nature, with a period of $P_{\oplus} = 5.899 \pm 0.062\,\text{yr}$, and an amplitude of $G_A = (1.619 \pm 0.103) \times 10^{-14}\,\text{m}^3\text{kg}^{-1}\text{s}^{-2}$, that is to say:

$$G_{\text{eff}}^{\oplus} = G + G_A \cos\left(\frac{2\pi t}{P} + \phi\right), \tag{1.1}$$

where ϕ is a phase term. Anderson et al. (2015) find that the mean-value crossings occur in 1994 and 1997 and have the value:

$$G = (6.673899 \pm 0.000069) \times 10^{-11}\text{m}^3\text{kg}^1\text{s}^2. \tag{1.2}$$

The maximum absolute value of the time rate of change of G_{eff} *i.e.* $\dot{G}_{\text{eff}}/G_{\text{eff}} = 2\pi G_A/(G + G_A)P$ is equal to $(2.60 \pm 0.20) \times 10^{-4}\,\text{yr}^{-1}$. At any rate, judged on the yardstick of our present knowledge, this result (of $\dot{G}_{\text{eff}}/G_{\text{eff}}$) is not only unprecedented but a complete surprise as it is much larger that has been inferred from cosmological and astronomical phenomenon. In this letter, we demonstrate that this seemingly strange oscillatory nature in the values of G is well explained in the gravitomagnetic model that we are currently working on (see Nyambuya 2010, 2015*a*, Nyambuya et al. 2015, Nyambuya 2015*b*,*c*, 2014*a*,*d*, for the said on-going work). Actually, Anderson et al. (2015)'s findings come as a most welcome

[1]Correspondence: E-mail: physicist.ggn@gmail.com

present to the aforementioned on-going work (Nyambuya 2010, 2015*a*, Nyambuya et al. 2015, Nyambuya 2015*b*,*c*, 2014*a*,*d*).

Instead of embracing this potentially important discovery, Anderson et al. (2015) distance themselves from claiming that their discovery is a discovery to do with the actual variation of the gravitational constant G. They conservatively hold that G can not actually vary by this much and this quickly, but instead that something in the measurement process is varying. Anderson et al. (2015)'s scepticism is not shared by all.

Though with a weakened correlation, Anderson et al. (2015)'s findings seem to be confirmed by Schlamminger et al. (2015)'s regression analysis which makes use of a much more complete set of 26 G-measurements made in the last 35 years. Schlamminger et al. (2015)'s regression analysis maintain the same period $P_{\oplus} = 5.899 \pm 0.062\,\mathrm{yr}$. In-order to seek an independent confirmation of this 'potential' discovery, one researcher of note, Professor Lorenzo Iorio, of the Ministero dell'Istruzione, dell'Universitàedella Ricerca in Italy; took the first bold step in accepting Anderson et al. (2015)'s potentially landmarking discovery by applying this discovery to the planetary ephemerides of Solar planetary orbits (Iorio 2015). This is perhaps the right step in the right direction, but, it is our view that the source of the variation must be investigated to see if it is intrinsic to Earth-based G-measurements. However, in a recent paper, Professor Iorio (2016) concludes that Solar planetary orbital motions do not support Anderson et al. (2015)'s variable-G observations. The ideas that we propagate herein suggest that this variation may be a gravitational effect intrinsic to the Earth and every gravitational body will have its own period of the variation of $G^{\oplus}_{\mathrm{eff}}$. It is for this reason that we have attached the Earth's symbols ($\oplus$) to $G^{\oplus}_{\mathrm{eff}}$ and $P_{\oplus}$.

Given the fundamental role played by G in the currently accepted theory of gravitation and the attempts to merge it with quantum mechanics, it is important to put to the test the hypothesis that the aforementioned harmonic variation may pertain G itself in a direct and independent way. The bounds on $\dot{G}/G$ existing in the literature inferred from Satellite Laser Ranging, Lunar Laser Ranging (cf. Williams et al. 2004, Müller & Biskupek 2007), astronomical and cosmological phenomenon (cf. Pitjeva & Pitjev 2012, 2013, Pitjev & Pitjeva 2013, Pitjeva 2013, Pitjeva & Pitjev 2014) are such that $\dot{G}/G \sim 10^{-13\pm1}\,\mathrm{yr}^{-1}$, that is $8 - 10$ orders of magnitude smaller than Anderson *at al.*'s signal. We are of the strong feeling and view that Anderson *at al.*'s variation is not to be linked to these secular measurements as this sinusoidal signal may very well be due a completely different physical phenomenon altogether. Since, we may not extended straightforwardly these G-variations to Anderson *at al.*'s G-variation. Within the confines of the ideas that we shall presents, our presentation will qualify our feelings and view points

