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Bulk Viscous String Cosmology with Hybrid Law Expansion in Modified Theory of Gravity

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Abstract

The present paper deals with the study of Bulk viscous string cosmological model in spatially homogeneous and anisotropic Bianchi-V space time in modified theory of gravity, specially $f(R, T)$ theory of gravity. To solve the Einstein's field equations, we consider that the expansion scalar θ is proportional to the shear scalar σ^2 and hybrid scalar factor which yields a time varying deceleration parameter. We also consider barotropic equation of state for pressure and density. Some physical properties of the model are also discussed in details.

Keywords: Bulk viscous string, Bianchi –V, space-time, theory of gravity, Barotropic equation of state.

1. Introduction

The genius Albert Einstein modified the special theory of relativity by introducing gravitation known as General theory of relativity. This theory of gravitation plays an important role in modern physics. Several authors have done their remarkable work in the General theory of relativity. This theory has solved many astronomical issues but it fails to answer the issue of accelerating expansion of the universe and dark energy problem. Recent observations [1-5] clear the picture of the universe and indicates that the universe is accelerating and filled with 2/3 parts with the component having large negative pressure called as dark energy. To explain the issue like accelerating expansion of universe and dark energy, there is a need to modify the General theory of relativity. Recently several modifications have been made and each one has its novel feature. Few modified theories are Brans-Dicke theory, scalar tenser theory of gravitation, self-creation theory, $f(R)$ theory of gravity, $f(R,T)$ theory of gravity.

We are interested in $f(R,T)$ theory of gravity proposed by Harko et al.[6] where the gravitational Lagrangian involves an arbitrary function of the scalar curvature R and the trace of

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the energy momentum tensor *T*. Pradhan et al. [7] reconstructed $f(R,T)$ theory of gravity by taking $f(R,T) = f_1(R) + f_2(T).$ Adhav [8] studied the exact solutions of locally rotationally symmetric Bianchi type - I space-time. M. Farasat Shamir et al.[9] obtained the exact solutions of Bianchi Types-I & V models in $f(R,T)$ gravity with the assumption of variation law of Hubble parameter which leads to constant declaration parameter. P.K. Sahoo et al. [10] investigated the Kaluza-Klein cosmological model in $f(R,T)$ gravity with $\Lambda(t)$ by considering constant deceleration parameter.

The bulk viscosity plays an important t role in the early phase evolution of the universe. G.L.Murphy [11] showed that the combination of cosmic fluid with bulk deceptive pressure can generate the accelerated expansion of universe. Moreover, Fabris et al. [12] discussed that the Bulk viscosity leading to an accelerated phase of the universe. Wang [13] have investigated the LRS Bianchi type-I model for a cloud string with bulk viscosity and same author [14] studied the Bianchi type-III model for a cloud string with bulk viscosity. Yadav et al. [15] have discussed the cosmic string in Bianchi type-III space-time in the presence of bulk viscous fluid. Yadav et al. [16] have studied some Bianchi type-I viscous string cosmological model for a cloud of string with bulk viscosity. Bali and Upadhaya [17] have discussed the Bianchi type-III string cosmological model with bulk viscosity. R.L. Naidu et.al.[18] have examined the FRW viscous fluid cosmological model in $f(R,T)$ gravity .

The non-existence of Bianchi type-III bulk viscous string cosmological model in $f(R,T)$ gravity have studied by M. Kiran et al. [19]. Reddy D. R. K. et.al. [20] have discussed the Kaluza-Klein universe with cosmic strings and bulk viscosity in $f(R,T)$ gravity. K.L. Mahanta [21] have investigated the bulk viscous cosmological model in $f(R,T)$ gravity. Reddy D. R. K et al. [22] have investigated the Kantowski-Sachs bulk viscous string cosmological model in $f(R,T)$ gravity. Same author [23] have also studied the LRS Bianchi type-II universe with cosmic string and bulk viscosity in a modified theory of gravity. H. R. Ghate et.al.[24] studied the Bianchi Types-IX viscous string cosmological model in $f(R,T)$ gravity with the special form of declaration parameter. R.L. Naidu et al.[25] have studied the Bianchi Types-V bulk viscous string cosmological model in $f(R,T)$ gravity. Reddy D. R. K [26-28] have investigated the bulk viscous string cosmological model in different modified theories.

