Article

Inhomogeneous Bianchi type-VIo String Cosmological Model for Stiff Perfect Fluid Distribution in General Relativity

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Abstract

We have investigated inhomogeneous Bianchi type VI_o string cosmological model for stiff perfect fluid in general relativity. To obtain the deterministic solution of Einstein's field equations, we assume that the isotropic pressure is equal to string rest density i.e. $p = \rho$. The model obtained is expanding, shearing and non-rotating universe. Some physical and geometrical features of the model are also discussed.

Keywords: Bianchi –$VI_{0,}$ space-time, cosmic string, stiff fluid, General Relativity.

1. INTRODUCTION

Inhomogeneous generalizations of the Friedmann-Lemaitre–Robertson–Walker (FLRW) cosmological models have gained interest in the astrophysical community and are mostly applied to study cosmological phenomena. However, in many papers the inhomogeneous cosmological models are treated as an alternative to the FLRW models. In fact, they are not an alternative, but an exact perturbation of the latter, and are gradually becoming a necessity in modern cosmology. The assumption of homogeneity is just a first approximation introduced to simplify equations. So far this assumption is commonly believed to have worked well, but future and more precise observations will not be properly analysed unless inhomogeneities are taken into account.

The important part of the present research is devoted to investigation of inhomogeneous cosmological model which generalize Bianchi type models. Considerable work has been done in obtaining various Bianchi type models and their inhomogeneous generalization. Stephani [1] has constructed a model which an inhomogeneous generalization of FRW models. Wainwright *et al.* [2] and Carmeli *et al.* [3,4] have obtained some exact solutions which generalize Bianchi type III, V and VI_h models for vacuum as well as matter filled. Roy and Narain [5, 6] have obtained solutions which generalize Bianchi type I, V and VI_0 models with perfect fluids.

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Stiff fluid cosmological models create more interest in the study of early universe because for these models velocity of sound is equal to the velocity of light so no material in this universe could be more stiff such stiffness is comprehensible at the very high densities just after the big bang. The equation of state ρ = p was first proposed by Zel'dovich [7] in the study of early universe. As importance of stiff fluid models, the cosmological models for stiff fluid distribution are also investigated by many authors, Barrow [8] Mak and Harko [9, 10] and Bali *et al.* [11,12] in different contexts.

String cosmology as cosmic strings play an important role in the study of the early universe. These strings appear after the big bang the universe may have experienced a number of phase transitions and its temperature lowered down. During the phase transition the symmetry of the universe is broken spontaneously. These phase transitions can produce vacuum domain structures such as domain walls, string and monopole Kibble [13].The existence of large scale network of strings in the early universe is not in contradiction with the present day observations of the universe. Moreover, galaxy formation can be explained by string theory Zel’dovich [14]. Letelier [15] has solved Einstein’s field equations for a cloud of massive strings and obtained cosmological models in Bianchi-I Kantowski-Sachs space times.

String cosmological models are also discussed by numbers of authors Banerjee *et al.* [16] Tikekarand Patel [17, 18], Roy and Banerjee [19], Patel and Maharaj [20], Bali *et al.* [21-25] in different context. Pradhan and Suman [26] have investigated Magnetized Bianchi type VI_o bulk viscous barotropic massive string universe with decaying vacuum energy density /\. Amirshchi [27] has derived String cosmology in Bianchi type-VI_0 dusty Universe with electromagnetic field. Verma and Shri Ram [28] have investigated Bianchi-Type VI_0 Bulk Viscous Fluid Models with Variable Gravitational and Cosmological Constants. Tyagi *et al.* [29] have obtained the solutions of field equations for inhomogeneous Bianchi type VI_0 string dust cosmological model of perfect fluid distribution.

In this paper, we have investigated inhomogeneous Bianchi type VI_o string cosmological model for stiff perfect fluid in general relativity. For the complete deterministic solution of the Einstein’s field equations, we assume that the isotropic pressure p is equal to string rest density ρ. The various physical and geometrical aspects of the models are also discussed.

