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Interacting LRS Bianchi Type-*I* Cosmological Model in *f*(*R*, *T*) Gravity with Modified Chaplygin Gas

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Abstract

The present study deals with spatially homogeneous LRS Bianchi type – *I* cosmological model in *f*(*R*, *T*) gravity with modified Chaplygin gas. To get the deterministic models of the Universe, we have used a power law between the metric potentials of the Universe. All the models obtained and presented here are expanding, non-rotating and accelerating. Also some important features of the models including look-back time, distance modulus and luminosity distance versus red shift with their significances are discussed.

Keywords: LRS Bianchi type-*I* metric, *f*(*R*, *T*) gravity, perfect fluid, Chaplygin gas.

1. Introduction

The dark energy is a prime candidate for explaining the recent cosmic observations. In view of the late time acceleration of the Universe and the existence of dark energy and dark matter, several modified theories of gravity have been developed and studied. Noteworthy amongst them is the $f(R)$ gravity theory Carroll (2004), Sotiriou and Faraoni (2010). Bertolami et al (2007) proposed a generalization of $f(R)$ theory of gravity by including in the theory an explicit coupling of an arbitrary function of the Ricci scalar R with the matter Lagrangian density L_m. Nojiri and Odintsov (2006) developed a general scheme for the modified $f(R)$ gravity reconstruction from any realistic FRW cosmology. They have shown that modified $f(R)$ gravity indeed represents a realistic alternative to general relativity, being more consistent in dark epoch. Nojiri et al. (2007) developed a general programme for the unification of matter-dominated era with acceleration epoch for scalar–tensor theory or dark fluid. Shamir (2010) proposed a physically viable $f(R)$ gravity model, which showed the unification of early time inflation and late time acceleration.

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Now days, there has been a lot of interest of cosmologists in alternative theories of gravitation Brans-Dicke (1961), Canuto (1977), Saez-Ballester (1986). A generalization of $f(R)$ modified theories of gravity was proposed in Takahashi and Soda (2010) by including in the theory an explicit coupling of an arbitrary function of the Ricci scalar R with the matter Lagrangian density L_m. As a result of the coupling the motion of the massive particles is non geodesic, and an extra-force orthogonal to the four velocities, arises. The connections with modified Newtonian dynamics and the pioneer anomaly were also explored. This model was extended to the case of the arbitrary coupling in both geometry and matter in Ilha and Lemos (1997) The astrophysical and cosmological implications of the non-minimal coupling matter-geometry coupling were extensively investigated in Ilha et al. (1999), Banados et al. (1994) and the Palatine formulation of the non-minimal geometry-coupling models was considered in Maeda (2006), In this context, a maximal extension of the Hilbert-Einstein action was proposed Jhingan and Ghosh (2010) by assuming that the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and of the matter Lagrangian L_m.

The gravitational field equations have been obtained in the metric formalism, as well as the equations of motion for test particles, which follow from the covariant divergence of the stress-energy tensor. The $f(R,T)$ gravity is the generalization of $f(R,T)$ gravity involving the dependence of the trace of energy-momentum tensor T. The dependence of T may be induced by exotic imperfect fluids or quantum effects. The cosmological reconstruction of $f(R,T)$ gravity has been studied in recent literature. Harko et al. (2011), Houndjo and Piattella (2012), Jamil et al. (2012), Sharif and Zubair (2012) in a paper, Harko et al. (2012) the reconstruction of FRW cosmology is presented for $f(R,T)=R+2f(T)$ model. Houndjo and Piattella (2012) constructed $f(R,T)$ models describing the unification as well as transition of matter dominated phase to late accelerating phase.

As we discussed, the acceleration can be consequence of the dark energy influence that this leads to some other models called Chaplygin gas Kamenshchik et al. (2001), Bento et al. (2002), Amani and Pourhassan (2013), Naji et al. (2014). Chaplygin gas is a fluid with negative pressure that begins to dominate the matter content and, at the end, the process of structure formation is driven by cold dark matter without affecting the previous history of the Universe. This kind of Chaplygin gas cosmology has an interesting connection to String Theory via the Nambu-Goto action for a D-brane moving in a (D + 2)-dimensional space-time, feature than can be regarded to the tachyonic panorama Ogawa (2000). The Chaplygin gas $f(R,T)$ models are investigated in Houndjo (2012), Jamil et al. (2012) and it is shown that dust fluid reproduces CDM, Einstein static Universe and phantom cosmology, Momeni et al. (2012). In our previous work, Sharif and Zubair (2012), we have reconstructed some explicit models of $f(R,T)$ gravity for anisotropic Universe and explored the phantom era of dark energy. We have also discussed the validity of first and second laws of thermodynamics in this modified gravity. Sharif and Zubair (2012) .The existence of exact power law solutions for FRW spacetime has been investigated in modified theories of gravity. Sharif and Zubair (2009), Rastkar et al. (2012), Setare and Darabi (2012).