Bianchi type models are important because these are homogeneous and anisotropic, from which the process of isotropization of the universe is studied through the passage of time. Bianchi space-times are useful to make the simplicity of the field equations in the constructing spatially homogeneous and anisotropic cosmological models**.** Moreover, from the theoretical point of view the anisotropic universe has a greater generality than isotropic universe. With this motivation, In this paper we present the Bianchi type-V space-time with the bulk viscous fluid containing one dimensional string in modified theory of gravity, *f(R,T)* theory of gravity, where *R* is the Ricci scalar and *T* is the trace of the energy-momentum tensor. To solve the Einstein's field equations, we consider that the expansion scalar θ is proportional to shear scalar σ^2 and hybrid scalar factor which yields a time varying deceleration parameter. We also consider barotropic equation of state for pressure and energy density. Some physical properties of the model are also discussed in details.

2. $f(R,T)$ theory of gravity

The action for $f(R,T)$ theory of gravity is given by

$$S = \frac{1}{16\pi}\int f(R,T)\sqrt{-g}\ d^4x + \int L_m\sqrt{-g}\ d^4x, \tag{1}$$

where R, T, g and L_m are the Ricci scalar, the trace of the stress-energy tensor of matter T_{ij}, the determinant of the metric tensor g_{ij} and the matter Lagrangian density respectively.

The stress-energy tensor of matter is defined as

$$T_{ij} = -\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\ L_m\right)}{\delta g^{ij}}, \tag{2}$$

and its trace by $T = g^{ij}T_{ij}$.

Assuming that the Lagrangian density L_m of matter depends only on the components of the metric tensor g_{ij} and not on its derivatives which yields

$$T_{ij} = g_{ij}L_m - 2\frac{\partial L_m}{\partial g^{ij}}, \tag{3}$$

Varying the action S with respect to the metric tensor g_{ij}, the field equations in $f(R,T)$ theory of gravity are given by

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} - (\nabla_i\nabla_j - g_{ij}\Box)f_R(R,T) = 8\pi T_{ij} - f_T(R,T)(T_{ij} + \Theta_{ij}), \tag{4}$$

where $\Theta_{ij} = g^{ij}\left(\frac{\delta T_{ij}}{\delta g^{ij}}\right)$, which follows from the equation $g^{ij}\left(\frac{\delta T_{ij}}{\delta g^{ij}}\right) = T_{ij} + \Theta_{ij}$ and $\Box \equiv \nabla^i\nabla_i$ is the De Alembert's operator, f_R and f_T are the ordinary derivatives with respect to R and T respectively.

Contraction of equation (4) gives

$$f_R(R,T)R+3\Box f_R(R,T)-2f(R,T)=8\pi T-f_T(R,T)(T+\Theta), \tag{5}$$

where $\Theta = g^{ij}\Theta_{ij} = \Theta_i^i$.

Assume that the source is regarded as a perfect fluid, the stress-energy tensor of the matter Lagrangian is derived as

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij}, \tag{6}$$

The problem of perfect fluids described by an energy density ρ, pressure p and four velocity $u^i = (0,0,0,1)$ are not an easy task to deal and there is no any unique definition of matter Lagrangian. Thus we can assume $L_m = -p$, which gives

$$\Theta_{ij} = -2T_{ij} - p\,g_{ij}, \tag{7}$$

Using equations (7) , the field equations (4) can be written as

$$f_R(R,T)\,R_{ij}-\frac{1}{2}f(R,T)g_{ij}-(\nabla_i\nabla_j-g_{ij}\Box))\,f_R(R,T)=8\pi T_{ij}+f_T(R,T)(T_{ij}+pg_{ij}), \tag{8}$$

Since the field equations in $f(R,T)$ gravity also depend on the physical nature of the matter field through the tensor g_{ij}, we obtain several models for each choice of f .Three explicit specification of the functional form f has been considered in Harko et. al. [6]

$$f(R,T)=\begin{cases} R+2f(T) \\ f_1(R)+f_2(T) \\ f_1(R)+f_2(R)f_3(T) \end{cases}. \tag{9}$$

Out of these three classes we used $f(R,T) = R + 2f(T)$. The Einstein's field equations (8) can be written as

$$R_{ij}-\frac{1}{2}Rg_{ij}=8\pi T_{ij}+2f'(T)T_{ij}+[f(T)+2p\,f'(T)]g_{ij}, \tag{10}$$

where overhead prime denotes the derivative w.r.to. *T*.

