Article

Two-Fluid Scenario for Higher Dimensional Dark Energy Cosmological Model in Saez-Ballester Theory of Gravitation

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Abstract

In this paper, we investigate the evolution of dark energy parameter within the scope of a spatially homogeneous five dimensional Kaluza-Klien universe filled with barotropic fluid and dark energy in the framework of scalar-tensor theory of gravitation formulated by Saez and Ballester [1]. To get the deterministic model of universe, we assume that the shear scalar (σ) in the model is proportional to expansion scalar (θ). This condition leads to $A = B^k$, where $A\&B$ are metric potentials and k is an arbitrary constant. It has been found that the anisotropic distribution of dark energy leads to the present accelerated expansion of universe. We consider both the cases when the dark energy is minimally coupled to barotropic fluid as well as direct interaction with it. In both the cases the equation of state(EoS) parameter ω_{de} changing from $\omega_{de} > -1$ to $\omega_{de} < -1$, which is consistent with recent observations. The physical aspects of the obtained models are also discussed.

Keywords: Kaluza-Klein metric, Saez-Ballester theory, two fluid, dark energy, EoS parameter.

1 Introduction:

Recent cosmological observations contradict the matter dominated universe with decelerating expansion indicating that our universe experiences accelerated expansion. Before the accelerating expansion of the universe was revealed by high red-shift supernovae Ia (SNe Ia) observations [2] it could hardly be presumed that the main ingredients of the universe are dark sectors. The concept of dark energy was proposed for understanding this currently accelerating expansion of the universe, and then its existence was confirmed by several high precision observational experiments [3, 4] especially the Wilkinson Microwave Anisotropy Probe(WMAP) satellite experiment. The WMAP shows that dark energy occupies about 73% of the energy of our universe, and dark matter about 23%. The usual baryon matter, which can be described by our known particle theory, occupies only about 4% of the total energy of the universe. Although we know that the ultimate fate of the universe is determined by the nature of dark energy, the information about its nature we can acquire is still very limited. So far the confirmed information about dark energy can be summarized as the following three items: it is a kind of exotic matter with negative pressure such that can drive the universe to expand post; it is spatially homogeneous and non-clustering; and it is in small part at the early times while dominates the universe very recently.

Koivisto and Mota [5, 6] proposed the mechanism of dark energy with anisotropic equation of state (EoS) parameter which is very attractive because cosmic anisotropy originates from the actual dominant component of the universe and then could be directly tested, for example, by either observations of the magnitude and redshift of type Ia supernovae or cosmic parallax effects of the distance source. Dark energy has been conventionally characterized by the equation of state(EoS) parameter $\omega_{de} = p_{de}/\rho_{de}$ which is not necessarily constant. The simplest dark energy candidate is the vacuum energy($\omega_{de} = -1$), which is argued to be equivalent to the cosmological constant(Λ) (Martins [7]). However, it is well known, there are two difficulties arising from the cosmological constant scenario, namely the two famous cosmological constant problems-the fine tuning and the cosmic coincidence. An alternative proposal is the concept of dynamical dark energy. Such a scenario is often realized by some scalar field mechanism and suggests that

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the energy form with negative pressure is provided by a scalar field evolving under a properly constructed potential.

So far, a large class of scalar-field dark energy models have been studied, including quintessence i.e $\omega_{de} > -1$ (Steinhardt et al.[8]), phantom i.e $\omega_{de} < -1$ (Caldwell [9]) and quintom i.e ω_{de} can across from quintessence region to phantom region(Feng et al. [10]). The quintom scenario of dark energy is designed to understand the nature of dark energy with ω_{de} across -1 (Setare [11]). Cai et al. [12], Setare and Saridakis [13, 14] have studied the dark energy models with EoS parameter across -1 which give a concrete theoretical justification for quintom paradigm. In addition, the other proposals on dark energy include interacting dark energy model (Setare [15]) and braneworld model (Setare and Saridakis [16]) etc. By combining data from seven CMB experiments with large scale structure data, the Hubble parameter measurement from the Hubble space-telescope and luminosity measurements of SN Ia, Melchiorri et al. [17] demonstrated the bound on ω_{de} to be $-1.38 < \omega_{de} < -0.82$ at 95% confidence level.

Dark energy models are investigated by many authors [18]-[21]. In particular, Rao et al. [22] have investigated LRS Bianchi type-I dark energy model in a scalar tensor theory of gravitation. Yadav and Saha [23] have studied LRS Bianchi type-I anisotropic cosmological model with dominance of dark energy. Rao et al. [24] have discussed Bianchi type-II, $VIII$, and IX perfect fluid dark energy cosmological models in Saez-Ballester theory and general theory of gravitation. Sharif and Zubair [25] have investigated dynamics of Bianchi type-I universe with magnetized anisotropic dark energy.