2 Gravitomagnetism

Currently, gravitatomagnetism is predominately understood (cf. Nordtvedt 1988, Ashby & Shahid-Saless 1990, Soffel et al. 2008, Chicone & Mashhoon 2011, Iorio 2011, Adler et al. 2012, Bini et al. 2015) in the context of Einstein (1915)'s linearised first order approximation of the General Theory of Relativity (GTR) which assumes that ($\dot{G}/G \equiv 0$). Our approach is different to this predominant approach. We consider gravitomagnetism as a exact theory independent of the GTR in much the same way it was conceived by Maxwell (1865) and HeaHeaviside (1893, 1894) and further championed (in modern times) *e.g.* by Jefimenko (2000), Behera (2006) amongst others. The present gravitomagnetic theory falls within the realm of a more ambition attempt that we are currently working on *i.e.*, an attempt at an all encompassing Unified Field Theory (UFT) of all the forces of *Nature* (Nyambuya 2014*a*,*d*). We shall say nothing about this attempt but direct the reader to these cited works. That said, shall now move to give a brief overview of our gravitomagnetic model, that is, in the subsequent subsections, we will make an attempt to give the reader our vision (version) of gravitomagnetism.

2.1 ASTG-model

We have presented (in Nyambuya 2010, 2015*a*) a new theory of gravitation that we have coined the Azimuthally Symmetric Theory *of* Gravitation (hereafter ASTG-model). This new theory is built from the ordinary azimuthally symmetric solutions of the well known Poisson-Laplace equation, namely:

$$\boldsymbol{\nabla}^2\Phi = 4\pi G\varrho, \tag{2.1}$$

where G is Newton's universal constant of gravitation, Φ is the gravitational potential, ϱ is the density of matter and $\boldsymbol{\nabla}^2$ is the usual Laplacian operator.

In its bare form, Newtonian gravitation is obtained as a radial solution [*i.e.* $\Phi = \Phi(r)$] of (2.1). In coming-up with the ASTG-model, we wondered what the azimuthal solutions [*i.e.* $\Phi = \Phi(r, \theta)$] of this equation really mean in so far a gravitation is concerned. While in our state of wonderment, it is then that we decided to investigate the meaning of the azimuthal solution [$\Phi = \Phi(r, \theta)$] of this equation with regard to the well known precession of the perihelia of Solar planetary orbits. As demonstrated in the readings Nyambuya (2010, 2015*a*), we obtained satisfactory results and these results enabled us to have a modicum of confidence that these azimuthal solutions may carry with them the potent seed to explain some of the known gravitational anomalies such as the $\sim 7-15\,\text{cm}$ annual drift of the Earth-moon system (Krasinsky & Brumberg 2004, Standish 2005) and as-well the $\sim 38\,\text{mm}$ annual drift of the Moon from the Earth (Williams et al. 2004, Williams & Boggs 2009), the Pioneer anomaly (Anderson et al. 1998, 2002) and the flyby anomaly problem (Antreasian & Guinn 1998, Anderson et al. 2007, 2008).

The azimuthal solution $\Phi = \Phi(r, \theta)$ of (2.1) that we obtained is such that:

$$\Phi(r,\theta) = -\frac{G\mathcal{M}}{r}\left[1 + \sum_{\ell=1}^{\infty}\lambda_\ell\left(\frac{G_{\ell 1}\mathcal{M}}{rc^2}\right)^{\ell}\mathcal{P}_\ell(\sin\theta)\right], \tag{2.2}$$

where $\mathcal{M}$ is the mass of the central gravitating body, c is the speed of light in vacuum, r is the radial distance from this gravitating body, and $\lambda_\ell : \ell = 1, 2, \ldots$ *etc* are some dynamic parameters which in the ASTG-model are assumed to be related to gravitating body in question and the explicit dependence of these λ-parameters on the gravitating body's spin was preliminarily worked-out in the reading Nyambuya (2010) and now has further been worked-out and made more clear in the reading Nyambuya (2015*a*). The function $\mathcal{P}_\ell(\sin\theta)$ in (2.2) is the (symmetric) Legendre polynomial and has been defined in an unusual manner in the reading Nyambuya (2015*a*).