One of the most generalizations of the flat Universe is Friedman Robertson–Walker (FRW) Universe. Similarly the simplest spatially homogeneous and anisotropic flat Universe is the Bianchi type-I Universe. FRW Universe has the same scale factor for each of the three spatial directions whereas Bianchi type-I Universe has different scale factors. Near the singularity Bianchi type-*I* Universe behave like Kanser Universe. It has been observed that a Universe filled with matter, the initial anisotropy in Bianchi type-I Universe quickly dies away and evolves into a FRW Universe. It has simple mathematical form and interesting because of the ability to explain the cosmic evolution of the early Universe. Due to its importance several authors have studied Bianchi type-I Universe from different aspects.

Adhav (2012) obtained exact solution so the field equations for LRS Bianchi type-I space-time with perfect fluid in the framework of $f(R,T)$ theory of gravity by applying the laws of variation of Hubble's parameter proposed by Berman (1983), Shamir et al. (2012) obtained exact solution of Bianchi type-I and type-V cosmological models in $f(R,T)$ gravity. Sharif and Zubair (2012a, b) have investigated thermodynamics and anisotropic Universe models with perfect fluid and scalar field in $f(R,T)$ gravity. Sharif and Zubair (2013b) have investigated cosmology of holographic and new age graphic $f(R,T)$ models.

Rao and Neelima (2013a, b, c) have discussed perfect fluid Einstein-Rosen, Bianchi type-VI_0 and non- static plane symmetric Universes respectively in this theory. Rao et al. (2013a) have obtained LRS Bianchi type-I perfect fluid model in this theory and established that the additional condition, special law of variation for the Hubble parameter proposed by Bermann (1983), taken by Adhav (2012), is superfluous. Rao et al. (2013b) have discussed perfect fluid cosmological models in GR and $f(R,T)$ gravity and it was again discussed by Sahoo et al. (2014).Chakroborthy (2013) has discussed $f(R,T)$ gravity by considering three different cases. Mishra and Sahoo (2014) have obtained Bianchi type VI_h perfect fluid cosmological model in $f(R,T)$ theory of gravity. Rao et al. (2014a) have investigated perfect fluid cosmological models in a modified theory of gravity. Recently, Rao et al. (2014b) have obtained Bianchi type - *III, V* and VI_0 bulk viscous string cosmological models in $f(R,T)$ gravity.

This paper is outlined as follows. In Sect. 2, we have obtained the $f(R,T)$ gravity field equations for LRS Bianchi type-*I* Universe filled with normal matter and modified Chaplygin gas. In Sect. 3, we have obtained the solution of the field equations. In Sect. 4, we have discussed the interaction between $f(R,T)$ gravity and the modified Chaplygin gas. In Sect. 5, we discuss reconstructing and parameterization of $f(R,T)$ gravity. In Sect.6, we have discussed some important properties of the model. Some conclusions are presented in the last section.

2. Metric and Energy Momentum Tensor

The action of $f(R,T)$ gravity theory coupled with matter is given by

$$S=\int d^4xe(f(R,T)+L_m) \tag{2.1}$$

where, $e=\det(e^i_\mu)=\sqrt{-g}$ in which g is the determinant of the metric tensor $g_{\mu\vartheta}$,and $\mathcal{L}_m$ is the matter Lagrangian. The $f(R,T)$ is an arbitrary function of curvature scalar R and torsion scalar. We note that curvature scalar represents gravity in general relativity, and torsion scalar represents gravity in tele parallel gravity by a different mathematical notations as Levi-Civita connection and Wienzbŏck connection, respectively. Despite this difference, independently both theories have similar result for equivalent descriptions of gravitation. Therefore, we consider a vierbein field $e_i(x^\mu)$ with index i running from 0 to 3, which one is an orthonormal basis for the tangent space at each point x^μ of the manifold in Wienzbŏck connection. Then, we can relate vierbein field to the metric as $g_{\mu\nu}=\eta_{ij}e^i_\mu e^j_\nu$ in which the Minkowski metric $\eta_{ij}=diag(-1,+1,+1,+1)$.

We consider the LRS Bianchi type-I line element can be taken as

$$ds^2=dt^2-A^2dx^2-B^2(dy^2+dz^2) \tag{2.2}$$

where A and B are functions of the cosmic time t only .