2. SOLUTION OF THE FIELD EQUATIONS

We consider generalizes Bianchi type VI_0 metric in the form:

$$ds^2 = A^2(x,t)\ (dt^2 - dx^2) - e^{2x}B^2(t)dy^2 - e^{-2x}C^2(t)dz^2, \tag{1}$$

The energy momentum tensor for perfect fluid distribution in the presence of massive string proposed by Letelier [30] is taken in the form

$$T_i^{\,j} = (\rho + p)v_i v^j - p g_i^{\,j} - \lambda x_i x^j \tag{2}$$

With $\rho = \rho_p + \lambda$ and v_i and x_i satisfy conditions

$v^i v_i = -x^i x_i = 1$, and $v^i x_i = 0$

Here, p denotes isotropic fluid pressure; ρ denotes proper energy density and λ the string tension density, ρ_p enters into the stress energy tensor as simply an additional dust component. The unit space-like vector x^i represents the string direction in the cloud, i.e. the direction of anisotropy and the unit time-like vector v^i describes the four-velocity vector of the matter satisfying the following conditions
$g_{ij} v^i v^j = 1$.

We assume that the coordinates system is co-moving and so that

$$v^i = \left(0,0,0,\frac{1}{A}\right) \text{ and } x^i = \left(\frac{1}{A},0,0,0\right)$$

The Einstein's field equations in the geometrized unit (c=1,8π G=1)

$$R_i^j - \frac{1}{2} R g_i^j = -T_i^j \tag{3}$$

The Einstein's field equations (3) for the line-element (1) lead to the following system of equations:

$$\frac{1}{A^2}\left[\frac{B_{44}}{B} + \frac{C_{44}}{C} - \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) + \frac{B_4 C_4}{BC} + 1\right] = -p + \lambda \tag{4}$$

$$\frac{1}{A^2}\left[\left(\frac{A_4}{A}\right)_4 - \left(\frac{A_1}{A}\right)_1 + \frac{B_{44}}{B} - 1\right] = -p \tag{5}$$

$$\frac{1}{A^2}\left[\left(\frac{A_4}{A}\right)_4 - \left(\frac{A_1}{A}\right)_1 + \frac{C_{44}}{C} - 1\right] = -p \tag{6}$$

$$\frac{1}{A^2}\left[1 - \frac{B_4 C_4}{BC} - \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right)\right] = -\rho \tag{7}$$

$$\left(\frac{B_4}{B} - \frac{C_4}{C}\right) - \frac{A_1}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) = 0 \tag{8}$$

The suffices 1 to 4 after A, B and C denote partial differentiation with respect to x and t respectively.

From equation (5) and (6) we have

$$\frac{B_{44}}{B} - \frac{C_{44}}{C} = 0 \tag{9}$$

On integration of equation (9), we get

$$\frac{B_4}{B}-\frac{C_4}{C}=\frac{b}{BC} \qquad (10)$$

where b is integration constant.

Equation (8) leads to

$$\frac{A_1}{A}=\frac{\left(\frac{B_4}{B}-\frac{C_4}{C}\right)}{\left(\frac{B_4}{B}+\frac{C_4}{C}\right)}=\gamma(t) \qquad (11)$$

where $\gamma(t)$ is some function of time t alone. Integrating equation (11), we have

$$\log A = x.\gamma(t)+\eta(t) \qquad (12)$$

where $\eta(t)$ is a function of the time.
Equation (10) and (11) give

$$\gamma=\frac{b}{(BC)_4} \qquad (13)$$

For stiff fluid $p=\rho \Rightarrow p-\rho=0$

From equations (6) and (7), we get

$$\left(\frac{A_4}{A}\right)_4-\left(\frac{A_1}{A}\right)_1+\frac{C_{44}}{C}-2+\frac{B_4C_4}{BC}+\frac{A_4}{A}\left(\frac{B_4}{B}+\frac{C_4}{C}\right)=0 \qquad (14)$$