For a sake of simplicity, we choose

$$f(T) = \mu T, \tag{11}$$

where μ is aconstant.

3. The Metric and Field Equations

The line element of Bianchi type-V space-time is given by

$$ds^2 = dt^2 - A^2(t)dx^2 - e^{2\alpha x}[B^2(t)dy^2 + C^2(t)dz^2], \tag{12}$$

where A, B and C are metric coefficients and $\alpha\,(\neq 0)$ is an arbitrary constant.

We consider the energy momentum tensor for a Bulk viscous fluid containing one dimensional cosmic string as

$$T_{ij} = (\rho + \bar{p})u_i u_j + \bar{p}g_{ij} - \lambda x_i x_j, \tag{13}$$

and

$$\bar{p} = p - \xi u^i_{;i}, = p - \xi\theta, \tag{14}$$

where ρ is the rest energy density of the system, ξ is the Bulk viscous coefficient, $\xi\theta$ is the known as bulk viscous pressure, θ is the expansion scalar, u^i is the four velocity vector, x^i is the direction of the string and λ is the string tension density.

The vectors u^i and x^i satisfy the conditions

$$u_i u^j = -x_i x^j = -1, \quad u^i x_i = 0. \tag{15}$$

In a commoving co-ordinates system, we have

$$u^i = (0,0,0,1), \quad x^i = \left(\frac{1}{A},0,0,0\right), \tag{16}$$

The corresponding Ricci scalar is

$$R = 2\left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{3\alpha^2}{A^2}\right], \tag{17}$$

where overhead dot means derivative with respect to t.

From equation (10), we have obtain field equations as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{\alpha^2}{A^2} = -(8\pi + 7\mu)\bar{p} + (8\pi + 3\mu)\lambda + \mu\rho, \tag{18}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}-\frac{\alpha^2}{A^2}=-(8\pi+7\mu)\bar{p}+\lambda\mu+\mu\rho, \tag{19}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{\alpha^2}{A^2}=-(8\pi+7\mu)\bar{p}+\lambda\mu+\mu\rho\,, \tag{20}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{C}\dot{A}}{CA}-\frac{3\alpha^2}{A^2}=(8\pi+3\mu)\rho-5\bar{p}\mu+\mu\lambda, \tag{21}$$

$$2\frac{\dot{A}}{A}-\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=0, \tag{22}$$

solving above field equations, we get

$$\frac{\ddot{B}}{B}+\frac{\ddot{A}}{A}+\frac{\dot{B}\dot{C}}{BC}-\frac{\dot{C}\dot{A}}{CA}=2\lambda(4\pi+\mu)\,, \tag{23}$$

$$\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{B}}{AB}-\frac{\dot{A}\dot{C}}{AC}=0, \tag{24}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{C}\dot{A}}{CA}-\frac{3\alpha^2}{A^2}=(8\pi+3\mu)\rho-5\bar{p}\mu+\mu\lambda, \tag{25}$$

$$A^2=kBC, \tag{26}$$

where k is a constant of integration.

Without loss of generality, we choose $k=1$, we have

$$A^2=BC, \tag{27}$$

The average scale factor and the volume scale factor are defined respectively as

$$a_s=(ABC)^{\frac{1}{3}}, \tag{28}$$

$$V=(a_s)^3=ABC, \tag{29}$$

The generalized mean Hubble parameter H is defined by

$$H=\frac{\dot{a}_s}{a_s}=\frac{1}{3}[H_1+H_2+H_3], \tag{30}$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$, $H_3 = \frac{\dot{C}}{C}$ are the directional Hubble parameters in the directions of x, y and z axes respectively.

The mean anisotropy parameter $\overline{A}$ is given by

$$\overline{A} = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 = 6\left(\frac{\sigma}{\theta}\right)^2, \tag{31}$$

where

$$\Delta H_i = H_i - H. \tag{32}$$

The expansion scalar θ and shear scalar σ^2 are defined as under

$$\theta = u^i_{;i} = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} = 3H, \tag{33}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{\theta^2}{3} - \left[\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC}\right], \tag{34}$$

where

$$\sigma_{ij} = \frac{1}{2}(\nabla_j u_i + \nabla_i u_j) - \frac{1}{3}g_{ij}\theta. \tag{35}$$

The important quantity is defined as

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1, \tag{36}$$

The sign of q indicates whether the model inflates or not. A positive sign of q corresponds to the standard decelerating model whereas the negative sign of q indicates inflation. The recent observations of SN Ia (Perlmutter et al. [1],Reiss et al [3]) reveal that the present universe is accelerating and the values of decelerating parameter in the range $-1 < q < 0$.

