Article

A Cosmological Model with Bulk Viscosity in Scalar Tensor Theory

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Abstract

In the present work, we have probed the significance of a new class of Hypersurface-homogeneous cosmological model containing bulk viscous fluid in the background of scalar tensor theory of gravitation formulated by Saez and Ballester (Phys. Lett. A 113:467, 1986). Some exact solutions of Einstein's modified field equations have been obtained explicitly by using two specific physically viable conditions. One is the proportionality relation between the shear scalar (σ) and the expansion scalar (θ) (Gen. Relativ. Gravit. 12:805, 1980). In the other condition, we assume that the bulk viscosity (ζ) is proportional to the expansion scalar (θ) (Int. J. Theor. Phys. 54:2652, 2015). A careful analysis of all the physical parameters of the model has been carried out.

Keywords: Hypersurface-homogeneous, bulk viscosity, scalar tensor theory.

1. Introduction

Current accelerating phase of the universe is considered as one of the major challenges in the cosmological sector. This accelerating expansion has been exposed by different experimental surveys such as Cosmic microwave background radiation (CMBR) [1-3], Supernovae Type Ia (SNIa) [4-6], Large scale structure (LSS) [7], Wilkinson microwave anisotropy probe (WMAP) [8], PLANK collaboration [9]. These observations also hint that the most dominant force behind driving this sudden acceleration is an exotic fluid which currently consists of about 68.3% dark energy, 26.8% dark matter. Normal matter content of the universe is only about 4.9%. The large negative pressure of dark energy plays an important role to accelerate the expansion of the universe.

However to know the exact nature of dark energy is a matter of open debate. A very first approach to form a scalar theory of gravity has been attempted by G.Nodstrom when he promoted Newtonian potential function to a Lorentz scalar. The scalar tensor theory was conceived originally by Jorden while embedding a four dimensional curved manifold in five-dimensional flat space-time (see Jorden et al. [20]). He exposed that a four dimensional scalar field can act as a constraint in formulating projective geometry which allows one to introduce a

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space-time-dependent gravitational constant in accordance with P.A.M Dirac's argument that gravitational constant should be a function of time. This is obviously beyond what can be understood within the scope of Einstein's standard theory. He also argued the possible connection of his theory with another five-dimensional theory which has been supplied by Th. Kaluza and O. Klein.

Saez and Ballester [21], in an attempt to modify standard theory of gravity, have proposed a scalar tensor theory in which the metric (g_{ij}) is coupled with a dimensionless scalar field (ϕ). This theory is known as Saez-Ballester theory and provides a satisfactory description of the weak fields resulting an antigravity regime despite the dimensionless character of the scalar field. The scalar tensor theory plays a crucial role to solve the missing mass problem in non-flat Friedmann models (FRW cosmologies). It also suggests a possible way to remove the graceful exit problem in the inflation era [22].

Scalar tensor theory of gravitation have been employed by a number of researchers [23-33] to study the universe models in distinct Bianchi cosmologies.For a perfect fluid distribution obeying a barotropic equation of state, Stewart and Ellis [44] have obtained some general solutions of Einstein's field equations in the background of hypersurface-homogeneous space-time. Hajj Boutros [45] has devised a method to find exact solutions of field equations in case of hypersurface-homogeneous space-time in presence of perfect fluid which add to rare solutions not satisfying the barotropic equation of state. Verma and Ram [46] have analyzed some hypersurface-homogeneous bulk viscous fluid cosmological models with time dependent Λ term. Most recently, Katore and Shaikh [47] have investigated hypersurface-homogeneous space-time with anisotropic dark energy in scalar tensor theory of gravitation.

Bulk viscosity plays a major role in the study of the dynamics of the universe. It is considered as the measure of pressure that is required to store the equilibrium which is broken when a thermodynamical fluid expands (or contracts). Ecart [48], Landau and Lifschitz [49], Israel [50], Israel and Stewart [51] have suggested and developed first and second order viscosity theories of relativistic fluids. In the light of homogeneous and isotropic cosmologies, bulk viscosity acts as a modeling of dissipative process within a thermodynamical approach. It provides an effective mechanism to study the entropy producing processes.

In reference to grand unified theories (GUTs), at the time of neutrino decoupling and during particle creation in the early universe, bulk viscosity comes into play leading to the formation of galaxies. Heller and Klimek [58], Murphy [59] have explored the role of viscosity in avoiding the initial big-bang singularity. Santos et al. [60] presented some exact solutions of the isotropic and homogeneous model with bulk viscosity being a power function of energy density. Singh [61] has investigated the possible effect of bulk viscosity on the early evolution of the universe for a spatially homogeneous and isotropic Robertson-Walker model and concluded that the introduction of viscosity term into the equation of Friedmann cosmology does not exclude automatically the appearance of singularity.