The matter tensor for perfect fluid is

$$\begin{aligned}&\Theta^i_j\equiv-2T^i_j-\delta^i_j p\\ &where T^i_j=(\rho+\overline{p})u_iu^j-\overline{p}g_{ij}-\lambda x_ix_j\end{aligned} \tag{2.3}$$

where $T^1_1=\lambda-\overline{p},T^2_2=T^3_3=-\overline{p},T^4_4=\rho,T=\rho-3\overline{p}+\lambda$

Now we intend to find the corresponding Lagrangian of the action (2.1) as

$$S=\int L\,dt \tag{2.4}$$

In that case, we expand expression $f(R,T)$ by Maclarian series, so that the Lagrangian is simplicity yielded as

$$L=a^3(F-TF_T-RF_R+vF_T+uF_R)-6(F_R+F_T)a\dot{a}^2-6(\dot{R}F_{RR}+\dot{T}F_{RT})+a^3L_m \tag{2.5}$$

where indices denote derivative with respect to R and T in corresponding locations, and the point denotes derivative with respect to cosmic time. The curvature and torsion scalars are find by

$$R = u + g^{\mu\nu} R_{\mu\nu} = u + 6(\dot{H} + 2H^2) \tag{2.6}$$

$$T = v - S_{\rho}^{\mu\nu} T_{\mu\nu}^{\rho} = v - 6H^2 \tag{2.7}$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter, and, $u = u(\dot{a}, a)$ and $v = v(a, \dot{a})$ be defined as the two arbitrary functions in terms of a and $\dot{a}$ and $R_{\mu\nu}$,$S_{\rho}^{\mu\nu}$ and $T_{\mu\nu}^{\rho}$ are the Ricci, antisymmetric and torsion tensor respectively, in the form

$$R_{\mu\nu} = \partial_{\lambda}\Gamma_{\mu\nu}^{\lambda} - \partial_{\mu}\Gamma_{\lambda\nu}^{\lambda} + \Gamma_{\mu\nu}^{\lambda}\Gamma_{\rho\lambda}^{\rho} - \Gamma_{\nu\rho}^{\lambda}\Gamma_{\mu\lambda}^{\rho} \tag{2.8}$$

$$S_{\rho}^{\mu\nu} = \frac{1}{2}\left(K_{\rho}^{\mu\nu} + \delta_{\rho}^{\mu} T_{\alpha}^{\alpha\nu} - \delta_{\rho}^{\nu} T_{\alpha}^{\alpha\mu}\right) \tag{2.9}$$

$$T_{\mu\nu}^{\lambda} = \Gamma_{\nu\mu}^{\lambda} - \Gamma_{\mu\nu}^{\lambda} = e_{A}^{\rho}\left(\partial_{\mu} e_{\nu}^{A} - \partial_{\nu} e_{\mu}^{A}\right) \tag{2.10}$$

where the contortion tensor $K_{\rho}^{\mu\nu}$ is

$$K_{\rho}^{\mu\nu} = -\frac{1}{2}\left(T_{\rho}^{\mu\nu} - T_{\rho}^{\nu\mu} - T_{\rho}^{\mu\nu}\right) \tag{2.11}$$

By using the equation (2.5), we can obtain the Friedman equations in the following form

$$2a^3\rho_{tot} = 6a^2\dot{a}\dot{R}F_{RR} - \left(6a^2\ddot{a} + a^3\dot{a}\frac{\partial u}{\partial \dot{a}}\right)F_R + 6a^2\dot{a}\dot{T}F_{RT} + \left(12a\dot{a}^2 - a^3\dot{a}\frac{\partial v}{\partial \dot{a}}\right)F_T + a^3 F \tag{2.12}$$

$$\begin{aligned} 6a^2 p_{tot} = {} & -6a^2\dot{R}^2F_{RRR} - \left(12a\dot{a}\dot{R} + 6a^2\ddot{R} - a^3\dot{R}\frac{\partial u}{\partial \dot{a}}\right)F_{RR} + \left(12\dot{a}^2 + 6a\ddot{a} + 3a^2\dot{a}\frac{\partial u}{\partial \dot{a}} + a^3\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial \dot{a}}\right) - a^3\frac{\partial u}{\partial a}\right)F_R \\ & - \left(12a\dot{a}\dot{T} - a^3\dot{T}\frac{\partial v}{\partial \dot{a}}\right)F_{TT} - \left(24\dot{a}^2 + 12a\ddot{a} - 3a^2\dot{a}\frac{\partial v}{\partial \dot{a}} - a^3\frac{\partial}{\partial t}\left(\frac{\partial v}{\partial \dot{a}}\right) + a^3\frac{\partial v}{\partial a}\right)F_T - \\ & \left(12a\dot{a}\dot{T} + 12a\dot{a}\dot{R} + 6a^2\ddot{T} - a^3\dot{R}\frac{\partial v}{\partial \dot{a}} - a^3\dot{T}\frac{\partial u}{\partial \dot{a}}\right)F_{RT} - 12a^2\dot{R}\dot{T}F_{RRT} - 6a^2\dot{T}^2F_{RTT} - 3a^2F \end{aligned} \tag{2.13}$$

where ρ_{tot} and p_{tot} are total energy density and total pressure of an Universe dominated with a perfect fluid, respectively. The continuity equation of the model becomes

$$\dot{\rho}_{tot} + 3H\left(\rho_{tot} + p_{tot}\right) = 0 \tag{2.14}$$

In this paper, we consider a simple particular model of $f(R,T)$ gravity as

$$f(R,T) = R + 2\mu T \tag{2.15}$$

where μ and v are constants.