2.2 Gravitational Four Poisson-Laplace Equation

In the case of dynamic gravitational fields, *i.e.*, non-static time-dependent gravitational fields, the Poisson-Laplace equation (2.1) upon whose shoulders the ASTG-model stands; this equation (2.1) is not Lorentz invariant and apart from this, it is obvious and clear that this equation will fail to describe a time-dependent gravitational field as it is not in its natural form equipped to do this. In-order to make it Lorentz invariant, one can add a time dependent term, in which event the resulting equation is the four Poisson-Laplace equation *i.e.*:

$$\boldsymbol{\nabla}^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = 4\pi G\varrho. \tag{2.3}$$

In the reading Nyambuya (2014*b*), equation (2.3) has been solved for five gravitational potentials and this has been done in the context of the gravitomagnetic theory given in the reading Nyambuya (2014*c*). It must be mentioned here that the gravitomagnetic theory proposed in the reading Nyambuya (2014*b*) is not championed in the same spirit as the gravitatomagnetic theory that emerges from the GTR in the weak field approximation – no! It is being championed from the vantage point of the proposed *Unified Field Theory* of all the forces of *Nature* (Nyambuya 2014*d*) *albeit*, in the original quest and spirit of Maxwell (1865) and Heaviside (1893, 1894) gravitomagnetic theory.

In this same aforementioned reading Nyambuya (2014*b*), it is shown in that the equation (2.3) admits five solutions, $\Phi_{(j)}$, which in the radial case *i.e.* $\Phi_{(j)} = \Phi_{(j)}(r) : j = 1, 2, 3, 4, 5$; these solutions are as follows – for the case $(j = 1)$:

$$\Phi_{(1)} = -\frac{G_{01}\mathcal{M}}{r}, \tag{2.4}$$

where $\Phi_{(1)}$ is the usual Newtonian gravitational potential and $(G_{01} = G)$ is the Newtonian gravitational constant. In the theory presented in the reading Nyambuya (2014*b*), it is assumed that the Newtonian gravitational constant G is not a time variable, it is an absoulte time and space constant. Therein the reading Nyambuya (2014*b*), its is seen that making G a time variable leads to a theory that at best can be said to be unphysical. So, we set this parameter to be a true constant with neither a space nor a time variation.

Now, for the case $(j = 2)$, we have:

$$\Phi_{(2)} = -\frac{G_{02}(t)\mathcal{M}e^{-\mu_{02}r}}{r}, \tag{2.5}$$

where $G_2(t) = G_2(0)e^{-\mu_2 ct}$ is the time variable gravitational constant associated with this potential and this constant has the same dimensions as the Newtonian gravitational constant G, and μ_2 is parameter with the dimensions of inverse length. This potential has been slated for investigation of the Pioneer anomaly (Anderson et al. 1998, 2002). The Yukawa potential (2.5) has been used to try and explain the Pioneer anomaly (see *e.g.* Brownstein & Moffat 2006, Iorio 2007*a*,*b*) and the rotation curves of spiral galaxies (see *e.g.* Moffat 1995, 2005). The fact that we derive this potential within the framework gravitomagnetism justifies not only its applications in gravitational physics but its existence in the gravitation physics.

Despite the fact that prevailing wisdom holds that the Pioneer anomaly (cf. Anderson et al. 1998, 2002) may not be a gravitational phenomenon but a result on-board problems to do with the heating systems of the spacecrafts (cf. Iorio 2007*a*, Turyshev & Toth 2010, Turyshev et al. 2011, 2012), in a future reading, the Yukawa potential (2.5) together with the potential $\Phi_{(4)}$ to the problem of the Pioneer anomaly. We are of the strong view that the resolution of the Pioneer anomaly may very well be far from resolved, all plausible causes must be considered and only a dedicated mission is going to decide which of the proposed mechanisms corresponds with reality. As said, in the future, we will be applying the Yukawa potential (2.5) together with the potential $\Phi_{(4)}$ to the Pioneer anomaly.