In the present paper, we have discussed a hypersurface-homogeneous cosmological model in presence of bulk viscosity in the framework of scalar tensor theory of gravitation proposed by Saez and Ballester. Einstein's modified field equations have been solved by using two physically viable conditions. We have carried out a careful analysis of all the important cosmological parameters in the model. The paper is arranged as per following plan: In Sec. 2, Model and field equations are shown, Sec. 3 is devoted to Solution of field equations, Discussion appears in Sec. 4 and finally Sec. 5 contains Conclusion.

2. Model and field equations

We consider the general metric for hypersurface-homogeneous space-time as

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)\{dy^2 + \sum^2(y,k)dz^2\} \tag{1}$$

where$\sum(y,k) = \sin y, y, \sinh y$ respectively when $k = 1,0,-1$.

The energy-momentum tensor for a bulk viscous fluid as a matter distribution of the universe is of the form

$$T_{ij} = (\rho + \bar{p})u_i u_j + \bar{p} g_{ij} \tag{2}$$

where

$$\bar{p} = p - \zeta u^i_{;i} \tag{3}$$

Here $\rho,\ p,\ \bar{p}$ and ζ are energy density, isotropic pressure, effective pressure and bulk viscous coefficient respectively, g_{ij} is the metric tensor and u_i is the four velocity vector given by

$$u_i u^i = -1 \tag{4}$$

Since the present model is based on scalar tensor theory of gravitation proposed by Saez and Ballester, therefore the generalized field equations in this theory have the form ($G = c = 1$)

$$G_{ij} - \omega\phi^h\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,l}\phi^{,l}\right) = -8\pi T_{ij} \tag{5}$$

$$2\phi^h\phi^{,i}_{;i} + h\phi^{h-1}\phi_{,l}\phi^{,l} = 0 \tag{6}$$

where $G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij}$ is the Einstein's tensor, ω and h are constants, ϕ is the scalar field. Comma (,) and semicolon (;) here and everywhere denote partial and covariant differentiation with respect to time t. Also it can be easily proved that the equations of motion

$$T^{ij}_{;i} = 0 \tag{7}$$

are consequences of the field equations (5) and (6).

For the metric (1) and the matter distribution (2), the modified Einstein's field equations (5) and (6) yield the following dynamical equations ($8\pi G = c = 1$)

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{k}{B^2} = -\bar{p} + \frac{1}{2}\omega\phi^h\dot{\phi}^2 \tag{8}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} = -\bar{p} + \frac{1}{2}\omega\phi^h\dot{\phi}^2 \tag{9}$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} + \frac{k}{B^2} = \rho - \frac{1}{2}\omega\phi^h\dot{\phi}^2 \tag{10}$$

$$\ddot{\phi} + \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)\dot{\phi} + \frac{h}{2}\frac{\dot{\phi}^2}{\phi} = 0 \tag{11}$$

Average scale factor (R), spatial volume (V) and the generalized Hubble parameter (H) for the metric have the form

$$R = (AB^2)^{\frac{1}{3}} \tag{12}$$

$$V = R^3 = AB^2 \tag{13}$$

$$H = \frac{1}{3}(H_1 + H_2 + H_3) \tag{14}$$

where $H_1 = \dot{A}/A$, $H_2 = \dot{B}/B$ and $H_3 = \dot{C}/C$ are the directional Hubble factors along x, y and z axes respectively.

From equations (12)-(14), we can obtain an important relation

$$H = \frac{\dot{R}}{R} = \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) \tag{15}$$

We define the expansion scalar (θ) and the shear scalar (σ) for the model as

$$\theta = 3H = \frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} \tag{16}$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - \frac{1}{3}\theta^2\right) \tag{17}$$

The general expressions for anisotropy parameter (A_{an}) and deceleration parameter (q) are given by

$$A_{an} = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 \tag{18}$$

where $\Delta H_i = H_i - H$ $(i = 1,2,3)$.

$$q = -\frac{R\ddot{R}}{\dot{R}^2} \tag{19}$$

3. Solution of field equations

Equations (8)-(11) are a system of four independent equations in six unknowns parameters A, B, p, ρ, ϕ and ζ. In order to close the system completely, we require two more conditions. Firstly we assume that shear scalar (σ) in the model is proportional to the expansion scalar (θ) [62]. This leads to the following relation between the metric potentials

$$A = B^m \tag{20}$$

where m is a positive constant. For anisotropic models, m should not be unity $(i.e. m \neq 1)$. As a second condition, we take the following relation between the expansion scalar (θ) and the bulk viscosity (ζ) [63]

$$\zeta = \zeta_0\theta \tag{21}$$

where ζ_0 is a constant.