Now for solving the model, we choose a power form for functions u_i and v as

$$u = \alpha\, a^n \tag{2.16}$$

$$v = \beta\, a^m \tag{2.17}$$

From equations (2.12),(2.13),(2.15) and (2.16), the Friedman equations are rewritten by

$$\rho_{tot} = 3(1+2\mu)H^2 + \frac{1}{2}\left(\alpha\, a^n + 2\mu\beta a^m\right) \tag{2.18}$$

$$-p_{tot} = (1+2\mu)\left(2\dot{H} + 3H^2\right) + \frac{1}{6}\alpha(n+3)a^n + \frac{1}{3}\mu\beta(m+3)a^m \tag{2.19}$$

3. Solutions of Field equations

Now the field equations for the metric (2.2) with the help of equation (2.3) can be written as

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} = (8\pi + 3\mu)\left(\bar{p} - \lambda\right) + 2\mu\,\bar{p} - \mu\rho - 2\mu\, p \tag{3.1}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = (8\pi + 5\mu)\,\bar{p} - \mu\rho - 2\mu p - \lambda\mu \tag{3.2}$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} = (8\pi + 3\mu)(-\rho) + 3\mu\bar{p} - \lambda\mu - 2\mu\rho \tag{3.3}$$

Here the overhead dot denotes differentiation with respect to 't'.

The field equations (3.1) to (3.3) are only three independent equations with four unknowns A, B, ρ and p. So, in order to get a deterministic solution we take the following plausible physical condition, the shear scalar σ is proportional to scalar expansion θ, which leads to the linear relationship between the metric potentials A and B, i.e.,

$$A = B^n \tag{3.4}$$

where n is an arbitrary constant.

From equations (3.1) - (3.4), we get

$$(1+\varepsilon)r\left(n\frac{\ddot{B}}{B} + n(n-1)\frac{\dot{B}^2}{B^2} - \frac{\ddot{B}}{B} + n\frac{\dot{B}^2}{B^2} - \frac{\dot{B}^2}{B^2}\right) + \left(n\frac{\dot{B}^2}{B^2} + \frac{\dot{B}^2}{B^2} - n\frac{\ddot{B}}{B} - n(n-1)\frac{\dot{B}^2}{B^2} - \frac{\ddot{B}}{B}\right) = 0 \tag{3.5}$$

Then we will get

$$A = \left[c_5\left(c_3 t + c_4\right)\right]^{\frac{n}{c_5}} \tag{3.6}$$

$$B = \left[c_5\left(c_3 t + c_4\right)\right]^{\frac{1}{c_5}} \tag{3.7}$$

Where $c_1 = (1+\varepsilon)r(n-1) - (n+1)$, $c_2 = (1+\varepsilon)r(n^2-1) + 2n + 1 - n^2$,

$c_5 = \left(\frac{c_1}{c_1+c_2}\right)^{-1}$ and c_3, c_4 are integrating constants.

From equations (3.1) and (3.2)

$$\lambda = \frac{\left[\left(n^2+n-2\right)+(1-n)c_5\right]\left[c_5\left(c_3 t + c_4\right)\right]^{-2} c_3^2}{8\pi + 2\mu} \tag{3.8}$$

From equation (3.2) and (3.3)

$$\rho = -\frac{c_3^2\left[n(1-n) + c_5(1+n)\right]\left[c_5\left(c_3 t + c_4\right)\right]^{-2}}{(8\pi+2\mu)(1+\varepsilon)} \tag{3.9}$$

Since $\bar{p} = \varepsilon\rho$

$$\bar{p} = -\frac{\varepsilon c_3^2\left[n(1-n) + c_5(1+n)\right]\left[c_5\left(c_3 t + c_4\right)\right]^{-2}}{(8\pi+2\mu)(1+\varepsilon)} \tag{3.10}$$

4. Interacting *f(R,T)* gravity with modified Chaplygin gas

One of the recent cosmological models which is based on the use of exotic type of perfect fluid suggest that our Universe filled with the following equation of state

$$p_{CG} = -\frac{B}{\rho_{CG}} \tag{4.1}$$

where B is a positive constant. The equation of state (4.1) generalized as

$$p_{CG} = -\frac{B}{\rho_{GCG}^{\gamma}} \tag{4.2}$$

which is known as generalized Chaplygin gas. Therefore, modified Chaplygin gas with the following equation of state introduced

$$p_{MCG} = A\rho_{MCG} - \frac{B}{\rho_{MCG}^{\gamma}} \tag{4.3}$$

where A is a positive constant, and p_{MCG} and ρ_{MCG} are the pressure and energy density in which $0 < \gamma < 1$.

