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Five-dimensional FRW String Cosmological Models with Bulk Viscosity in Brans-Dicke Theory of Gravitation

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Abstract

This paper deals with spatially homogeneous anisotropic five dimensional FRW cosmological models in a scalar tensor theory of gravitation proposed by Brans and Dicke [1] when the source for energy momentum tensor is a bulk viscous fluid containing one dimensional cosmic string. To obtain a determinate solution of the field equations we have used a power law between scalar field and the scale factor of the universe. All the models obtained and presented here are expanding, non-rotating and accelerating. Also some important features of the models, thus obtained, have been discussed.

Keywords: Five-dimensional, FRW metric, bulk viscosity, cosmic strings, Brans– Dicke theory.

1. Introduction

Investigation of relativistic cosmological models usually involves the energy momentum tensor of matter generated by a perfect fluid. One must take into account viscosity mechanism to consider more realistic model, since viscosity mechanism has attracted the attention of many researches. Misner [2, 3] suggested the strong dissipation due to neutrino viscosity may considerably reduce the anisotropy of the black body radiation. Weinberg [4, 5] suggested that a viscosity mechanism in cosmology can explain the unusual high entropy per baryon in present events. Waga et al. [6] , Pacher et al. [7], Guth [8] and Murphy [9] have shown that bulk viscosity associated with the grand unified theory phase transition. In the early evolution of the universe the bulk viscosity is very important. There are many circumstances in the evolution of the universe in which bulk viscosity could arise. The bulk viscosity coefficient determines the magnitude of the viscous stress relative to the expansion.

Spatially homogeneous and anisotropic cosmological models play a significant role in the description of the large scale behavior of the universe. In search of a realistic picture of the early universe such models have been widely studied within a framework of General Relativity. In

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order to study the evolution of the universe, many authors constructed cosmological models containing viscous fluid. Cosmological models with bulk-viscosity are important since bulk-viscosity has a greater role in getting accelerated expansion of the universe popularly known as inflationary phase. The role of bulk viscosity in the cosmic evolution, especially as its early stages, seems to be significant. The distribution of matter can be satisfactorily described by a perfect fluid due to the large scale distribution of galaxies in our universe. However, observed physical phenomena such as the large entropy per baryon and the remarkable degree of isotropy of the cosmic microwave background radiation, suggest analysis of dissipative effects in cosmology.

Furthermore, there are several processes which are expected to give rise to viscous effects. These are the decoupling of neutrinos during the radiation era and the decoupling of radiation and matter during the recombination era. Bulk viscosity is associated with the GUT phase transition and string creation. The presence of viscosity in the fluid gives many interesting features in the dynamics of homogeneous cosmological models. The possibility of bulk viscosity leading to inflationary like solutions in general relativistic FRW models has been discussed by several authors. Barrow [10] , Padmanabhan and Chitre [11], Pavon et al. [12], Martens [13] , Lima et al. [14] , Wang [15,16,17] , Bali and Dave [18], Bali and Pradhan [19] , Tripathy et al. [20, 21] and Rao et al. [22] have studied various Bianchi type cosmological models in the presence of cosmic strings and bulk viscosity. Rao and Sireesha [23] have discussed axially symmetric string cosmological model with bulk viscosity in self-creation theory of gravitation.

It is well known that viscosity plays an important role in cosmology (Singh and Devi [24], Singh and Kale [25], Setare and Sheykhi [26] and Misner [27]). Also, bulk viscosity appears as the only dissipative phenomenon occurring in FRW models and has a significant role in getting accelerated expansion of the universe popularly known as inflationary space. Bulk viscosity contributes negative pressure term giving rise to an effective total negative pressure stimulating repulsive gravity. The repulsive gravity overcomes attractive gravity of matter and gives an impetus for rapid expansion of the universe hence cosmological models with bulk viscosity have gained importance in recent years.

The study of the cosmic strings has gained significant attention for explaining structure formation and evolution of the universe. Gravitational effects of cosmic strings in general relativity have been studied by several authors (Stachel [28], Vilenkin [29], Letelier [30] & Goetz [31]). Currently, cosmological models evolving from a string-dominated era and ending up in a particle-dominated era are of great interest, since there is no direct evidence of strings in the present day universe. The study of string theory has received considerable attention in cosmology. String cosmological models are attracting more and more attention of research workers since cosmic strings are important in the early stages of evaluation of the universe before the practical creation. Spontaneous symmetry breaking in elementary practical physics has given rise to topological defects known as cosmic strings. The gravitational effects of such objects are of practical interest since they are considered as possible seeds for galaxy formation and gravitational lenses. Rao et al. [32] have obtained Bianchi type-II,VIII & IX string cosmological models in Brans-Dicke theory of gravitation. Rao and Sireesha [33] have discussed higher dimensional string cosmological model in a scalar tensor theory of gravitation.