The astrophysical data indicates that the universe is spatially flat and is dominated by 73% of dark energy, 4% other cosmic matter and 23% dark matter. Hence the investigation of interacting and non-interacting two fluid (dark energy and cosmic barotropic fluid) models have become important in the discussion of the cosmic acceleration of the universe. Reddy et al. [26, 27] have discussed two fluid scenario for dark energy model in Saez-Ballester and Brans-Dicke theories of gravitation. Venkateswarlu [28] has investigated Kaluza-Klein mesonic cosmological model with two-fluid source. Liange et al. [29] have investigated the cosmological evolution of a two-field dilation model of dark energy. Chimento et al. [30] have discussed two fluid scenario for dark energy models and they have shown that such an interaction may help alleviate the coincidence problem. Saha et al. [32] have revisited the two fluid scenario in FRW universe investigated by Amirhashchi et al. [33]. Recently, Amirhashchi et al. [34] have investigated interacting two-fluid viscous dark energy models in a non-flat universe. Pradhan [35] has generalised the two-fluid atmosphere from decelerating to accelerating FRW dark energy models studied by Amirhashchi et al. [36]. Very recently, Amirhaschi et al. [37] have investigated interacting and non-interacting two-fluid atmosphere for dark energy in FRW universe.

Saez and Ballester [1] formulated a scalar tensor theory of gravitation in which the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of the weak fields. In spite of the dimensionless character of the scalar field an anti-gravity regime appears. This theory also suggests a possible way to solve missing matter problem in non-flat FRW cosmologies. The field equations given by Saez-Ballester [1] for the combined scalar and tensor fields (with $8\pi G = 1$ and $c = 1$) are

$$R_{ij} - \frac{1}{2}g_{ij}R - \omega\phi^n(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}) = -T_{ij} \tag{1.1}$$

and scalar field satisfies the equation

$$2\phi^n\phi^{;i}_{;i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0, \tag{1.2}$$

also we have energy-conservation equation

$$T^{ij}_{;j} = 0, \tag{1.3}$$

where ω and n are constants, comma and semicolon denote partial and covariant differentiation respectively.

The study of cosmological models in the framework of scalar tensor theories has been the active area of research for the last few decades. In particular, Rao et al. [38, 39, 40] are some of the authors who have investigated several aspects of the cosmological models in Saez-Ballester [1] theory. Naidu et al. [41, 42] have discussed various aspects of Bianchi space times in Saez-Ballester [1] theory.

Motivated by the above investigations and discussions, in this paper we study the evolution of dark energy parameter in five dimensional Kaluza-Klein space-time filled with two fluids (barotropic fluid and dark energy) in Saez-Ballester theory of gravitation. The paper is organized as follows: In section (2) we discuss metric and field equations. In sections (3) and (4) we discuss non-interacting and interacting two fluid models with their physical significances respectively. Physical stability of corresponding solutions are analyzed in section (5). Finally, the conclusions of the obtained models are presented in section (6).

2 Metric and field equations:

We consider the five dimensional Kaluza-Klein metric in the form

$$ds^2 = dt^2 - A^2(t)(dx^2 + dy^2 + dz^2) - B^2(t)d\psi^2 \tag{2.1}$$

where the fifth coordinate ψ is assumed to be space like coordinate.
The energy momentum tensor is given by

$$T^i_j = T^i_{j(de)} + T^i_{j(m)} \tag{2.2}$$

where $T^i_{j(de)}$ and $T^i_{j(m)}$ are the energy momentum tensors of dark energy and ordinary matter(barotropic fluid) respectively. These are given by

$$\begin{aligned} T^i_{j(de)} &= diag[-p_{de}, -p_{de}, -p_{de}, \rho_{de}, -p_{de}] \\ &= [-(\omega_{de}+\delta), -(\omega_{de}+\delta), -(\omega_{de}+\delta), 1, -(\omega+\gamma)]\rho_{de} \end{aligned} \tag{2.3}$$

and

$$\begin{aligned} T^i_{j(m)} &= diag[-p_m, -p_m, -p_m, \rho_m, -p_m] \\ &= [-\omega_m, -\omega_m, -\omega_m, 1, -\omega_m]\rho_m \end{aligned} \tag{2.4}$$

where ρ_{de} and p_{de} are respectively the energy density and pressure of the dark energy component matter while $\omega_{de} = \frac{p_{de}}{\rho_{de}}$ is its EoS parameter. Similarly, ρ_m and p_m are respectively the energy density and pressure of the barotropic fluid component while $\omega_m = \frac{p_m}{\rho_m}$ is the corresponding EoS parameter. Where, δ and γ are the deviations from ω_{de} on x, y, z and ψ axes respectively.