Therefore we have considered the total energy density and pressure of Universe as the combination of components of dark energy and Chaplygin gas in the following form

$$\rho_{tot} = \rho_{MCG} + \rho_{DE} \tag{4.4}$$

$$p_{tot} = p_{MCG} + p_{DE} \tag{4.5}$$

Now, we take an energy flow between the dark energy and the modified Chaplygin gas. So the energy flow is introduced as an interaction between two components. In this way, the continuity equation (2.14) can be written to two separate continuity equations in the following form

$$\dot{\rho}_{DE} + 3H\left(p_{DE} + \rho_{DE}\right) = -Q \tag{4.6}$$

$$\dot{\rho}_{MCG} + 3H\left(p_{MCG} + \rho_{MCG}\right) = Q \tag{4.7}$$

and the energy density modified Chaplygin gas as

$$\rho_{MCG} = \left[\frac{B}{\eta} + c_0 a^{-3\eta(1+\gamma)}\right]^{\frac{1}{1+\gamma}} \tag{4.8}$$

$$p_{MCG} = A\left[\frac{B}{\eta} + c_0 a^{-3\eta(1+\gamma)}\right]^{\frac{1}{1+\gamma}} - \frac{B}{\left[\frac{B}{\eta} + c_0 a^{-3\eta(1+\gamma)}\right]^{\frac{\gamma}{1+\gamma}}} \tag{4.9}$$

From equations (2.17), (2.18), (3.6) and (3.7) we get the total energy density and pressure are

$$\rho_{tot} = \frac{1}{3}(1+2\mu)(2n+1)^2 c_3^2\left[c_5\left(c_3 t + c_4\right)\right]^{-2} + \frac{\alpha}{2}\left[c_5\left(c_3 t + c_4\right)\right]^{\frac{2n(2n+1)}{3c_5}} + \mu\beta\left[c_5\left(c_3 t + c_4\right)\right]^{\frac{2m(2n+1)}{3c_5}} \tag{4.10}$$

$$\begin{aligned} - p_{tot} = \frac{1}{3}(1+2\mu)(2n+1)c_3^2\left[c_5\left(c_3 t + c_4\right)\right]^{-2}\left(2n - 2c_5 + 1\right) + \frac{1}{6}\alpha(n+3)\left[c_5\left(c_3 t + c_4\right)\right]^{\frac{2n(2n+1)}{3c_5}} + \\ \frac{1}{3}\mu\beta\left[c_5\left(c_3 t + c_4\right)\right]^{\frac{2m(2n+1)}{3c_5}} \end{aligned} \tag{4.11}$$

Using equations (3.6) and (3.7) in equations (4.8) and (4.9), we get

The energy density and pressure of modified Chaplygin gas are

$$\rho_{MCG} = \left[\frac{B}{\eta} + c_0\left[c_5\left(c_3 t + c_4\right)\right]^{\frac{-6\eta(1+\gamma)(1+2n)}{3c_5}}\right]^{\frac{1}{1+\gamma}} \tag{4.12}$$

$$p_{MCG} = A\left[\frac{B}{\eta} + c_0\left[c_5\left(c_3 t + c_4\right)\right]^{\frac{-2(2n+1)\eta(1+\gamma)}{c_5}}\right]^{\frac{1}{1+\gamma}} - \frac{B}{\left[\frac{B}{\eta} + c_0\left[c_5\left(c_3 t + c_4\right)\right]^{\frac{-2\eta(2n+1)(1+\gamma)}{c_5}}\right]^{\frac{\gamma}{1+\gamma}}} \tag{4.13}$$

Substitute equations (4.10) and (4.12) in equation (4.4) we get

The dark energy density and pressure are

$$\rho_{DE} = \frac{1}{3}(1+2\mu)(2n+1)^2 c_3^2\left[c_5\left(c_3 t + c_4\right)\right]^{-2} + \frac{\alpha}{2}\left[c_5\left(c_3 t + c_4\right)\right]^{\frac{2n(2n+1)}{3c_5}} + \mu\beta\left[c_5\left(c_3 t + c_4\right)\right]^{\frac{2m(2n+1)}{3c_5}} - \left[\frac{B}{\eta} + c_0\left[c_5\left(c_3 t + c_4\right)\right]^{\frac{-6\eta(1+\gamma)(1+2n)}{3c_5}}\right]^{\frac{1}{1+\gamma}} \tag{4.14}$$