Brans-Dicke [1] theory of gravitation is a natural extension of general relativity which introduces an additional scalar field ϕ besides the metric tensor g_{ij} and dimensionless coupling constant ω. The Brans - Dicke field equations for combined scalar and tensor field are given by

$$G_{ij} = -8\pi\phi^{-1}T_{ij} - \omega\phi^{-2}\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) - \phi^{-1}(\phi_{i;j} - g_{ij}\phi_{;k}{}^{,k}) \tag{1.1}$$

and $$\phi_{;k}{}^{,k} = 8\pi(3+2\omega)^{-1}T \tag{1.2}$$

where $G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij}$ is an Einstein tensor, R is the scalar curvature, ω and n are constants, T_{ij} is the stress energy tensor of the matter and comma and semicolon denote partial and covariant differentiation respectively.

Also, we have energy – conservation equation

$$T^{ij}_{;j} = 0 \tag{1.3}$$

This equation is a consequence of the field equations (1.1) and (1.2).

Several aspects of Brans-Dicke cosmology have been extensively investigated by many authors. Rao et al. [34] have obtained axially symmetric string cosmological models in Brans – Dicke theory of gravitation. Rao and Sireesha [35] have studied Bianchi type-II, VIII and IX string cosmological models with bulk viscosity in Brans-Dicke theory of gravitation. Rao et al. [36] have obtained LRS Bianchi type - I dark energy cosmological model in Brans-Dicke theory of gravitation. Rao and Sireesha [37] have investigated Bianchi type-II, VIII & IX cosmological models with strange quark matter attached to string cloud in Brans-Dicke and General theory of gravitation.

Higher dimensional cosmology is important because it has physical relevance to the early stages of evolution of the universe before it has undergone compactification transitions. Study of five dimensional space times is important because of the fact that cosmos at its early stage of evolution might have had a higher dimensional era. Marciano [38] suggested that the experimental detection of time variation of fundamental constants provide strong evidence for the existence of extra dimension. The extra dimension in the space time contracted to a very small size of Planck length or remains invariant. Further, during contraction process extra dimensions produce large amount of entropy which provides an alternative resolution to the flatness and horizon problem (Guth [8]). Witten [39], Appelquist et al. [40] and Chodos and Detweiler [41] were attracted to the study of higher dimensional cosmology because it has physical relevance to the early times before the universe has undergone compactification transitions. Banerjee et al. [42] have obtained Bianchi type-I cosmological models with viscous fluid in higher dimensional space time. Mohanty et al. [43] have obtained higher dimensional string cosmological model with bulk viscous fluid in Lyra manifold. Ramprasad et al. [44] have investigated five dimensional FRW bulk viscous cosmological models in Brans-Dicke theory of gravitation.

This paper is organised as follows: In section 2, we discuss metric and energy momentum tensor. In section 3, we obtain the solutions of the field equations. In section 4, we discuss some important properties of the universe including look-back time, distance modulus and luminosity distance versus red shift. Finally, the conclusions of the obtained model are presented in section 5.

2. Metric and Energy Momentum Tensor

We consider spatially homogeneous five dimensional FRW metric in the form

$$ds^2 = -dt^2 + R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right] + A^2(t)d\mu^2 \tag{2.1}$$

where $R(t)$ is the scale factor and $k = 0, -1 \, or \, +1$ is the curvature parameter for flat, open and closed Universe, respectively. The fifth coordinate μ is also assumed to be space like coordinate.

The energy momentum tensor for a bulk viscous fluid containing one dimensional string is given by

$$T_{ij} = (\rho + \bar{p})u_i u_j + \bar{p} g_{ij} - \lambda x_i x_j \tag{2.2}$$

$$\text{and} \quad \bar{p} = p - 3\xi H \,, \quad p = \epsilon_0 \rho \quad (0 \le \epsilon_0 \le 1) \tag{2.3}$$

where $\bar{p}$ is the total pressure which includes the proper pressure, ρ is the rest energy density of the system , λ is the tension in the string , $\xi(t)$ is the coefficient of bulk viscosity, $3\xi H$ is usually known as bulk viscous pressure, H is the Hubble parameter, u^i is the four velocity vector and x^i is a space-like vector which represents the anisotropic directions of the string.