From equations (8) and (9), we have a single equation

$$\frac{\ddot{B}}{B} - \frac{\ddot{A}}{A} + \frac{\dot{B}^2}{B^2} - \frac{\dot{A}\dot{B}}{AB} + \frac{k^2}{B^2} = 0 \tag{22}$$

It is worth to mention here that Ram and Verma [64] have analyzed the above equation for $k = -1, 1$ to obtain some interesting consequences. We shall explore the equation for $k = 0$. For $k = 0$, equations (20) and (22) give

$$\frac{\ddot{B}}{B} + (m+1)\frac{\dot{B}^2}{B^2} = 0 \tag{23}$$

Equation (23) integrates to give

$$B = (at+b)^{\frac{1}{m+2}} \tag{24}$$

where a, b are constants due to first and second integrals.

From equation (20), we obtain

$$A = (at+b)^{\frac{m}{m+2}} \tag{25}$$

From equations (11) and (15), we have

$$\phi(t) = \left[\frac{\alpha(h+2)}{2}\int R^{-3}dt\right]^{\frac{2}{h+2}} \tag{26}$$

where α is constant due to first integral. Without any loss of generality, we take constant due to second integral as zero for sake of convenience.

Corresponding to the equations (24) and (25), the Hypersurface-homogeneous metric (1) reduces to the following form

$$ds^2 = -dt^2 + (at+b)^{\frac{2m}{m+2}} + (at+b)^{\frac{2}{m+2}}\{dy^2 + y^2dz^2\} \tag{27}$$

Average scale factor(R), proper volume (V), generalized Hubble parameter (H), expansion scalar (θ), anisotropy parameter (A_{an}) and shear scalar (σ) for the model have the form

$$R = (at+b)^{\frac{1}{3}} \tag{28}$$

$$V = (at+b) \tag{29}$$

$$H = \frac{a}{3(at+b)} \tag{30}$$

$$\theta = \frac{a}{(at+b)} \tag{31}$$

$$A_{an} = \frac{2(m-1)^2}{(m+2)^2} \tag{32}$$

$$\sigma^2 = \frac{1}{3}\frac{a^2(m-1)^2}{(m+2)^2}(at+b)^{-2} \tag{33}$$

From equation (26), the scalar field $\phi(t)$ is given by

$$\phi(t) = \left[\frac{\alpha(h+2)}{2a}\log(at+b)\right]^{\frac{2}{h+2}} \tag{34}$$

which implies that

$$\phi^h\dot{\phi}^2 = \alpha^2(at+b)^{-6} \tag{35}$$

Isotropic pressure (p), energy density (ρ), deceleration parameter (q) and the bulk viscous coefficient (ζ) for the given model (27) are

$$p = \frac{a^2(2m+1)+\zeta_0 a^2(m+2)^2}{(m+2)^2(at+b)^2} + \frac{\omega\alpha^2}{2}\log(at+b) \tag{36}$$

$$\rho = \frac{a^2(2m+1)}{(m+2)^2(at+b)^2} + \frac{\omega\alpha^2}{2}\log(at+b) \tag{37}$$

$$q = 2 \tag{38}$$

$$\zeta = \frac{\zeta_0 a}{at+b} \tag{39}$$

The equation of state parameter (γ) for the model is given by

$$\gamma = \frac{p}{\rho} = 1 + \frac{\zeta_0 a^2}{\left[\frac{a^2(2m+1)}{(m+2)^2} + \frac{\omega\alpha^2\log(at+b)(at+b)^2}{2}\right]} \tag{40}$$

Here,

$$\frac{dp}{d\rho} = \frac{P}{Q} \tag{41}$$

where

$$P = -\left[\frac{2a^3\{(2m+1)+\zeta_0(m+2)^2\}}{(m+2)^2(at+b)^3} - \frac{\omega a\alpha^2}{2(at+b)}\right]$$

and

$$Q = -\left[\frac{2a^3(2m+1)}{(m+2)^2(at+b)^3} - \frac{\omega a\alpha^2}{2(at+b)}\right]$$

For physical reality, we must require $(dp/d\rho) \leq 1$. It can be seen that in our model, the physical reality condition is satisfied.