Moving on ... for the case $(j = 3)$, we have:

$$\Phi_{(3)} = -\frac{G_{03}(t)\mathcal{M}\cos(\mu_3 r)}{r}, \tag{2.6}$$

where $G_{03}(t) = G_{03}(0)\cos(\mu_3 ct)$ is the time variable gravitational potential associated with this potential and this constant has the same dimensions as the Newtonian gravitational constant G, and μ_3 constant parameter with the dimensions of inverse length. The potential (2.6) is a new gravitational potential and this potential has been slated for investigation of the existence of rings system around planets. In the present reading we shall make use of this potential where we shall make the endeavour to identify it with the sinusoidal variation of the Newtonian gravitational constant discovered by Anderson et al. (2015).

For the case $(j = 4)$, we have:

$$\Phi_{(4)} = -\frac{G_4\mathcal{M}}{\mathcal{R}}\left[\left(\frac{\mathcal{R}}{r}\right)^{\alpha_4} + \kappa_4\left(\frac{\mathcal{R}}{r}\right)^{1-\alpha_4}\right], \tag{2.7}$$

where as before $G_4(r, t)$ is a space and time variable gravitational potential associated with this potential and $\mathcal{R}$ is the radius of the gravitating object in-question and α is constant while κ_4 is a dimensionless parameter which is such that $(|\kappa_4| \geq 0)$. The potential (2.7) is a new gravitational potential and this potential has been slated for investigation of the flat rotation curves of spiral galaxies.

Lastly, for the case $(j = 5)$, we have:;

$$\Phi_{(5)} = -\frac{G_5(r,t)\mathcal{M}}{\mathcal{R}}\left(\frac{\mathcal{R}}{r}\right)^{\frac{1}{2}}\cos\left[\ln\left(\frac{\mathcal{R}}{r}\right)^{\alpha_5}\right], \tag{2.8}$$

and again $G_5(r,t)$ is the time and space variable gravitational potential associated with this potential. Like the other potentials, this potential (2.8) is a new gravitational potential; it has been slated for investigation of the origins of the Titius-Bode Law.

The potentials (2.4), (2.5), (2.6), (2.7) and (2.8) all have a radial dependence. Expect for (2.4), the azimuthal gravitational components of (2.5), (2.6), (2.7) and (2.8) have up to now not been worked out. Be that it may, these potentials will suffice for the work we intent to carry out here. All the potentials (2.4), (2.5), (2.6), (2.7) and (2.8) are assumed to act simultaneously on any gravitating body so that the total or resultant gravitational potential Φ is such that:

$$\Phi_{\text{eff}} = \sum_{j=1}^{5}\Phi_{(j)}. \tag{2.9}$$

In the next section, we will justify the existence of equation (2.3) as a gravitational equation and thereafter, we shall demonstrate how the potential Φ_3 does account – in principle – for the sinusoidal time variation of the Newtonian gravitational constant.

2.3 Fundamental Basis for Gravitomagnetism

The first to consider the possibility of a formal analogy between gravitation and electromagnetism is James Maxwell (1831 – 1879), in his treatise on "*A Dynamical Theory of the Electromagnetic Field*" (Maxwell 1865). After failing to justify (to himself) the implied negative energies associated with the gravitational field, Maxwell abandoned this line of thought (Behera 2006). Twenty eight years had to pass before Oliver Heaviside (1850 – 1925) reconsidered Maxwell's imaginative and brilliant but forgotten thoughts (Heaviside 1893, 1894). However, both Maxwell (1865) and Heaviside (1893, 1894)'s work were speculative with no real justification from the then known fundamental principles of either physics or logic but rather from the intuitive power of human reason and imagination. Only in recent times, has a justifiable fundamental physical basis for Maxwell-Heaviside's speculation (hereafter, the Maxwell-Heaviside Gravitomagnetic Theory) been heralded (in *e.g.*, Behera 2006, Jefimenko 2000, Hera 2007).