Here u^i and x^i satisfy the equations

$$g_{ij}u^i u^j = -1,\ g_{ij}x^i x^j = 1, \text{ and } u^i x_i = 0. \tag{2.4}$$

We assume that the string be lying along the μ -axis. The one dimensional strings are assumed to be loaded with particles and the particle energy density is $\rho_p = \rho - \lambda$.

In a commoving coordinate system, we get

$$T_1^1 = T_2^2 = T_3^3 = \bar{p}, T_5^5 = \bar{p} - \lambda,\ T_4^4 = \rho \tag{2.5}$$

where $\rho, \lambda, \bar{p}$ and ϕ are functions of time 't' only.

3. Solutions of Field equations

Now with the help of (2.2) to (2.5), the field equations (1.1) for the metric (2.1) can be written as

$$\frac{\ddot{A}}{A} + \frac{2\ddot{R}}{R} + \frac{2\dot{R}\dot{A}}{RA} + \frac{\dot{R}^2}{R^2} + \frac{\omega\dot{\phi}^2}{2\phi^2} + \frac{\ddot{\phi}}{\phi} + \frac{2\dot{R}\dot{\phi}}{R\phi} + \frac{\dot{A}\dot{\phi}}{A\phi} + \frac{k}{R^2} = -8\pi\phi^{-1}\bar{p} \tag{3.1}$$

$$\frac{3\dot{R}\dot{A}}{RA} + \frac{3\dot{R}^2}{R^2} - \frac{\omega\dot{\phi}^2}{2\phi^2} + \frac{3\dot{R}\dot{\phi}}{R\phi} + \frac{\dot{A}\dot{\phi}}{A\phi} + \frac{3k}{R^2} = 8\pi\phi^{-1}\rho \tag{3.2}$$

$$\frac{3\ddot{R}}{R} + \frac{3\dot{R}^2}{R^2} + \frac{\omega\dot{\phi}^2}{2\phi^2} + \frac{\ddot{\phi}}{\phi} + \frac{3\dot{R}\dot{\phi}}{R\phi} + \frac{3k}{R^2} = -8\pi\phi^{-1}(\bar{p} - \lambda) \tag{3.3}$$

$$\ddot{\phi} + \dot{\phi}\left(\frac{3\dot{R}}{R} + \frac{\dot{A}}{A}\right) = 8\pi(3 + 2\omega)^{-1}(\lambda + \rho - 4\bar{p}) \tag{3.4}$$

$$\dot{\rho} + (\rho + \bar{p})(3\frac{\dot{R}}{R} + \frac{\dot{A}}{A}) - \lambda\frac{\dot{A}}{A} = 0 \tag{3.5}$$

Here the over head dot denotes differentiation with respect to't'.

Using the transformation $dt = R^3 A dT$, the above field equations (3.1) to (3.5) will reduce to

$$\frac{A''}{A} + \frac{2R''}{R} - \frac{3R'A'}{RA} - \frac{5R'^2}{R^2} - \frac{A'^2}{A^2} + \frac{\omega\phi'^2}{2\phi^2} + \frac{\phi''}{\phi} - \frac{R'\phi'}{R\phi} + kR^4A^2 = -8\pi\phi^{-1}\bar{p}(R^3A)^2 \tag{3.6}$$

$$\frac{3R'A'}{RA} + \frac{3R'^2}{R^2} - \frac{\omega\phi'^2}{2\phi^2} + \frac{3R'\phi'}{R\phi} + \frac{A'\phi'}{A\phi} + 3kR^4A^2 = 8\pi\phi^{-1}\rho(R^3A^2) \tag{3.7}$$

$$\frac{3R''}{R}-\frac{6R'^2}{R^2}-\frac{3R'A'}{RA}+\frac{\omega\phi'^2}{2\phi^2}+\frac{\phi''}{\phi}-\frac{2A'\phi'}{A\phi}+3kR^4A^2=-8\pi\phi^{-1}(\bar{p}-\lambda)(R^3A)^2 \tag{3.8}$$

$$\phi''=8\pi(3+2\omega)^{-1}(\lambda+\rho-4\bar{p})(R^3A)^2 \tag{3.9}$$

$$\rho'+(\rho+\bar{p})(3\frac{R'}{R}+\frac{A'}{A})-\lambda\frac{A'}{A}=0 \tag{3.10}$$

Here the overhead dash denotes differentiation with respect to 'T'.