In a co-moving coordinate system Saez-Ballester field equations (1.1)-(1.3) for the metric (2.1), in the two fluid scenario, lead to

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{B}}{B} - \frac{\omega}{2}\phi^n\dot{\phi}^2 = -(\omega_{de}+\delta)\rho_{de} - \omega_m\rho_m \tag{2.5}$$

$$3\frac{\ddot{A}}{A} + 3\frac{\dot{A}^2}{A^2} - \frac{\omega}{2}\phi^n\dot{\phi}^2 = -(\omega_{de}+\gamma)\rho_{de} - \omega_m\rho_m \tag{2.6}$$

$$3\frac{\ddot{A}}{A} + 3\frac{\dot{A}\dot{B}}{AB} + \frac{\omega}{2}\phi^n\dot{\phi}^2 = \rho_{de} + \rho_m \tag{2.7}$$

$$\ddot{\phi} + \dot{\phi}\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) + \frac{n}{2}\frac{\dot{\phi}^2}{\phi} = 0. \tag{2.8}$$

The energy conservation equation, $T^{ij}_{;j} = 0$ leads to

$$\dot{\rho}_m + \left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)(\rho_m + p_m) + \dot{\rho}_{de} + \left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)(\rho_{de} + p_{de}) + \rho_{de}\left(3\delta\frac{\dot{A}}{A} + \gamma\frac{\dot{B}}{B}\right) = 0. \tag{2.9}$$

where overhead dot denotes ordinary differentiation with respect to time.

The fifth term of (2.9) arises due to the deviation from ω_{de} while the first and second terms of (2.9) are deviation free part of $T^i_{j(de)}$. According to (2.9), the behavior of ρ_{de} is controlled by the deviation free part of EoS parameter of dark energy but deviation will affect ρ_{de} indirectly, since as can be seen later, they affect the value of EoS parameter. But we are looking for physically viable models consistent with observations. Hence we constrained $\delta(t)$ and $\gamma(t)$ by assuming the special dynamics which is consistent with (2.9). The dynamics of skewness parameter on x, y, z axes as $\delta(t)$, and on ψ-axis as $\gamma(t)$ are given by

$$\delta(t) = \frac{\alpha}{4\rho_{de}}\frac{\dot{B}}{B}\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) \tag{2.10}$$

$$\gamma(t) = -\frac{3\alpha}{4\rho_{de}}\frac{\dot{A}}{A}\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) \tag{2.11}$$

where $\delta(t)$ and $\gamma(t)$ are dimensionless parameters and α is the real dimensionless constant that parameterizes the deviation from EoS parameter. The anisotropy of the dark energy is measured using the relation $\frac{\delta(t)-\gamma(t)}{\omega_{de}(t)}$ and for $\alpha = 0$, dark energy is found to be isotropic.

3 Non-interacting two-fluid model:

First we consider that two fluids do not interact with each other. Hence the general form of conservation equation (2.9) leads us to write the conservation equations for the dark energy and barotropic fluid separately as

$$\dot{\rho}_{de} + \left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)(\rho_{de} + p_{de}) + \rho_{de}\left(3\delta\frac{\dot{A}}{A} + \gamma\frac{\dot{B}}{B}\right) = 0 \tag{3.1}$$

$$\dot{\rho}_m + \left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)(\rho_m + p_m) = 0. \tag{3.2}$$

The EoS parameter of barotropic fluid ω_m is constant (Akarsu [43] and Kilinc [44]), that is

$$\omega_m = \frac{p_m}{\rho_m} = const., \tag{3.3}$$

while ω_{de} has been allowed to be a function of time since the current cosmological data from SNIa, CMB and large scale structures mildly favor dynamically evolving dark energy crossing the phantom divide line (PDL).

In order to solve highly non-linear field equations (2.5)-(2.8), we assume that the shear scalar (σ^2) is proportional to expansion scalar (θ). This condition leads to

$$A = B^k \tag{3.4}$$

where A and B are the metric potentials and k is positive constant.
Now, from (2.5) and (2.6), we get

$$\frac{\ddot{A}}{A}+2\frac{\dot{A}^2}{A^2}-2\frac{\dot{A}\dot{B}}{AB}-\frac{\ddot{B}}{B}=(\delta-\gamma)\rho_{de}. \tag{3.5}$$

Using (3.4), (2.10) and (2.11) in (3.5), we obtain

$$\frac{\ddot{B}}{B}+\left(\frac{12k(k-1)-\alpha(3k+1)^2}{4(k-1)}\right)\frac{\dot{B}^2}{B^2}=0. \tag{3.6}$$