4. Solution of Field Equations

The system of these four non-linear differential equations consist of six undefined functions i.e. A, B, C, ρ, p, and λ. Hence to find deterministic solution of the model, two additional conditions relating to these parameters are required. We first assume that the expansion scalar θ is proportional to shear scalar σ^2. This condition yields the following relation between the metric coefficients B and C:

$$B = C^m, \tag{37}$$

where $m \neq 0$ is an arbitrary constant.

Secondly, we consider the average scale factor

$$a_s = e^{at} t^b, \tag{38}$$

where a and b are positive constant.

This above equation is the combination of two types of expansion i) exponential law and
ii) power law, so it is called as hybrid scale factor. When $a = 0,$ the power law is recovered and for $b = 0,$ the scale factor reduces to exponential law.

Using equation (27), (28) ,(37), (38), the metric coefficients are

$$A = e^{at} t^b, \tag{39}$$

$$B = (e^{at} t^b)^{\frac{2m}{m+1}},, \tag{40}$$

$$C = (e^{at} t^b)^{\frac{2}{m+1}},, \tag{41}$$

The equation (12) takes the form as

$$ds^2 = dt^2 - (e^{at} t^b)^2\, dx^2 - e^{2\alpha x}\left[(e^{at} t^b)^{\frac{4m}{m+1}}\, dy^2 + (e^{at} t^b)^{\frac{4}{m+1}}\, dz^2\right], \tag{42}$$

which represents the Bianchi -V Bulk viscous cosmological model in $f(R,T)$ gravity.

5. Physical behavior of model

The following physical properties play a important role in the study of cosmology. By using this physical quantities the physical behavior of model as discuss below

The volume scale factor becomes

$$V = (e^{at} t^b)^3, \tag{43}$$

The directional Hubble parameters and the mean generalized Hubble parameter are given by

$$H_1 = a + \frac{b}{t}, \tag{44}$$

$$H_2 = \frac{2m}{m+1}\left[a + \frac{b}{t}\right], \tag{45}$$

$$H_3 = \frac{2}{m+1}\left[a + \frac{b}{t}\right], \tag{46}$$

$$H = a + \frac{b}{t}, \tag{47}$$

The expansion scalar θ is given by

$$\theta = 3\left(a + \frac{b}{t}\right), \tag{48}$$

The shear scalar σ^2 becomes

$$\sigma^2 = \left[\frac{m-1}{m+1}\left(a + \frac{b}{t}\right)\right]^2, \tag{49}$$

It is to be noted that for $m = 1$ the model is shear free and nature of shear scalar will be retained for $m \neq 1$.

The mean anisotropy parameter $\overline{A}$ turns out to be

$$\overline{A} = \frac{2}{3}\left(\frac{m-1}{m+1}\right)^2, \tag{50}$$

A parameter m is considered to take care of the anisotropic behavior of the model in the sense that, if $m = 1$, we get isotropic model and for $m \neq 1$, anisotropic nature will be retained.

The time varying deceleration parameter q is given by

$$q = \frac{b}{(at+b)^2} - 1, \tag{51}$$

From equation (51), it is clear that $q>0$ for $t<\frac{\sqrt{b}-b}{a}$ and $q<0$ for $t>\frac{\sqrt{b}-b}{a}$. At an early phase of cosmic evolution $t\to 0$, $q=\frac{1}{b}-1$ and at late phase of cosmic evolution $t\to\infty$, $q=-1$. Thus for $0<b<1$, q has a positive value at early time and at late time q has a negative value. This suggests that the universe is undergoing decelerating at early stage to accelerating at late stage.

3) For a barotropic fluid, the combined effect of the proper pressure and the bulk viscous pressure can be expressed as

$$\bar{p}=p-3\xi H=\varepsilon\rho \tag{52}$$

Where

$$p=\varepsilon_0\rho,\quad \varepsilon_0\in[0,1] \tag{53}$$

The string density for the present model is

$$\lambda=\frac{(m-1)}{2(m+1)(4\pi+\mu)}\left[\frac{3(at+b)^2-b}{t^2}\right], \tag{54}$$

Also parameter m is look after survivor of the sting. If $m=1$, the sting does not survive for present model otherwise the nature of the string will be retained.