Substitute equations (4.11) and (4.13) in equation (4.5) we get

$$p_{DE} = \frac{1}{3}(1+2\mu)(2n+1)c_3^2\left[c_5\left(c_3 t + c_4\right)\right]^{-2}\left(-2n + 2c_5 - 1\right) - \frac{1}{6}\alpha(n+3)\left[c_5\left(c_3 t + c_4\right)\right]^{\frac{2n(2n+1)}{3c_5}} - \frac{1}{3}\mu\beta\left[c_5\left(c_3 t + c_4\right)\right]^{\frac{2m(2n+1)}{3c_5}} - A\left[\frac{B}{\eta} + c_0\left[c_5\left(c_3 t + c_4\right)\right]^{\frac{-2(2n+1)\eta(1+\gamma)}{c_5}}\right]^{\frac{1}{1+\gamma}} + \frac{B}{\left[\frac{B}{\eta} + c_0\left[c_5\left(c_3 t + c_4\right)\right]^{\frac{-2\eta(2n+1)(1+\gamma)}{c_5}}\right]^{\frac{\gamma}{1+\gamma}}} \tag{4.15}$$

$$w_{DE}=\frac{p_{DE}}{\rho_{DE}}=\frac{\left\{\begin{array}{l}\frac{1}{3}(1+2\mu)(2n+1)c_3^2\left[c_5\left(c_3t+c_4\right)\right]^{-2}\left(-2n+2c_5-1\right)-\frac{1}{6}\alpha(n+3)\left[c_5\left(c_3t+c_4\right)\right]^{\frac{2n(2n+1)}{3c_5}}-\\ \frac{1}{3}\mu\beta\left[c_5\left(c_3t+c_4\right)\right]^{\frac{2m(2n+1)}{3c_5}}-A\left[\frac{B}{\eta}+c_0\left[c_5\left(c_3t+c_4\right)\right]^{\frac{-2(2n+1)\eta(1+\gamma)}{c_5}}\right]^{\frac{1}{1+\gamma}}+\frac{B}{\left[\frac{B}{\eta}+c_0\left[c_5\left(c_3t+c_4\right)\right]^{\frac{-2\eta(2n+1)(1+\gamma)}{c_5}}\right]^{\frac{\gamma}{1+\gamma}}}\end{array}\right\}}{\left\{\begin{array}{c}\frac{1}{3}(1+2\mu)(2n+1)^2c_3^2\left[c_5\left(c_3t+c_4\right)\right]^{-2}+\frac{\alpha}{2}\left[c_5\left(c_3t+c_4\right)\right]^{\frac{2n(2n+1)}{3c_5}}+\mu\beta\left[c_5\left(c_3t+c_4\right)\right]^{\frac{2m(2n+1)}{3c_5}}-\\ \left[\frac{B}{\eta}+c_0\left[c_5\left(c_3t+c_4\right)\right]^{\frac{-6\eta(1+\gamma)(1+2n)}{3c_5}}\right]^{\frac{1}{1+\gamma}}\end{array}\right\}} \tag{4.16}$$

The coefficient of bulk viscosity is

$$\xi=\frac{p-\bar{p}}{3H}=\frac{\begin{array}{c}\left(\varepsilon-\varepsilon_0\right)\frac{1}{3}(1+2\mu)(2n+1)^2c_3^2\left[c_5\left(c_3t+c_4\right)\right]^{-2}+\frac{\alpha}{2}\left[c_5\left(c_3t+c_4\right)\right]^{\frac{2n(2n+1)}{3c_5}}+\mu\beta\left[c_5\left(c_3t+c_4\right)\right]^{\frac{2m(2n+1)}{3c_5}}-\\ \left[\frac{B}{\eta}+c_0\left[c_5\left(c_3t+c_4\right)\right]^{\frac{-6\eta(1+\gamma)(1+2n)}{3c_5}}\right]^{\frac{1}{1+\gamma}}\end{array}}{(2n+1)c_3\left[c_5\left(c_3t+c_4\right)\right]^{-1}} \tag{4.17}$$

5. Reconstructing and parameterization

In this section, we first will reconstruct the model as a scalar field and a potential entitled the quintessence model. For this purpose, we suppose that the origin of interacted $f(R,T)$ gravity with Chaplygin gas is a real scalar field in which the corresponding Friedmann equations is written in the following form Dent et al. (2011), Copeland et al. (2006).

$$\rho_\phi=\frac{3}{k^2}H^2=\frac{1}{2}\dot{\phi}^2+V(\phi) \tag{5.1}$$

$$p_\phi=-\frac{1}{k^2}\left(3H^2+2\dot{H}\right)=\frac{1}{2}\dot{\phi}^2-V(\phi) \tag{5.2}$$