The field equations (3.6) to (3.9) are only four independent equations with seven unknowns $R, A, \rho, \lambda, \bar{p}, \xi$ & ϕ which are functions of 'T'.Since these equations are non-linear in nature, in order to get a deterministic solution we take the following plausible physical conditions:

(1). The shear scalar σ is proportional to scalar expansion θ, so that we can take a linear relationship between the metric potentials R and A, i.e.,

$$A=R^n \tag{3.11}$$

where n is an arbitrary constant.

(2). The relation between the scalar field ϕ and the scale factor of the universe $a(t)$ given by (Pimental [45], Johri and Kalyani [46])

$$\phi=\phi_0 a^m \tag{3.12}$$

where ϕ_0 and $m>0$ are constants.

From equations (3.6) - (3.12), we get

$$\frac{R''}{R}+\frac{k_2}{k_1}\frac{R'^2}{R^2}+9\frac{k}{k_1}R^{4+2n}=0 \tag{3.13}$$

where $k_1=(3+n)\left[\frac{m(1-2\omega)+12}{4}\right]$ & $k_2=\frac{m}{4}(3+n)\left[\frac{m(3+n)(4-\omega)}{8}-(n+1-2\omega)\right]-6(2n+3)$

FRW Open cosmological model (k = -1):

If k = -1, from equation (3.13) we get

$$\frac{R''}{R}+\frac{k_2}{k_1}\frac{R'^2}{R^2}-9\frac{1}{k_1}R^{4+2n}=0 \qquad (3.14)$$

Let

$$R'=f(R),\ R''=f\,f'\,where\ f'=\frac{df}{dR} \qquad (3.15)$$

With the help of equation (3.15), equation (3.14) becomes

$$f^2={k_3}^2R^{2(n+3)} \qquad (3.16)$$

But

$$R'=f(R) \qquad (3.17)$$

Using (3.17), equation (3.16) becomes

$$R^{-(n+3)}\,dR=k_3dT \qquad (3.18)$$

From equation (3.18), we get

$$R=[(2-n)(k_3T+k_4)]^{1/2-n}\ ,\quad n\neq 2 \qquad (3.19)$$

where $k_3=\left[\frac{9}{(n+3)k_1+k_2}\right]$ and k_4 is an integrating constant.

From equations (3.11) & (3.19), we get

$$A=[(2-n)(k_3T+k_4)]^{n/2-n} \qquad (3.20)$$

From equations (3.12), (3.19)& (3.20), we get

$$\phi=\phi_0[(2-n)(k_3T+k_4)]^{\frac{m(3+n)}{4(2-n)}} \qquad (3.21)$$

From equations (3.7) & (3.19) -(3.21), we get

the energy density

$$\rho=k_5{k_3}^2[(2-n)(k_3T+k_4)]^{\frac{(n+3)(m-8)}{4(2-n)}-2}-3\phi_0[(2-n)(k_3T+k_4)]^{\frac{(n+3)(m-8)+8(n+2)}{4(2-n)}} \qquad (3.22)$$

where $k_5=\phi_0\left[3(n+1)+\frac{m}{4}(n+3)^2(1-\frac{m\omega}{8})\right]$.

From equations (3.6) & (3.19) - (3.21), we get

the total pressure

$$\bar{p} = k_6 k_3{}^2 [(2-n)(k_3T+k_4)]^{\frac{(n+3)(m-8)}{4(2-n)}-2} + \phi_0 [(2-n)(k_3T+k_4)]^{\frac{(n+3)(m-8)+8(n+2)}{4(2-n)}} \quad (3.23)$$

where $k_6 = \phi_0 \left\{ \frac{m}{4}(n+3)\left[\frac{m}{4}(n+3) - (2-n) - 1\right] + (n^2 - 3n + 7) + \frac{m^2\omega}{32}(n+3)^2 \right\}.$

The proper pressure is given by

$$p = \in_0 \rho = \in_0 k_5 k_3{}^2 [(2-n)(k_3T+k_4)]^{\frac{(n+3)(m-8)}{4(2-n)}-2} - 3\in_0 \phi_0 [(2-n)(k_3T+k_4)]^{\frac{(n+3)(m-8)+8(n+2)}{4(2-n)}} \quad (3.24)$$