The energy density of the universe becomes

$$\rho=\frac{1}{[8\pi+\mu(3-5\varepsilon)]t^2}\left[\frac{[m^2(16\pi-\mu)+16m(4\pi+\mu)+7\mu+16\pi](at+b)^2}{2(m+1)^2(4\pi+\mu)}+\frac{(m-1)\mu b}{2(m+1)(4\pi+\mu)}\right], \tag{55}$$

The pressure of the universe becomes

$$p=\frac{\varepsilon_0}{[8\pi+\mu(3-5\varepsilon)]t^2}\left[\frac{[m^2(16\pi-\mu)+16m(4\pi+\mu)+7\mu+16\pi](at+b)^2}{2(m+1)^2(4\pi+\mu)}+\frac{(m-1)\mu b}{2(m+1)(4\pi+\mu)}\right], \tag{56}$$

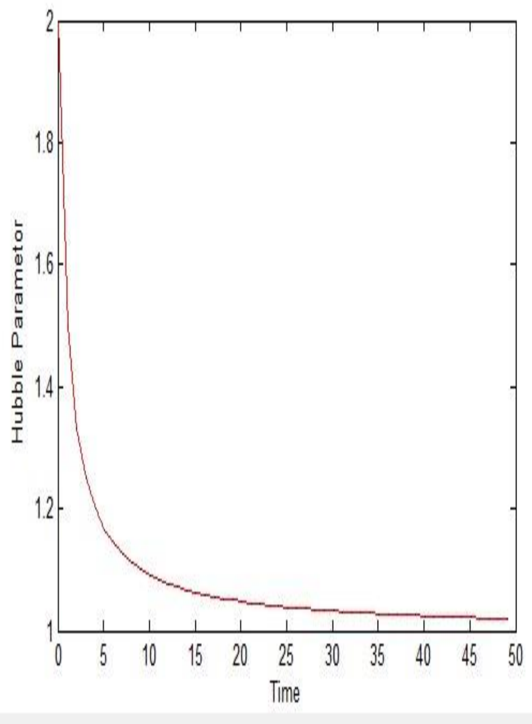
The Bulk viscous pressure is

$$\bar{p}=\frac{\varepsilon}{[8\pi+\mu(3-5\varepsilon)]t^2}\left[\frac{[m^2(16\pi-\mu)+16m(4\pi+\mu)+7\mu+16\pi](at+b)^2}{2(m+1)^2(4\pi+\mu)}+\frac{(m-1)\mu b}{2(m+1)(4\pi+\mu)}\right], \tag{57}$$

The coefficient of bulk viscosity

$$\xi = \frac{\varepsilon_0 - \varepsilon}{3[8\pi + \mu(3-5\varepsilon)](at+b)t}\left[\frac{[m^2(16\pi - \mu) + 16m(4\pi + \mu) + 7\mu + 16\pi](at+b)^2}{2(m+1)^2(4\pi + \mu)} + \frac{(m-1)\mu b}{2(m+1)(4\pi + \mu)}\right], \tag{58}$$

From the above results , we observe that the volume scale factor vanishes initially and increases with time t. The expansion scalar θ tends to infinity at $t \to 0$. These show that the universe starts evolving with zero volume at $t \to 0$ and expands with the cosmic time t. All metric coefficients are vanishes at $t = 0$. So that model has initial singularity. All the three directional Hubble's parameters, Hubble parameter H, shear scalar σ^2 diverge at $t = 0$. The other physical quantities $\rho, \bar{p}, p$ and ξ are well behaved and decreasing function of time t. These approach to zero for $t \to \infty$. The anisotropy parameter $\bar{A}$ is constant throughout the passage of time, therefore the model is anisotropic at any time.