The general solution of (3.6) is

$$B=(c_4t+c_5)^{\frac{4(k-1)}{k_2}} \tag{3.7}$$

where $k_2=4k_1-\alpha(3k+1)^2\neq 0$ with $k_1=(3k+1)(k-1)$.
From (3.4) and (3.7), we get

$$A=(c_4t+c_5)^{\frac{4k(k-1)}{k_2}} \tag{3.8}$$

From (1.2) scalar field is obtained as

$$\phi^{\frac{n+2}{2}}=\phi_0\frac{n+2}{2}\left[\frac{k_2}{k_2-4k_1}(c_4t+c_5)^{\frac{k_2-4k_1}{k_2}}+c_6\right] \tag{3.9}$$

where c_4, c_5, c_6 and ϕ_0 are integration constants.
Now, the metric (2.1) can be written as

$$ds^2=dt^2-(c_4t+c_5)^{\frac{8k(k-1)}{k_2}}(dx^2+dy^2+dz^2)-(c_4t+c_5)^{\frac{8(k-1)}{k_2}}d\psi^2. \tag{3.10}$$

In view of the assumption $\omega(m)=$ constant, (3.2) can be integrated to obtain

$$\rho_m=\rho_0(A^3B)^{-(1+\omega_m)} \tag{3.11}$$

where ρ_0 is an integration constant.
Using (3.7) and (3.8) in (3.11), we get

$$\rho_m=\rho_0(c_4t+c_5)^{\frac{-4(1+\omega_m)k_1}{k_2}}. \tag{3.12}$$

By using (3.12) in (2.5)-(2.8), we obtain density(ρ_{de}) and pressure(p_{de}) of dark energy as

$$\rho_{de}=\frac{48k(k+1)(k-1)^2c_4^2}{k_2^2(c_4t+c_5)^2}+\frac{\omega\phi_0^2}{2(c_4t+c_5)^{8k_1/k_2}}-\frac{\rho_o}{(c_4t+c_5)^{4k_1(1+\omega_m)/k_2}} \tag{3.13}$$

$$p_{de}=-\left[\frac{(8(k-1)^2(9k^2+2k+1)-2k_2(k^2-1))c_4^2}{k_2^2(c_4t+c_5)^2}-\frac{\omega\phi_0^2}{2(c_4t+c_5)^{8k_1/k_2}}+\frac{\omega_m\rho_o}{(c_4t+c_5)^{4k_1(1+\omega_m)/k_2}}\right]. \tag{3.14}$$

By using (3.13) and (3.14), we find the EoS ($\omega_{de}=p_{de}/\rho_{de}$) parameter of dark energy as

$$\omega_{de}=-\left[\frac{\frac{(8(k-1)^2(9k^2+2k+1)-2k_2(k^2-1))c_4^2}{k_2^2(c_4t+c_5)^2}-\frac{\omega\phi_0^2}{2(c_4t+c_5)^{8k_1/k_2}}+\frac{\omega_m\rho_o}{(c_4t+c_5)^{4k_1(1+\omega_m)/k_2}}}{\frac{48k(k+1)(k-1)^2c_4^2}{k_2^2(c_4t+c_5)^2}+\frac{\omega\phi_0^2}{2(c_4t+c_5)^{8k_1/k_2}}-\frac{\rho_o}{(c_4t+c_5)^{4k_1(1+\omega_m)/k_2}}}\right]. \tag{3.15}$$

Thus, the metric (3.10) together with (3.13), (3.14), (3.15) and (3.9) represents two fluid non-interacting dark energy model in Saez-Ballester theory.

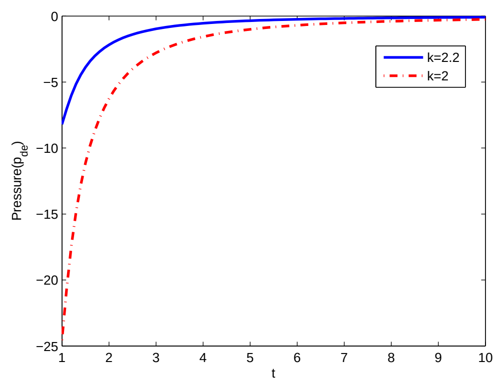

Figure 1: The plot of p_{de} versus t for ω=2, $\omega_m = m = 0.5$ and $\rho_0 = \phi_0 = 1$ in non-interacting two fluid model.

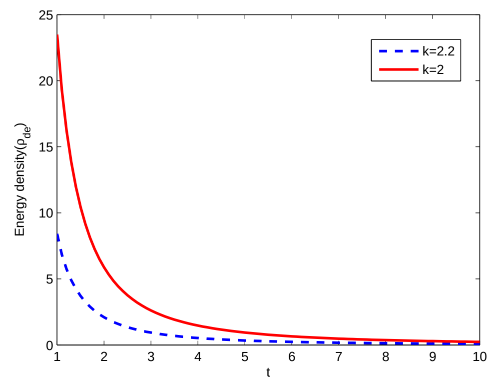

Figure 2: The plot of ρ_{de} versus t for ω=2, $\omega_m = m = 0.5$ and $\rho_0 = \phi_0 = 1$ in non-interacting two fluid model.