From equations (3.8), (3.19),(3.20), (3.21)& (3.23), we get

the string tension density

$$\lambda = (k_6+k_7) k_3{}^2 [(2-n)(k_3T+k_4)]^{\frac{(n+3)(m-8)}{4(2-n)}-2} - 2\phi_0 [(2-n)(k_3T+k_4)]^{\frac{(n+3)(m-8)+8(n+2)}{4(2-n)}} \quad (3.25)$$

where
$$k_6 = \phi_0 \left\{ \frac{m}{4}(n+3)\left[\frac{m}{4}(n+3) - (2-n) - 1\right] + (n^2 - 3n + 7) + \frac{m^2\omega}{32}(n+3)^2 \right\} \&$$
$$k_7 = \phi_0 \left\{ \frac{m}{4}(n+3)\left[\frac{m}{4}(n+3) + \frac{\omega m}{8}(n+3) - (n+2)\right] - 9 \right\}.$$

The particle energy density is given by

$$\rho_p = \rho - \lambda = (k_5 - k_6 - k_7) k_3{}^2 [(2-n)(k_3T+k_4)]^{\frac{(n+3)(m-8)}{4(2-n)}-2} - \phi_0 [(2-n)(k_3T+k_4)]^{\frac{(n+3)(m-8)+8(n+2)}{4(2-n)}} \quad (3.26)$$

The coefficient of bulk viscosity is given by

$$\xi = \frac{1}{(n+3)k_3} \left\{ \begin{array}{l} k_3{}^2 [\in_0 k_5 - k_6][(2-n)(k_3T+k_4)]^{\frac{(n+3)(m-8)}{4(2-n)}-1} - \\ [3\in_0 +1]\phi_0 [(2-n)(k_3T+k_4)]^{\frac{(n+3)(m-8)+8(n+2)}{4(2-n)}+1} \end{array} \right\} \quad (3.27)$$

The metric (2.1), in this case, can be written as

$$ds^2 = -dt^2 + [(2-n)(k_3T+k_4)]^{2/2-n} \left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right] + [(2-n)(k_3T+k_4)]^{2n/2-n} d\mu^2$$
(3.28)

Thus (3.28) together with (3.21) - (3.27) constitutes an open five dimensional FRW string cosmological model with bulk viscosity in Brans-Dicke [1] scalar tensor theory of gravitation.

FRW Closed cosmological model (k = 1):

If k = 1, from equation (3.13) we get

$$\frac{R''}{R}+\frac{k_2}{k_1}\frac{R'^2}{R^2}+9\frac{1}{k_1}R^{4+2n}=0 \tag{3.29}$$

Let

$$R'=f(R),\ R''=f\ f'\ where\ f'=\frac{df}{dR} \tag{3.30}$$

With the help of equation (3.30), equation (3.29) becomes

$$f^2={k_3}^2R^{2(n+3)} \tag{3.31}$$

But

$$R'=f(R) \tag{3.32}$$

Using (3.32), equation (3. 31) becomes

$$R^{-(n+3)}\,dR=k_3dT \tag{3.33}$$

From equation (3.33), we get

$$R=[(2-n)(k_3T+k_4)]^{1/2-n}\ ,\quad n\neq 2 \tag{3.34}$$

where $k_3=\left[\frac{-9}{(n+3)k_1+k_2}\right]$ and k_4 is an integrating constant.

From equations (3.11) & (3.34), we get

$$A=[(2-n)(k_3T+k_4)]^{n/2-n} \tag{3.35}$$

From equations (3.12), (3.34)& (3.35), we get

$$\phi=\phi_0[(2-n)(k_3T+k_4)]^{\frac{m(3+n)}{4(2-n)}} \tag{3.36}$$

From equations (3.7) & (3.34) -(3.36), we get

the energy density

$$\rho=k_5{k_3}^2[(2-n)(k_3T+k_4)]^{\frac{(n+3)(m-8)}{4(2-n)}-2}+3\phi_0[(2-n)(k_3T+k_4)]^{\frac{(n+3)(m-8)+8(n+2)}{4(2-n)}} \tag{3.37}$$

where $k_5=\phi_0\left[3(n+1)+\frac{m}{4}(n+3)^2(1-\frac{m\omega}{8})\right].$

From equations (3.6) & (3.34) - (3.36), we get

the total pressure

$$\bar{p} = k_6 k_3{}^2[(2-n)(k_3T+k_4)]^{\frac{(n+3)(m-8)}{4(2-n)}-2} - \phi_0[(2-n)(k_3T+k_4)]^{\frac{(n+3)(m-8)+8(n+2)}{4(2-n)}} \quad (3.38)$$

where $k_6 = \phi_0\left\{\frac{m}{4}(n+3)\left[\frac{m}{4}(n+3)-(2-n)-1\right]+(n^2-3n+7)+\frac{m^2\omega}{32}(n+3)^2\right\}.$