4. Discussion

From the equations of the physical parameters of the model, we see that the radius scale factor R and the proper volume V are zero at $t = t_c = -\frac{b}{a}$. Therefore the observed universe under the considered model has a finite-time big-bang singularity at $t = t_c$, which shifts to initial singularity by setting $b = 0$. Hubble parameter H, expansion scalar θ, shear σ^2, pressure p and density ρ all diverge at the time $t = t_c$. In the limit of sufficiently large t, all the physical parameters $H, \theta, \sigma^2, p, \rho$ converge to zero whereas the scale factor R and the volume V become infinite.

Therefore the current model shows a cosmological scenario in which the universe starts from a finite-time big-bang singular state and expands with cosmic time t. For $m = 1$, the anisotropy parameter A_{an} equals to zero. This depicts that the universe in the model turns isotropic for $m = 1$. From equation (38), we observe that the value of deceleration parameter q is positive which shows the decelerating phase of expansion of the observed universe that is mainly responsible for structure formation [65, 36]. Also, recent astronomical observations of SNIa and CMBR favor accelerating models i.e. $q < 0$. But both these do not altogether rule out the decelerating ones which are also consistent with recent observations [66]. Interestingly, from equation (39), we infer that the bulk viscosity ζ decreases with the increase in cosmic time t. Thus it contributes to pressure and plays the role of accelerating the expansion of the universe in the considered model [46, 67-70].

Therefore our model universe is also fit for accelerating. In view of equation (40), the equation of state parameter γ is a function of cosmic time t. γ shows a decline as time t increases and as t tends to infinite, γ approaches to 1, which signifies stiff fluid era. Generally, it is to note that using a constant value of γ is not at all obligatory. Usually, the equation of state parameter is taken as constant with phase wise values $-1, 0, 1/3$ and $+1$ for vacuum fluid, dust fluid, radiation fluid and stiff fluid respectively. But in general, γ is a function of time or red-shift [71-73]. From equation (34), we observe that the scalar field ϕ shows an increase with the cosmic time t and becomes immensely large for late times. Note that if we consider ϕ as a function of red-shift z. Then the nature of the scalar field ϕ decreases as z increases [74, 47].

It deserves to mention here that in the study of the present day cosmological models, classical scalar fields are essential in view of the fact that in recent years there is an increasing interest in general relativity and alternative gravitational theories in the context of inflationary universe and they are very important to describe the early stages of evolution of the universe.

5. Conclusion

In this study, we have investigated the significance of a new class of Hypersurface-homogeneous cosmological model containing bulk viscous matter as a source of cosmic fluid in the context of scalar tensor theory of gravitation proposed by Saez and Ballester. Einstein's modified field equations have been solved by using two nice physically plausible conditions. Physical and kinematical parameters of the model have been discussed in detail. In the model, we observe that the universe starts with a finite-time big-bang singular state and expands with the cosmic time t [75, 76]. The model possesses initial singularity for $b = 0$. Radius scale factor R and proper volume V start of with negligible values at $t = t_c = -\frac{b}{a}$, then increase with cosmic time t and become infinitely large for late epochs (i.e.as $t \to \infty$). The parameters $H, \theta, \sigma^2, p, \rho$ diverge at the time $t = t_c$. As t increases, these parameters decrease and become negligible for sufficiently large value of cosmic time t. The anisotropy parameter A_{an} in the model vanishes for $m = 1$. Thus the universe observed in the model becomes isotropic for $m = 1$. This is in agreement with recent astronomical observations.

The deceleration parameter found in our model is positive which indicates the decelerating phase of expansion. No doubt, recent observations favor accelerating models but they do not rule out decelerating models which are also consistent with current observations. Decelerating phase of expansion is believed to be mainly responsible for structure formation of the universe. It is interesting to observe that bulk viscosity decreases with the increase in cosmic time t therefore plays the role of accelerating the expansion of the universe [77]. It has been suggested that viscosity plays a crucial role in getting the accelerated expansion of the universe popularly known as inflationary phase [78-82].

Ellis [83] has shown that in the evolution of the universe bulk viscosity could arise in many circumstances leading to inflationary scenario. Misner [84] and Hu [85] have exposed that at the time of neutrino decoupling, formation of galaxies and during particle creation in the early universe viscosity arises. Thus our model is fit for accelerating. The equation of state parameter γ is a function of cosmic time t and its value decreases with the increase in time t. We also infer that the scalar field ϕ increases as time t increases.

If we consider ϕ as a function of red-shift z. Then ϕdecreases with the increase in red-shift.In the present day cosmological studies, Classical scalar fields are attracting the attention of researchers as they are very important to describe the early stages of evolution of the universe in the context of inflationary models. We also observe that the physical reality condition $(dp/d\rho) \leq 1$ is satisfied in the present model.