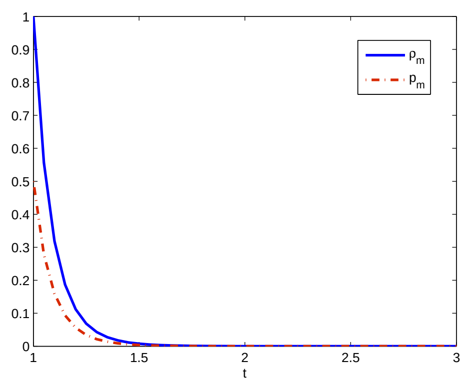

Figure 3: The plot of p_m and ρ_m versus t for ω=2, $\omega_m = 0.5$, $m = 0.5$ and $\rho_0 = \phi_0 = 1$ in non-interacting two fluid model.

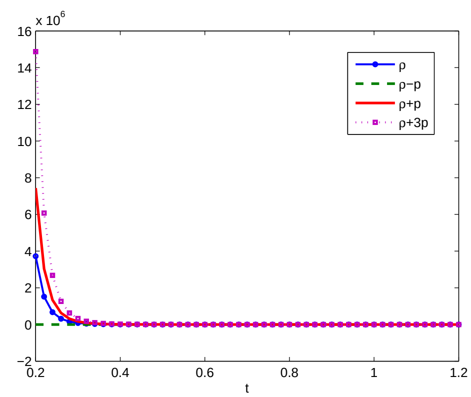

Figure 4: The plot of energy conditions versus t for ω=2, $\omega_m = m = 0.5$ and $\rho_0 = \phi_0 = 1$ in non-interacting two fluid model.

The behavior of pressure p_{de} for dark fluid versus t is shown in Fig. 1. It is observed that p_{de} is always negative and finally tends to zero as expected. Fig. 2 depicts the variation of energy density of dark fluid ρ_{de} versus t. Here, we observe that ρ_{de} decreases as time increases.

Fig. 3 shows the behavior of ρ_m and p_m versus cosmic time t. Both are positive decreasing functions of time and converges to zero for large time. Fig. 4 depicts the L.H.S of energy conditions given by Ellis [46] (i) $(\rho + p) > 0$, (ii) $(\rho + 3p) > 0$, (iii) $\rho > 0$ and the dominant energy conditions given by Hawking and Ellis [47] (i) $(\rho - p) \geq 0$, (ii) $(\rho + p) \geq 0$. We observed that both energy conditions are satisfied.

The behavior of EoS parameter for dark energy(ω_{de}) in terms of cosmic time t is shown in Fig. 5. It is observed that the EoS parameter of dark energy(ω_{de}) is a decreasing function of time and it is crossing -1 near the past i.e., from quintessence region($\omega_{de} > -1$) to phantom region($\omega_{de} < -1$), it is well-known Quintom dark energy scenario by Xin [45]. The rapidity of its decrease at the early stage depends on the type of the universe, while later on it tends to the constant value -1. Feng et al. [10] suggested an oscillating Quintom for unifying the early inflation and current acceleration of the universe. This oscillating Quintom can avoid leading to an event horizon. Also such a component($\omega_{de} < -1$) is found to be compatible with most classical tests of cosmology based on current data, including the recent type Ia SNe data as well as the cosmic microwave background anisotropy and mass power spectrum. So the model obtained and presented here is represent not only the early stages of evolution but also the present universe.

The expressions for the matter-energy density Ω_m and dark-energy density Ω_{de} are given by

$$\Omega_m = \frac{\rho_0 k_2^2}{3k_1^2 c_4^2 (c_4 t + c_5)^{(4(1+\omega_m)k_1/k_2)-2}} \tag{3.16}$$

$$\Omega_{de} = \frac{16k(k+1)(k-1)^2}{k_1^2} + \frac{\omega\phi_0^2 k_2^2}{2c_4^2 k^2 (c_4 t + c_5)^{(8k_1/k_2)-2}} - \frac{\rho_0 k_2^2}{3k_1^2 c_4^2 (c_4 t + c_5)^{(4(1+\omega_m)k_1/k_2)-2}} \tag{3.17}$$

where as the total density parameter(Ω) is given by

$$\begin{aligned} \Omega &= \Omega_m + \Omega_{de} \\ &= \frac{16k(k+1)(k-1)^2}{k_1^2} + \frac{\omega\phi_0^2 k_2^2}{2c_4^2 k^2 (c_4 t + c_5)^{(8k_1/k_2)-2}} \end{aligned} \tag{3.18}$$

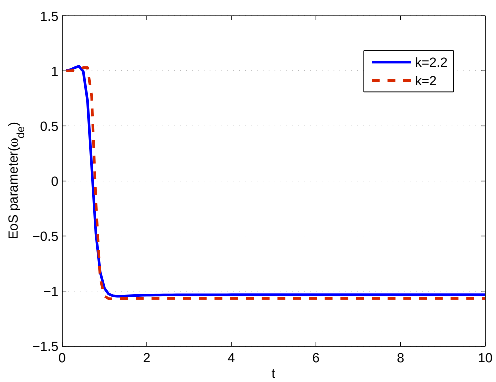

Figure 5: The plot of EoS parameter(ω_{de}) versus t for ω=2, $\omega_m = m = 0.5$ and $\rho_0 = \phi_0 = 1$ in non-interacting two fluid model.