From equations (11), (12) and (14), we have

$$x\left[\gamma_{44}+\gamma_4\left(\frac{B_4}{B}+\frac{C_4}{C}\right)\right]+\left[\eta_{44}+\eta_4\left(\frac{B_4}{B}+\frac{C_4}{C}\right)+\frac{B_4C_4}{BC}+\frac{C_{44}}{C}-2\right]=0 \qquad (15)$$

From which we conclude that

$$\gamma_{44}+\gamma_4\left(\frac{B_4}{B}+\frac{C_4}{C}\right)=0 \qquad (16)$$

and

$$\eta_{44}+\eta_4\left(\frac{B_4}{B}+\frac{C_4}{C}\right)+\frac{B_4C_4}{BC}+\frac{C_{44}}{C}-2=0 \qquad (17)$$

Equation (16) on integration yields

$$\gamma_4=\frac{a}{BC} \qquad (18)$$

where a is a constant.

From equations (13) and (18), we get

$$\gamma = \beta [BC]^{\frac{a}{b}} \tag{19}$$

where β is a constant.

From equations (13) and (19), we get

$$BC = LT^{\frac{b}{a+b}} \tag{20}$$

where $L = \left\{ \dfrac{a+b}{\beta} \right\}^{\frac{b}{a+b}}$, $T = t + t_0$, t_0 being a constant.

Equations (10) and (20) give

$$\frac{B}{C} = \alpha \exp\left[\frac{b(a+b)}{aL} T^{\frac{a}{a+b}} \right] \tag{21}$$

where α is a constant.

Equations (17) on integration yields

$$\eta = \frac{a+b}{a+2b} T^2 + \frac{\ell(a+b)}{aL} T^{\frac{a}{a+b}} + \ell_1 - \log C \tag{22}$$

where ℓ and ℓ_1 are integration constant.

From equations (20) and (21), we get

$$\log C = -\frac{b(a+b)}{2aL} T^{\frac{a}{a+b}} + \frac{b}{2(a+b)} \log T - \frac{1}{2} \log \frac{\alpha}{L} \tag{23}$$

From equation (22) and (23), we have□

$$\eta = \frac{a+b}{a+2b} T^2 + \left\{ \frac{\ell(a+b)}{aL} + \frac{b(a+b)}{2aL} \right\} T^{\frac{a}{a+b}} - \frac{b}{2(a+b)} \log T + \ell_2 \tag{24}$$

where $\ell_2 = \frac{1}{2} \log \frac{\alpha}{L} + \ell_1$ is a new constant.

From equations (12), (19) and (24), we get

$$A = T^{-\frac{n}{2}} \exp\left[(Xm + K) T^{1-n} + \frac{T^2}{n+1} + \ell_2 \right] \tag{25}$$

where $X = x,\ m = \beta L^{\frac{a}{b}}$ $, K = \dfrac{(b + 2a\ell)(a+b)}{2aL}$ and $n = \dfrac{b}{a+b}$.

From equations (18) and (19), we have

$$B = (\alpha L)^{1/2} T^{n/2} \exp\left(\frac{b}{2L(1-n)} T^{1-n}\right), \quad n \neq 1 \tag{26}$$

and

$$C = \left(\frac{L}{\alpha}\right)^{1/2} T^{n/2} \exp\left(-\frac{b}{2L(1-n)} T^{1-n}\right), \quad n \neq 1 \tag{27}$$

By suitable transformation of coordinates and remaining constants the line element (1) reduces to the form

$$ds^2 = T^{-n} \exp\left[2\left\{(Xm+S)T^{1-n} + \frac{1}{n+1}T^2 + \ell_2\right\}\right](dT^2 - dX^2) - T^n \exp\left(2X + \frac{b}{L(1-n)}T^{1-n}\right)dY^2$$
$$-T^n \exp\left\{-\left(2X + \frac{b}{L(1-n)}T^{1-n}\right)\right\}dZ^2 \tag{28}$$

which may be considered as an inhomogeneous Bianchi type-VI_0 string cosmological model for stiff perfect fluid distribution.