The proper pressure is given by

$$p = \in_0 \rho = \in_0 k_5 k_3{}^2[(2-n)(k_3T+k_4)]^{\frac{(n+3)(m-8)}{4(2-n)}-2} + 3\in_0 \phi_0[(2-n)(k_3T+k_4)]^{\frac{(n+3)(m-8)+8(n+2)}{4(2-n)}}$$

(3.39)

From equations (3.8), (3.34),(3.35), (3.36)& (3.38), we get

the string tension density

$$\lambda = (k_6+k_7)k_3{}^2[(2-n)(k_3T+k_4)]^{\frac{(n+3)(m-8)}{4(2-n)}-2} + 2\phi_0[(2-n)(k_3T+k_4)]^{\frac{(n+3)(m-8)+8(n+2)}{4(2-n)}}$$

(3.40)

where $k_6 = \phi_0\left\{\frac{m}{4}(n+3)\left[\frac{m}{4}(n+3)-(2-n)-1\right]+(n^2-3n+7)+\frac{m^2\omega}{32}(n+3)^2\right\}$ &
$k_7 = \phi_0\left\{\frac{m}{4}(n+3)\left[\frac{m}{4}(n+3)+\frac{\omega m}{8}(n+3)-(n+2)\right]-9\right\}.$

The particle energy density is given by

$$\rho_p = \rho - \lambda = (k_5-k_6-k_7)k_3{}^2[(2-n)(k_3T+k_4)]^{\frac{(n+3)(m-8)}{4(2-n)}-2} + \phi_0[(2-n)(k_3T+k_4)]^{\frac{(n+3)(m-8)+8(n+2)}{4(2-n)}} \quad (3.41)$$

The coefficient of bulk viscosity is given by

$$\xi = \frac{1}{(n+3)k_3}\left\{ k_3{}^2[\in_0 k_5 - k_6][(2-n)(k_3T+k_4)]^{\frac{(n+3)(m-8)}{4(2-n)}-1} + [3\in_0+1]\phi_0[(2-n)(k_3T+k_4)]^{\frac{(n+3)(m-8)+8(n+2)}{4(2-n)}+1} \right\} \quad (3.42)$$

The metric (2.1), in this case, can be written as

$$ds^2 = -dt^2 + [(2-n)(k_3T+k_4)]^{2/2-n}\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2\right] + [(2-n)(k_3T+k_4)]^{2n/2-n} d\mu^2 \tag{3.43}$$

Thus (3.43) together with (3.36) - (3.42) constitutes a closed five dimensional FRW string cosmological model with bulk viscosity in Brans-Dicke [1] scalar tensor theory of gravitation.

FRW flat cosmological model (k = 0):

If k = 0, from equation (3.13) we get

$$\frac{R''}{R} + \frac{k_2}{k_1}\frac{R'^2}{R^2} = 0 \tag{3.44}$$

From equation (3.44), we get

$$R = [(\tfrac{k_1+k_2}{k_1})(k_3T+k_4)]^{k_1/k_1+k_2} \tag{3.45}$$

where k_3 and k_4 are integrating constants.

From equations (3.11) & (3.45), we get

$$A = [(\tfrac{k_1+k_2}{k_1})(k_3T+k_4)]^{nk_1/k_1+k_2} \tag{3.46}$$

From equations (3.12), (3.45) & (3.46), we get

$$\phi = [(\tfrac{k_1+k_2}{k_1})(k_3T+k_4)]^{\frac{m(3+n)k_1}{4(k_1+k_2)}} \tag{3.47}$$

From equations (3.7) & (3.45) - (3.47), we get

the energy density

$$\rho = k_5 k_3{}^2[(\tfrac{k_1+k_2}{k_1})(k_3T+k_4)]^{\frac{(n+3)(m-8)k_1}{4(k_1+k_2)}-2} \tag{3.48}$$

where $k_5 = \phi_0\left[3(n+1) + \frac{m}{4}(n+3)^2(1-\frac{m\omega}{8})\right]$.