Therefore our model is a realistic cosmological model. It can also be seen that (ρ/θ^2) remains to be a constant which shows that matter and expansion are comparable during the whole evolution of the universe. We would like to mention here that the present model can be utilized to discuss the evolution of the universe at the earlier epochs and structure formation of galaxies. It is worth mentioning here that the Murphy's conclusion about the disappearance of singularity in the finite past in presence of bulk viscosity does not good hold in case of our model.The evolutionary behaviors of some important parameters are plotted in the Figs. [1-4].

Fig. 1

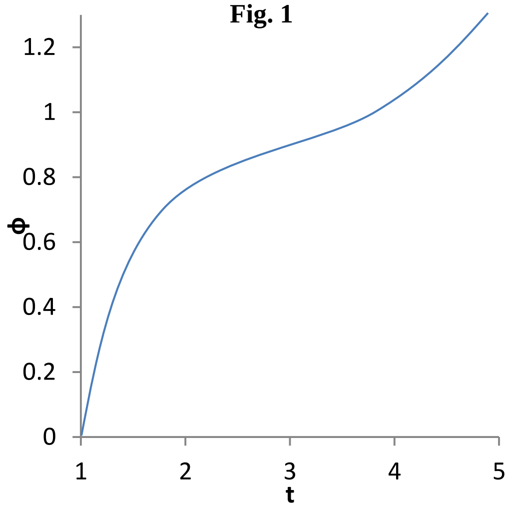

Fig.2 shows the plot of scalar field (ϕ) verses cosmic time (t)

Fig. 2

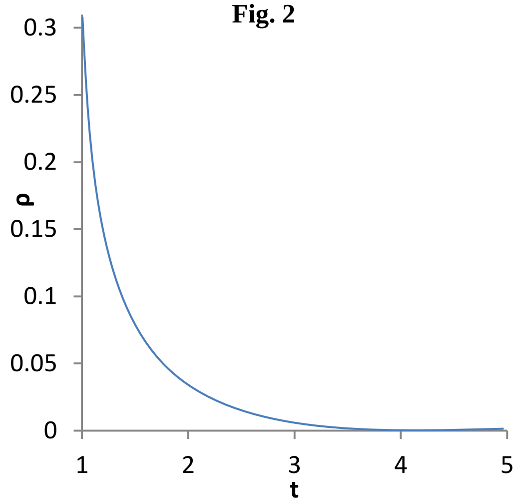

Fig. 2 shows the plot of energy density (ρ) verses cosmic time (t)

Fig. 3

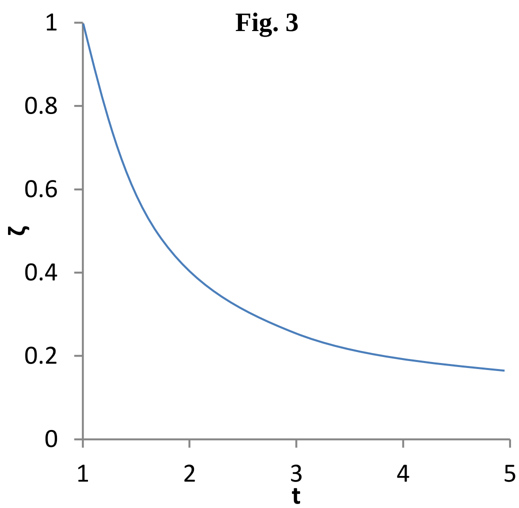

Fig. 3 shows the plot of bulk viscosity (ζ) verses cosmic time (t)

Fig. 4

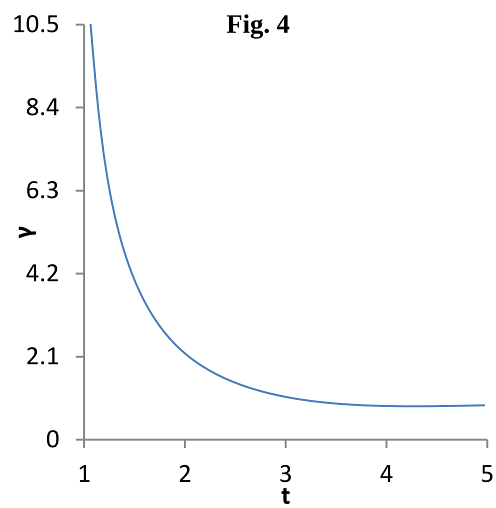

Fig. 4 shows the plot of equation of state parameter (γ) verses cosmic time (t)

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