3. SOME PHYSICAL AND GEOMETRICAL PROPERTIES

The physical and geometrical properties of the model are given as fallows.
Pressure p and rest density ρ of the model are given by

$$p = \frac{\left[\frac{(Xm+K)n(1-n)}{T^{n+1}} + \frac{n-1}{n+1} - \frac{n^2}{4T^2} - \frac{b^2}{4L^2T^{2n}}\right]}{T^{-n} \exp\left[2\left\{(Xm+K)T^{1-n} + \frac{1}{n+1}T^2 + \ell_2\right\}\right]} = \rho \tag{29}$$

Magnitude of rotation ω is zero i.e.

$$\omega = 0 \tag{30}$$

The Expansion scalar θ of the model is given by

$$\theta = \frac{\frac{(Xm+K)(1-n)}{T^n} + \frac{2T}{n+1} + \frac{n}{2T}}{T^{-\frac{n}{2}} \exp\left[(Xm+K)T^{1-n} + \frac{1}{n+1}T^2 + \ell_2\right]} \tag{31}$$

The Shear σ of the model is given by

$$\sigma = \frac{\left[\frac{4T^2}{(n+1)^2} + \left\{(n-1)^2(Xm+K)^2 + \frac{3b^2}{4L^2}\right\}\frac{1}{T^{2n}} - \frac{4(n-1)(Xm+K)}{(n+1)T^{n-1}} - \frac{2(Xm+K)n(n-1)}{T^{n+1}} + \frac{n^2}{T^2} - \frac{4n}{n+1}\right]^{\frac{1}{2}}}{\sqrt{3}T^{n/2}\exp\left[(Xm+K)T^{1-n} + \frac{1}{n+1}T^2 + \ell_2\right]} \tag{32}$$

String tension density λ of the model is given by

$$\lambda = \frac{n(n-1)T^{n-2}}{\exp\left[2\left\{(Xm+K)T^{1-n} + \frac{1}{n+1}T^2 + \ell_2\right\}\right]} \tag{33}$$

The deceleration parameter q of the model is given by

$$q = -1 + \frac{3\left\{\frac{(Xm+K)n(1-n)}{T^{n+1}} - \frac{2(n-1)}{(n+1)} - \frac{3n^2}{4T^2} - \frac{(n-1)^2(Xm+K)^2}{T^{2n}} - \frac{4T^2}{(n+1)^2} - \frac{4(1-n)(Xm+K)}{(n+1)T^{n-1}}\right\}}{T^{\frac{n}{2}}\left[(Xm+K)(1-n)T^{-n} + \frac{2T}{n+1} + \frac{n}{2T}\right]^2 \exp\left[-\left\{(Xm+K)T^{1-n} + \frac{1}{n+1}T^2 + \ell_2\right\}\right]} \tag{34}$$

4. CONCLUSIONS

The model (28) starts with a big bang at T = 0 when $0 < n < 1$ and goes on expanding till T → ∞, θ becomes zero when $-1 < n < 1$. It is clear that as T increases, the ratio of the shear σ and expansion θ tends to finite value i.e. $\frac{\sigma}{\theta} \to \frac{1}{\sqrt{3}}$ *as* $T \to \infty$. Hence model does not approach isotropy for large value of T. The fluid flow is irrotational and it is observed that as T→0 and X→0, the energy density ρ →∞ when $n < -2$ and as T→∞ and X→∞, ρ→0. As T→0 and X→0 string tension density λ →∞ when $n < 1$ and as T → ∞ & X→∞, λ→0 when $n < 1$ therefore the string will be disappear from the universe at later time. As T→0 the deceleration parameter q approaches to -1 when $n < -4$ as in De-Sitter universe. The model represents accelerating universe. In general the model represents expanding, shearing and non-rotating universe.

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