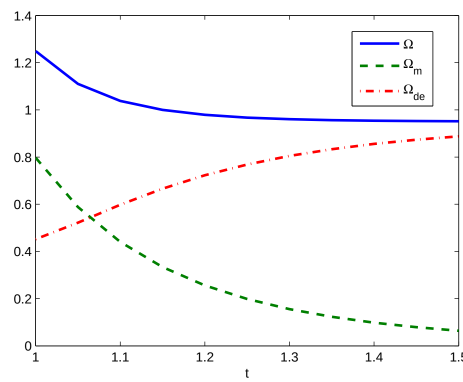

Figure 6: The plot of density parameters versus t in non-interacting two fluid model.

Fig. 6 demonstrates the behavior of density parameters in the evolution of universe with appropriate choice of constants of integration and other physical parameters. We observe that initially the ordinary matter density(Ω_m) dominates the universe. But later on, the dark energy density(Ω_{de}) dominates the evolution which is probably responsible for the accelerated expansion of present day universe. Also, we observe that density parameter $\Omega \to 1$ at late times. This result is compatible with the observational results. Since our model predicts a flat universe for large times and present day universe is very close to flat universe, the derived model is also in agreement with the observational results.

Now using (3.13), (3.7) and (3.8) in (2.10) and (2.11), the deviation parameters $\delta(t)$ and $\gamma(t)$ are obtained as

$$\delta(t) = \frac{4\alpha k_1 (k-1) c_4^2}{48k(k+1)(k-1)^2 c_4^2 + \omega\phi_0^2 k_2^2 (c_4 t + c_5)^{2-(8k_1/k_2)} - \rho_o k_2^2 (c_4 t + c_5)^{2-(4k_1(1+\omega_m)/k_2)}} \tag{3.19}$$

$$\gamma(t) = \frac{12\alpha k k_1 (1-k) c_4^2}{48k(k+1)(k-1)^2 c_4^2 + \omega\phi_0^2 k_2^2 (c_4 t + c_5)^{2-(8k_1/k_2)} - \rho_o k_2^2 (c_4 t + c_5)^{2-(4k_1(1+\omega_m)/k_2)}}. \tag{3.20}$$

The anisotropy measure of anisotropic fluid(dark energy) is given by

$$\frac{\delta-\gamma}{\omega_{de}} = \frac{8\alpha k_1(k^2-1)c_4^2}{\omega\phi_0^2 k_2^2(c_4t+c_5)^{2-\frac{8k_1}{k_2}} - (8(k-1)^2(9k^2+2k+1)-2k_2(k^2-1))c_4^2 - \omega_m\rho_0 k_2^2(c_4t+c_5)^{2-\frac{4k_1(1+\omega_m)}{k_2}}} \tag{3.21}$$

Since $\frac{k_1}{k_2} > \frac{1}{4}$ & $(1+\omega_m) > 2$, from (3.19) and (3.20) the deviation parameters $\delta(t)$ and $\gamma(t)$ are finite at $t = 0$ and converges to $\frac{mk_1}{12k(k^2-1)}$ & $\frac{mk_1}{4(1-k^2)}$ respectively as $t \to \infty$. The anisotropic measure of dark energy $(\delta-\gamma)/\omega_{de}$ is constant at $t = 0$ and converges to $\frac{4mk_1(k^2-1)}{4(k-1)^2(9k^2+2k+1)-k_2(k^2-1)}$ for the later times of the universe. We note that the anisotropic distribution of dark energy doesn't vanish throughout the evolution of the universe.

The other important properties of the model (3.10) are

Volume(V) and average sacale factor (a) are given by

$$V = A^3B = (c_4t+c_5)^{\frac{4k_1}{k_2}} \tag{3.22}$$

$$a = V^{\frac{1}{4}} = (c_4t+c_5)^{\frac{k_1}{k_2}} \tag{3.23}$$

Hubble parameter(H), expansion scalar(θ), shear scalar(σ^2) and average anisotropic parameter(A_m) are given by

$$H = \frac{\dot{a}}{a} = \frac{k_1c_4}{k_2(c_4t+c_5)} \tag{3.24}$$

$$\theta = 4H = \frac{4k_1c_4}{k_2(c_4t+c_5)} \tag{3.25}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{3c_4^2(k-1)^2(21k^2-18k+13)}{2k_2^2(c_4t+c_5)^2} \tag{3.26}$$

$$A_h = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{\triangle H_i}{H}\right)^2 = \frac{7}{3}\left(\frac{k-1}{3k+1}\right)^2. \tag{3.27}$$

where $\triangle H_i = H_i - H$,$(i = 1,2,3,4)$.