From equations (3.6) & (3.45) - (3.47), we get

the total pressure

$$\bar{p} = k_6 k_3^{\ 2}[(\frac{k_1+k_2}{k_1})(k_3T + k_4)]^{\frac{(n+3)(m-8)k_1}{4(k_1+k_2)}-2} \tag{3.49}$$

where $k_6 = \phi_0 \left\{ \frac{m}{4}(n+3)[1 - \frac{m}{4}(n+3) - \frac{(k_1+k_2)}{k_1}] + \frac{(2+n)((k_1+k_2)}{k_1} - \frac{m^2\omega}{32}(n+3)^2 + 3(n+1) \right\}$.

The proper pressure is given by

$$p = \in_0 \rho = \in_0 k_5 k_3^{\ 2}[(\frac{k_1+k_2}{k_1})(k_3T + k_4)]^{\frac{(n+3)(m-8)k_1}{4(k_1+k_2)}-2} \tag{3.50}$$

From equations (3.8), (3.45),(3.46), (3.47)& (3.49), we get

the string tension density

$$\lambda = (k_6 + k_7)k_3^{\ 2}[(\frac{k_1+k_2}{k_1})(k_3T + k_4)]^{\frac{(n+3)(m-8)k_1}{4(k_1+k_2)}-2} \tag{3.51}$$

where

$k_6 = \phi_0 \left\{ \frac{m}{4}(n+3)[1 - \frac{m}{4}(n+3) - \frac{(k_1+k_2)}{k_1}] + \frac{(2+n)((k_1+k_2)}{k_1} - \frac{m^2\omega}{32}(n+3)^2 + 3(n+1) \right\}$ &

$k_7 = \phi_0 \left\{ \frac{m}{4}(n+3)[\frac{m}{4}(n+3) - \frac{(k_1+k_2)}{k_1} - 2n] - \frac{3(k_1+k_2)}{k_1} + \frac{m^2\omega}{32}(n+3)^2 - 3(n+1) \right\}$.

The particle energy density is given by

$$\rho_p = \rho - \lambda = (k_5 - k_6 - k_7)k_3^{\ 2}[(\frac{k_1+k_2}{k_1})(k_3T + k_4)]^{\frac{(n+3)(m-8)k_1}{4(k_1+k_2)}-2} \tag{3.52}$$

The coefficient of bulk viscosity is given by

$$\xi = \frac{1}{(n+3)k_3}\left\{ k_3^{\ 2}[\in_0 k_5 - k_6][(2-n)(k_3T + k_4)]^{\frac{(n+3)(m-8)}{4(2-n)}-1} \right\} \tag{3.53}$$

The metric (2.1), in this case, can be written as

$$ds^2 = -dt^2 + [(\tfrac{k_1+k_2}{k_1})(k_3T+k_4)]^{2k_1/k_1+k_2}\left[\frac{dr^2}{1-kr^2}+r^2d\theta^2+r^2\sin^2\theta d\phi^2\right]+ [(\tfrac{k_1+k_2}{k_1})(k_3T+k_4)]^{2nk_1/k_1+k_2}d\mu^2 \tag{3.54}$$

Thus (3.54) together with (3.47) - (3.53) constitutes a flat five dimensional FRW string cosmological model with bulk viscosity in Brans-Dicke [1] scalar tensor theory of gravitation.

4. Some other important properties of the models

The spatial volume for the model is

$$V = (-g)^{\frac{1}{2}} = [(2-n)(k_3T+k_4)]^{(3+n)/(2-n)} \tag{4.1}$$

The average scale factor for the model is

$$a(t) = V^{\frac{1}{4}} = [(2-n)(k_3T+k_4)]^{(3+n)/4(2-n)} \tag{4.2}$$

The expression for expansion scalar θ calculated for the flow vector u^i is given by

$$\theta = u^i{}_{,i} = \frac{(3+n)}{(2-n)}\frac{k_3}{(k_3T+k_4)} \tag{4.3}$$

and the shear scalar σ is given by

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{7}{18}\frac{(3+n)^2}{(2-n)^2}\frac{k_3{}^2}{(k_3T+k_4)^2} \tag{4.4}$$

The deceleration parameter q is given by

$$q = (-3\theta^{-2})(\theta_{,i}u^i + \frac{1}{3}\theta^2) = \frac{-(4n-3)}{(3+n)} \tag{4.5}$$

The deceleration parameter appears with negative sign for $n \in (-\infty,-3)\cup(1,\infty)$. This implies that the accelerated expansion of the universe, which is consistent with the present day observations.