The continuity equation and the EoS parameter are given by

$$\dot{\rho}_\phi+3H\left(1+\omega_\phi\right)\rho_\phi=0 \tag{5.3}$$

$$\omega_\phi=-1-\frac{2}{3}\frac{\dot{H}}{H^2}=\frac{\dot{\phi}^2-2V(\phi)}{\dot{\phi}^2+2V(\phi)} \tag{5.4}$$

Now let's replace equations (5.3) and (5.4) by transforming $\rho_\phi \to \rho_{DE}$ and $p_\phi \to p_{DE}$ into equation (4.14) and (4.15), so that we find the kinetic energy term and potential as

$$\dot{\phi}^2 = -\frac{2}{3}(1+2\mu)(2n+1)c_5 c_3^2\left[c_5(c_3t+c_4)\right]^{-2} - \frac{\alpha}{6}n\left[c_5(c_3t+c_4)\right]^{\frac{2n(2n+1)}{3c_5}} - \frac{1}{3}\mu\beta\, m\left[c_5(c_3t+c_4)\right]^{\frac{2m(2n+1)}{3c_5}} - \left[\frac{B}{\eta}+c_0\left[c_5(c_3t+c_4)\right]^{\frac{-2\eta(2n+1)(1+\gamma)}{c_5}}\right]^{\frac{1}{1+\gamma}}\left[1+A-\frac{B}{\frac{B}{\eta}+c_0\left[c_5(c_3t+c_4)\right]^{\frac{-2\eta(2n+1)(1+\gamma)}{c_5}}}\right] \tag{5.5}$$

$$V(\phi) = \frac{1}{3}(1+2\mu)\left[(2n+1)^2 c_3^2\left[c_5(c_3t+c_4)\right]^{-2} - (2n+1)c_5 c_3^2\left[c_5(c_3t+c_4)\right]^{-2}\right] + \frac{1}{12}\alpha(n+6)\left[c_5(c_3t+c_4)\right]^{\frac{2n(2n+1)}{3c_5}} + \frac{1}{6}\mu\beta(m+6)\left[c_5(c_3t+c_4)\right]^{\frac{2m(2n+1)}{3c_5}} - \frac{1}{2}\left[\frac{B}{\eta}+c_0\left[c_5(c_3t+c_4)\right]^{\frac{-2\eta(2n+1)(1+\gamma)}{c_5}}\right]^{\frac{1}{1+\gamma}}\left[1+A-\frac{B}{\frac{B}{\eta}+c_0\left[c_5(c_3t+c_4)\right]^{\frac{-2\eta(2n+1)(1+\gamma)}{c_5}}}\right] \tag{5.6}$$

The metric (2.1) can be written as

$$ds^2 = dt^2 - \left[c_5(c_3t+c_4)\right]^{\frac{2n}{c_5}}dx^2 - \left[c_5(c_3t+c_4)\right]^{\frac{2}{c_5}}(dy^2+dz^2) \tag{5.7}$$

Thus (5.7) together with (3.4) - (3.10) & (4.10) – (4.16) and (5.5) and (5.6) constitutes interacting LRS Bianchi type – I cosmological model in $f(R,T)$ gravity with modified Chaplygin gas.

6. Physical and geometrical properties

The spatial volume for the model is

$$V = (-g)^{\frac{1}{2}} = \left[c_5(c_3t+c_4)\right]^{\frac{2n+1}{c_5}} \tag{6.1}$$

The average scale factor for the model is

$$a(t) = V^{\frac{1}{4}} = \left[c_5(c_3t+c_4)\right]^{\frac{2n+1}{3c_5}} \tag{6.2}$$

The expression for expansion scalar θ calculated for the flow vector u^i is given by

$$\theta = u^i{}_{,i} = (2n+1)c_3\left[c_5(c_3t+c_4)\right]^{-1} \tag{6.3}$$

Look-back time-red shift: The look-back time, $\Delta t = t_0 - t(z)$ is the difference between the age of the Universe at present time (*z=0*) and the age of the Universe when a particular light ray at red shift z, the expansion scalar of the Universe $a(t_z)$ is related to a_0 by $1+z=\frac{a_0}{a}$, where a_0 is the present scale factor. Therefore from (3.2), we get

$$1+z=\frac{a_0}{a}=\left[\frac{(c_3t_0+c_4)}{(c_3t+c_4)}\right]^{\frac{2n+1}{3c_5}} \tag{6.4}$$

This equation can also be expressed as

$$H_0\Delta t=\frac{2n+1}{3}\left[1-(1+Z)\right]^{\frac{3c_5}{2n+1}} \tag{6.5}$$

where H_0 is the Hubble's constant.