From a fundamental axiomatic approach, José Hera (2007) formulated an important *Existence Theorem* that states that, given any space and time-dependent localized scalar (ϱ) and vector sources ($\boldsymbol{J}$) satisfying the continuity equation *i.e.*:

$$\frac{\partial\varrho}{\partial t} = -\boldsymbol{\nabla}\cdot\boldsymbol{J}, \tag{2.10}$$

as is the case with gravitation and electromagnetism; then, there exists two retarded fields ($\boldsymbol{X}, \boldsymbol{Y}$) that satisfy a set of four field equations, namely:

$$\boldsymbol{\nabla}\cdot\boldsymbol{X} = \alpha\varrho, \tag{2.11}$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{Y} = 0, \tag{2.12}$$

$$\boldsymbol{\nabla}\times\boldsymbol{X} + \gamma\frac{\partial\boldsymbol{Y}}{\partial t} = 0, \tag{2.13}$$

$$\boldsymbol{\nabla}\times\boldsymbol{Y} - \frac{\beta}{\alpha}\frac{\partial\boldsymbol{X}}{\partial t} = \beta\boldsymbol{J}, \tag{2.14}$$

where α, β, γ are arbitrary positive constants and are related to the speed of light c by the equation ($\alpha = \beta\gamma c^2$). By applying the theorem to the usual electromagnetic charge and current densities, the retarded fields are identified with the electric ($\boldsymbol{X} := \boldsymbol{E}$) and magnetic ($\boldsymbol{Y} := \boldsymbol{B}$) fields and the associated field equations with Maxwell's equations provided the following holds true: ($\alpha = 1/\varepsilon_0$), ($\beta = \mu_0$) and ($\gamma = 1$).

Clearly, this axiomatic approach of deriving Maxwell's field equations strongly suggests that electric charge conservation – and nothing else; can be considered to be the most fundamental assumption underlying Maxwell's equations of Electrodynamics. As has been argued in the readings Nyambuya (2015*b*,*c*), this Existence Theorem of José Hera (2007) can easily be extended to gravitomagnetism thus justify the Maxwell-Heaviside hypothesis (Maxwell 1865, Heaviside 1893, 1894). In this way, the four Poisson-Laplace equation (2.3) finds its just reason for existence and this existence is independent of the fact that we are working on our gravitomagnetic theory from a completely different approach (Nyambuya 2014*a*,*d*) to the conventional (Nordtvedt 1988, Ashby & Shahid-Saless 1990, Soffel et al. 2008, Chicone & Mashhoon 2011, Iorio 2011, Adler et al. 2012). Now, we shall move to identify with the framework of our gravitomagnetic theory, the sinusoidal G-component of Anderson et al. (2015).

3 Anderson *at al.*'s Sinusoidally Time Varying G

The 13 measurements that were used by Anderson et al. (2015) in their unearthing of the sinusoidal time variation of G, these measurements have been made in Earth-based laboratories, thus, if the origins of this sinusoidal time varying component in the measurements are of a gravitational nature, they must have something to with the gravitational field of the Earth having a sinusoidal time varying component. We can readily identify this signal from the potential given in (2.6).

If say the gravitational field of the Earth is dominated by two potentials, *i.e.*, the Newtonian potential (2.4) and the sinusoidal potential (2.6), then the gravitational field of the Earth ($\boldsymbol{g} = \boldsymbol{F}_{\text{eff}}/\text{m}$) will be given is in equation (3.1).

$$\frac{\boldsymbol{F}_{\text{eff}}}{\text{m}} = -\frac{G\mathcal{M}}{r^2}\hat{\boldsymbol{r}} - \left[\frac{G_{03}(t)\mathcal{M}\cos(\mu_3 r + \phi_3)}{r^2} + \frac{\mu_3 G_{03}(t)\mathcal{M}\cos(\mu_3 r + \phi_3)}{r}\right]\hat{\boldsymbol{r}} \tag{3.1}$$