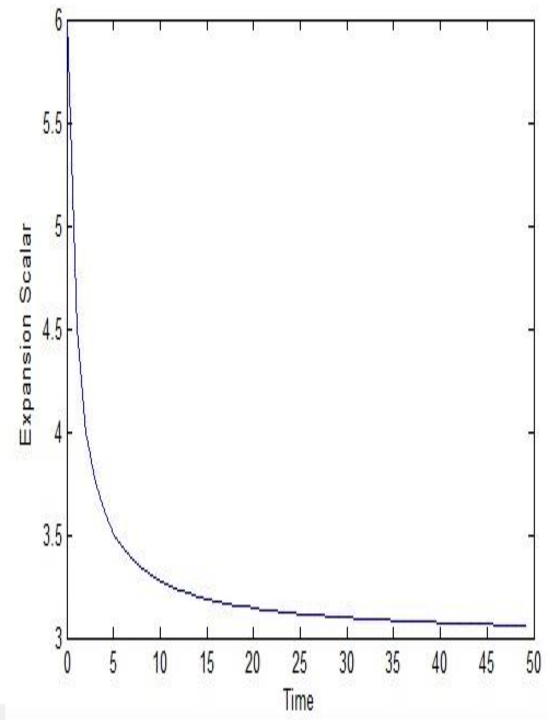


Fig (1) Hubble parameter vs Time **Fig (2)** Expansion scalar vs.Time

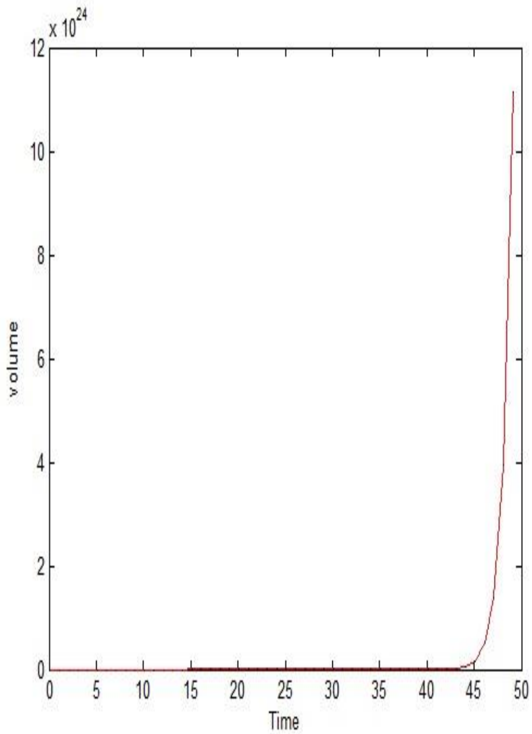


Fig (3) Volume vs Time

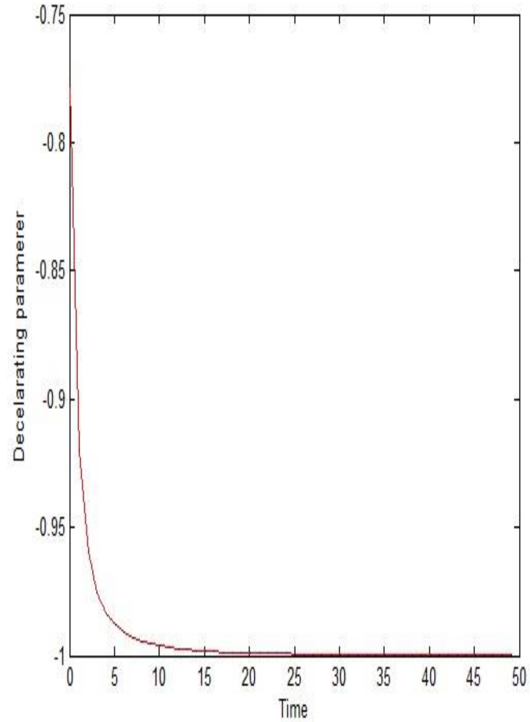


Fig (4) Decelerating parameter vs Time

6. Conclusion

The present paper devoted to the study of Bulk viscous string cosmological model in spatially homogeneous and anisotropic Bianchi-V space time in modified theory of gravity, specially $f(R,T)$ theory of gravity. The Einstein's field equations are solved by assuming the expansion scalar θ is proportional to shear scalar σ^2 and hybrid scalar factor which yields a time varying deceleration parameter. We also consider barotropic equation of state for pressure and energy density. We have also discussed the physical properties of this model and variation of some physical parameter have been shown graphically. We show that the universe starts evolving with zero volume at $t=0$ and expands with the cosmic time t.

Thus this model describes a shearing , expanding universe. We hope that our model will be helpful in study of structure formation and accelerating expansion of the universe at present.

Received Feb. 21, 2016; Accepted March 1, 2016

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