Deceleration parameter:

The deceleration parameter q is given by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{k_2}{k_1} - 1 \tag{3.28}$$

the sign of q indicates whether the model inflates or not. A positive sign of q, i.e. $k_2/k_1 > 1$ corresponds to standard decelerating model whereas negative sign of q, i.e. $0 < k_2/k_1 < 1$ indicates acceleration. The recent observations SN Ia, reveal that the present universe is accelerating and the value of deceleration parameter lies somewhere in the range $-1 < q < 0$. It follows that in the derived model, one can choose the value of deceleration parameter consistent with observations.

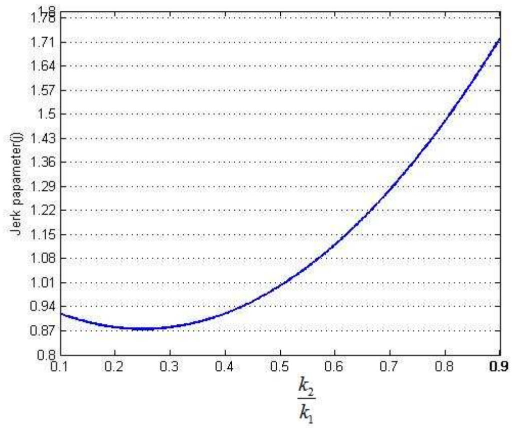

Figure 7: The plot of jerk parameter(j) versus $\frac{k_2}{k_1}$

Jerk parameter:

Jerk parameter in cosmology is defined as the dimensionless third derivative of scale factor with respect to cosmic time and is given by

$$j(t) = \frac{1}{H^3}\frac{\dddot{a}}{a}, \tag{3.29}$$

where a is the cosmic scale factor, H is the Hubble parameter. This parameter appears in the fourth term of a Taylor expansion of the scale factor around a_0:

$$\frac{a(t)}{a_0} = 1 + H_0(t-t_0) - \frac{1}{2}q_0H_0^2(t-t_0)^2 + \frac{1}{6}j_0H_0^3(t-t_0)^3 + O[(t-t_0)^4], \tag{3.30}$$

where the subscript 0 denotes the present value.

We can rewrite equation (3.29) as

$$j = q + 2q^2 - \frac{\dot{q}}{H}, \tag{3.31}$$

where q is deceleration parameter.
Hence the expression for jerk parameter is given by

$$j = 2\frac{k_2^2}{k_1^2} - \frac{k_2}{k_1} + 1. \tag{3.32}$$

From Fig. 7 it is observed that the jerk parameter value lies within the interval $(0.85, 1.75)$ for $0 < (k_2/k_1) < 1$. It is in agreement with the results given by Blandford [48] in which it has been approved that for $\omega_{de} < -1$, the jerk parameter should be greater than one.

4 Interacting two fluid model:

In this section, we consider the interaction between dark and barotropic fluids. For this purpose we can write the continuity equations for dark fluid and barotropic fluids as

$$\dot{\rho}_{de} + \left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)(\rho_{de} + p_{de}) = Q \tag{4.1}$$

$$\dot{\rho}_{m} + \left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)(\rho_{m} + p_{m}) = -Q. \tag{4.2}$$

The quantity Q expresses the interaction between the dark components. Since we are interested in an energy transfer from the dark energy to dark matter. Also $Q > 0$ ensure that the second law of thermodynamics is satisfied [49]. Following Amendola et al. [50] and Guo et al. [51], we consider

$$Q = 3H\sigma\rho_m \tag{4.3}$$

where σ is a coupling constant. Using (4.3) in (4.2) and integrating we obtain

$$\rho_m = \rho_0 (A^3 B)^{-(1+\omega_m - \frac{3\sigma}{4})} \tag{4.4}$$

where ρ_0 is an integration constant.
In view of (3.7) and (3.8), above equation (4.4) takes the form

$$\rho_m = \rho_0 (c_4 t + c_5)^{\frac{-4(1+\omega_m - \frac{3\sigma}{4})k_1}{k_2}}. \tag{4.5}$$