If we are going to have ($r \ll 1/\mu_3$), then, in magnitude, the term $[\mu_3 G_3(r,t)\mathcal{M}\text{m}/r]$ will be much smaller than the other two terms, in-which event, we will have:

$$\frac{\boldsymbol{F}_{\text{eff}}}{\text{m}} = -\frac{G\mathcal{M}}{r^2}\hat{\boldsymbol{r}} - \frac{G_3(t)\mathcal{M}\cos(\mu_3 r + \phi_3)}{r^2}\hat{\boldsymbol{r}}, \tag{3.2}$$

and this can be written compactly as:

$$\frac{\boldsymbol{F}_{\text{eff}}}{\text{m}} = -\frac{G_{\text{eff}}\mathcal{M}}{r^2}\hat{\boldsymbol{r}}, \tag{3.3}$$

where $[G_{\text{eff}} = G + G_{03}(t)\cos(\mu_3 r + \phi_3)]$. Since ϕ_3 is a phase term, we can always conveniently set to equal zero. With this as given, we know that if $\mu_3 r$ is small – which is the case when ($r \ll 1/\mu_3$); then, $[\cos(\mu_3 r) \simeq 1]$, so that $[G_{\text{eff}} = G + G_{03}(t)\cos(\mu_3 r + \phi_3)]$ becomes:

$$G_{\text{eff}} \simeq G + G_{03}(t). \tag{3.4}$$

Now, if the above described is what is happening in reality, then, the sinusoidal G-component emanating from the third gravitational component (Φ_3) is here superimposed on any measurement of the Newtonian gravitational constant. Under the given circumstances, the term $G_A\cos(2\pi t/P + \phi)$ in (1.1) must equal $G_{03}(t)$ in (3.4). We know that ($\mu_3 c = 2\pi/P$), and from this it follows that:

$$\mu_3^{\oplus} = \frac{2\pi}{cP_{\oplus}} = (1.13 \pm 0.01) \times 10^{-16}\,\text{m}^{-1} \tag{3.5}$$

The value of μ_3 may defer depending perhaps on the mass ($\mathcal{M}$) and the radius ($\mathcal{R}$) of the gravitating body in question *i.e.* $\mu_3 = \mu_3(\mathcal{M}, \mathcal{R})$. This is something that can only be verified by observational data and our on-going work on the origins of planetary ring systems is taking this approach with the hope of finding a correlation.

4 General Discussion

Within the gravitomagnetic model (Nyambuya 2010, 2015*a*, Nyambuya et al. 2015, Nyambuya 2015*b*,*c*, 2014*a*,*d*) that we are currently working on, we have shown that the sinusoidal component discovered by Anderson *at al.* Anderson et al. (2015) in the Newtonian gravitational has a place. We have not made any effort to justify this using any data, but merely pointed out that this sinusoidal signal might proof to be a real gravitational effect whose origins is that third component of the force. In our model, the gravitational field has five components (presented in Equations 2.4, 2.5, 2.6, 2.7 and 2.8). These five components seem to hold the potential to explain structure formation in gravitational systems such as planetary rings seen in the *Solar Gas Giants* planets Jupiter, Saturn, Uranus and Jupiter, and the emergence of the Titius-Bode Law.

We are currently working on the gravitational component Φ_3 in-order to try and explain planetary ring systems. Surely, Anderson et al. (2015) give our ideas a reason to believe that we may very well be on the right path, for how were we to justify all these potentials (2.4, 2.5, 2.6, 2.7 and 2.8) except if we are have physical phenomenon that can be explained by them such as this sinusoidal sign evident in Newton's gravitational constant?

In closing, we state that since this sinusoidal G-signal is hypothesised here to be of gravitational in nature, independent confirmations should be sought in the ephemerides of test bodies that are affected by the Earth's gravitational field such as the orbiting satellites. For these test bodies, a signal with the same period $\sim 5.90\,\mathrm{yr}$ should be'visible' in, e.g., the variation of the eccentricity of the orbit. Such kind of evidence will be a clear indication of this sinusoidal G-signal to be of gravitational in nature. For now, researchers can only speculate.

Acknoweledgements: We are grateful to the National University *of* Science *&* Technology (NUST)'s Research *&* Innovation Department and Research Board for their unremitting support rendered toward our research endeavours; of particular mention, Prof. Dr. P. Mundy, Dr. P. Makoni, Dr. D. J. Hlatywayo and Prof. Dr. Y. S. Naik's unwavering support.

Received Feb. 29, 2016; Accepted March 4, 2016

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