The Hubble's parameter H is given by

$$H = \frac{(3+n)}{4(2-n)} \frac{k_3}{(k_3 T + k_4)} \tag{4.6}$$

The mean anisotropy parameter A_m is given by

$$A_m = \frac{1}{4} \sum_{i=1}^{4} \left(\frac{H_i - H}{H} \right)^2 = \frac{(n-1)^2}{(n+3)^2}, \quad \text{where } \Delta H_i = H_i - H \ \ (i = 1,2,3,4) \tag{4.7}$$

Look-back time-red shift:

The look-back time, $\Delta t = t_0 - t(z)$ is the difference between the age of the universe at present time (z=0) and the age of the universe when a particular light ray at red shift z, the expansion scalar of the universe $a(t_z)$ is related to a_0 by $1 + z = \frac{a_0}{a}$, where a_0 is the present scale factor. Therefore from (4.2), we get

$$1 + z = \frac{a_0}{a} = \left(\frac{k_3 T_0 + k_4}{k_3 T + k_4} \right)^{(3+n)/4(2-n)} \tag{4.8}$$

This equation can also be expressed as

$$H_0 \Delta T = \frac{(3+n)}{4(2-n)} \left[1 - (1+z)^{(3+n)/4(2-n)} \right] \tag{4.9}$$

where H_0 is the Hubble's constant.

Luminosity distance:

Luminosity distance is defined as the distance which will preserve the validity of the inverse law for the fall of intensity and is given by

$$d_L = r_1 (1+z) a_0 \tag{4.10}$$

where r_1 is the radial coordinate distance of the object at light emission and, is given by

$$r_1 = \int_T^{T_0} \frac{1}{a} dT = \frac{4(2-n)}{5(1-n)} [(2-n)(k_3 T_0 + k_4)]^{\frac{5(1-n)}{4(2-n)}} \left[1 - (1+z)^{5(n-1)/(3+n)} \right] \tag{4.11}$$

From equations (4.10) and (4.11), we get
The luminosity distance

$$d_L = \frac{4(2-n)}{5(1-n)\,k_3} a_0 (1+z)[(2-n)(k_3 T_0 + k_4)]^{\frac{5(1-n)}{4(2-n)}} \left[1-(1+z)^{5(n-1)/(3+n)}\right] \tag{4.12}$$

From equations (4.11) and (4.12), we get
The distance modulus

$$D(z) = 5\log\left\{\frac{4(2-n)}{5(1-n)\,k_3} a_0 (1+z)[(2-n)(k_3 T_0 + k_4)]^{\frac{5(1-n)}{4(2-n)}} \left[1-(1+z)^{5(n-1)/(3+n)}\right]\right\} + 25 \tag{4.13}$$

The tensor of rotation $w_{ij} = u_{i,j} - u_{j,i}$ is identically zero and hence this universe is non-rotational.

5. Discussion and Conclusions

In this paper we have presented Spatially homogeneous anisotropic Five dimensional FRW cosmological models is obtained in a scalar tensor theory of gravitation proposed by Brans and Dicke [1] when the source for energy momentum tensor is a bulk viscous fluid containing one dimensional cosmic strings.

The following are the observations and conclusions:

- For all the three models have singularity at $T = \frac{-k_4}{k_3}$.
- For all the three models, the spatial volume vanishes at $T = \frac{-k_4}{k_3}$ and increases with time for $-3 < n < 2$. This shows that at the initial epoch, the universe starts with zero volume and expands uniformly.
- For all the three models, the expansion scalar θ, shear scalar σ and the Hubble parameter H decreases with the increase of time.
- From (4.7), one can observe that the average anisotropy parameter $A_m \neq 0$ for $n \neq 1$, which indicates that the universe is anisotropic. The experiments show that there is a certain amount of anisotropy in the universe and hence anisotropic space-times are important.

- For all the three models, the energy density, the total pressure, the string tension density and the coefficient of bulk viscosity are increases with the increase of time'T'.

- The deceleration parameter appears with negative sign for $n \in (-\infty, -3) \cup (1, \infty)$. This implies that the accelerated expansion of the universe, which is consistent with the present day observations.

- We have obtained expressions for look-back time ΔT, distance modulus $D(z)$ and luminosity distance d_L versus red shift and discussed their significance.
- All the models presented here are anisotropic, non-rotating, expanding and also accelerating. Hence they represent not only the early stage of evolution but also the present universe.

Received Feb. 25, 2016; Accepted March 3, 2016

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