Luminosity distance:

Luminosity distance is defined as the distance which will preserve the validity of the inverse law for the fall of intensity and, is given by

$$d_L=r_1(1+z)a_0 \tag{6.6}$$

where r_1 is the radial coordinate distance of the object at light emission and is given by

$$r_1=\int_T^{T_0}\frac{1}{a}dT=\frac{3\left\{\left[c_5(c_3t_0+c_4)\right]^{-\frac{2n+1+3c_5}{3c_5}}-\left[c_5(c_3t+c_4)\right]^{-\frac{2n+1+3c_5}{3c_5}}\right\}}{c_3\left[3c_5-2(2n+1)\right]} \tag{6.7}$$

From equations (7.9) - (7.12), we get

The luminosity distance

$$d_L=\frac{3a_0\left\{\left[c_5(c_3t_0+c_4)\right]^{-\frac{2n+1+3c_5}{3c_5}}-\left[c_5(c_3t+c_4)\right]^{-\frac{2n+1+3c_5}{3c_5}}\right\}}{c_3\left[3c_5-2(2n+1)\right]}\left[\frac{(c_3t_0+c_4)}{(c_3t+c_4)}\right]^{\frac{2n+1}{3c_5}} \tag{6.8}$$

The distance modulus (*D*) is given by

$$D(z)=5\log d_L+25 \tag{6.9}$$

where d_L stands for the luminosity distance.

From equations (6.8) and (6.9), we get

The distance modulus

$$D(z)=5\log\left\{\frac{3a_0\left\{\left[c_5\left(c_3t_0+c_4\right)\right]^{-\frac{2n+1+3c_5}{3c_5}}-\left[c_5\left(c_3t+c_4\right)\right]^{-\frac{2n+1+3c_5}{3c_5}}\right\}}{c_3\left[3c_5-2(2n+1)\right]}\left[\frac{\left(c_3t_0+c_4\right)}{\left(c_3t+c_4\right)}\right]^{\frac{2n+1}{3c_5}}\right\}+25 \tag{6.10}$$

The tensor of rotation

$w_{ij}=u_{i,j}-u_{j,i}$ is identically zero and hence this Universe is non-rotational.

The shear scalar σ is given by

$$\sigma^2=\frac{1}{2}\sigma^{ij}\sigma_{ij}=\frac{7}{18}(2n+1)^2c_3^2\left[c_5\left(c_3t+c_4\right)\right]^{-2} \tag{6.11}$$

The deceleration parameter q is given by

$$q=(-3\theta^{-2})(\theta_{,i}u^i+\frac{1}{3}\theta^2)=-\frac{(2n+1)(c_1+c_2)-3c_1}{(2n+1)(c_1+c_2)} \tag{6.12}$$

The deceleration parameter appears with negative sign implies accelerating expansion of the Universe, which is consistent with the present day observations.

The Hubble's parameter H is given by

$$H=\frac{c_3(2n+1)}{3\left[c_5\left(c_3t+c_4\right)\right]} \tag{6.13}$$

The mean anisotropy parameter A_m is given by

$$A_m=\frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i-H}{H}\right)^2=\frac{2(n-1)^2}{(2n+1)^2}, \tag{6.14}$$

The jerk parameter is given by

$$j=\frac{18c_1^2}{(2n+1)^2(c_1+c_2)^2}-\frac{9c_1}{(2n+1)(c_1+c_2)}+1 \tag{6.15}$$

7. Discussion and Conclusions

The issue of accelerated expansion of the Universe can be explained by taking into account the modified theories of gravity such as $f(R,T)$ gravity. The $f(R,T)$ gravity provides an alternative way to explain the current cosmic acceleration with no need of introducing either the existence of extra spatial dimension or an exotic component of DE. In this modified gravity, cosmic acceleration may result not only due to geometrical contribution to the total cosmic energy density but it also depends on matter contents. This theory depends upon matter source term, so each choice of matter Lagrangian L_m would generate a specific set of field equations. The various forms of Lagrangian in this gravity give rise to question how to constrain the $f(R,T)$ gravity theories on physical grounds.

In this paper we have presented spatially homogeneous anisotropic LRS Bianchi type – I cosmological model in $f(R,T)$ gravity interacting with modified Chaplygin gas. For this model we can observe that the spatial volume increases with the increase of time $'t'$ and also the models have no initial singularity at $t=0$. We can see that the expansion scalar θ, shear scalar σ and the Hubble parameter H decrease with the increase of time $'t'$. From equation (6.12) we can see that the deceleration parameter appears with negative sign implies accelerating expansion of the Universe, which is consistent with the present day observations. From equations (4.10) - (4.15), we can observe that energy density and the pressure are decreases with the increase of time t. From (6.14) we can observe that $A_m \neq 0$ and this indicates that the Universe is anisotropic. We have also obtained expressions for look-back time Δt, distance modulus $D(z)$ and luminosity distance d_L versus red shift and discussed their significance.

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