By using (4.5) in (2.5)-(2.8) we obtain density(ρ_{de}) and pressure(p_{de}) of dark energy as

$$\rho_{de} = \frac{48k(k+1)(k-1)^2 c_4^2}{k_2^2 (c_4 t + c_5)^2} + \frac{\omega\phi_0^2}{2(c_4 t + c_5)^{8k_1/k_2}} - \frac{\rho_o}{(c_4 t + c_5)^{\frac{4k_1(1+\omega_m - \frac{3\sigma}{4})}{k_2}}} \tag{4.6}$$

$$p_{de} = -\left[\frac{(8(k-1)^2(9k^2+2k+1) - 2k_2(k^2-1))c_4^2}{k_2^2 (c_4 t + c_5)^2} - \frac{\omega\phi_0^2}{2(c_4 t + c_5)^{8k_1/k_2}} + \frac{\omega_m \rho_o}{(c_4 t + c_5)^{\frac{4k_1(1+\omega_m - \frac{3\sigma}{4})}{k_2}}}\right]. \tag{4.7}$$

The behavior of pressure p_{de} for dark fluid versus t is shown in Fig. 8. It is observed that p_{de} is always negative and finally tends to zero as expected. Fig. 9 depicts the variation of energy density of dark fluid ρ_{de} versus t. Here, we observe that ρ_{de} decreases as time increases. Fig. 10 shows the behavior of ρ_m and p_m versus cosmic time t. Both are positive decreasing functions of time and converges to zero for large time. Fig. 11 depicts the L.H.S of energy conditions (i) $(\rho + p) > 0$, (ii) $(\rho + 3p) > 0$, (iii) $\rho > 0$ and the dominant energy conditions (i) $(\rho - p) \geq 0$, (ii) $(\rho + p) \geq 0$. We observed that both energy conditions are satisfied.

By using (4.6) and (4.7) we find the EoS parameter($\omega_D = p_{de}/\rho_{de}$) of dark energy as

$$\omega_{de} = -\left[\frac{\frac{(8(k-1)^2(9k^2+2k+1) - 2k_2(k^2-1))c_4^2}{k_2^2(c_4t+c_5)^2} - \frac{\omega\phi_0^2}{2(c_4t+c_5)^{8k_1/k_2}} + \frac{\omega_m\rho_o}{(c_4t+c_5)^{4k_1(1+\omega_m-\frac{3\sigma}{4})/k_2}}}{\frac{48k(k+1)(k-1)^2c_4^2}{k_2^2(c_4t+c_5)^2} + \frac{\omega\phi_0^2}{2(c_4t+c_5)^{8k_1/k_2}} - \frac{\rho_o}{(c_4t+c_5)^{4k_1(1+\omega_m-\frac{3\sigma}{4})/k_2}}}\right]. \tag{4.8}$$

Thus, the metric (3.10) together with (4.6), (4.7), (4.8) and (3.9) represents two fluid interacting dark energy model in Saez-Ballester theory.

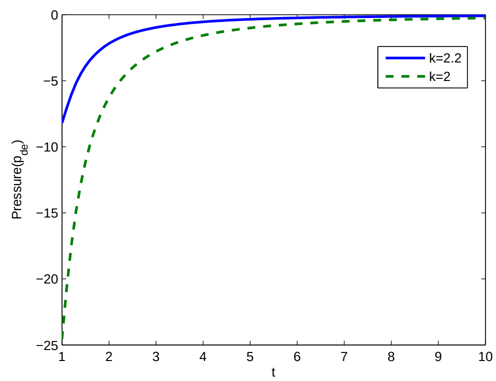

Figure 8: The plot of p_{de} versus t for ω=2, $\omega_m = m = 0.5$, $\rho_0 = \phi_0 = 1$ and $\sigma = 0.4$ in interacting two fluid model.

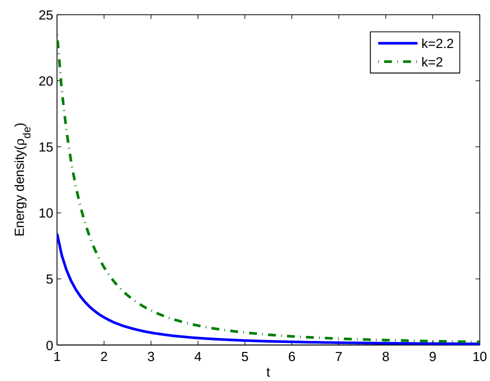

Figure 9: The plot of ρ_{de} versus t for ω=2, $\omega_m = m = 0.5$ and $\rho_0 = \phi_0 = 1$ in interacting two fluid model.

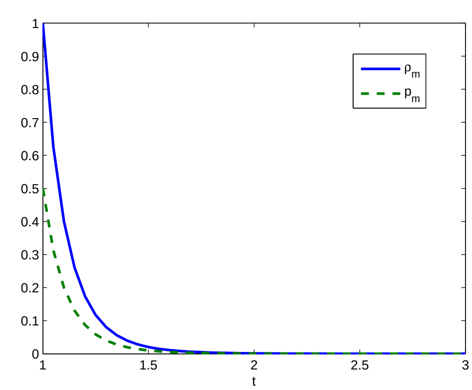

Figure 10: The plot of p_m and ρ_m versus t for ω=2, $\omega_m = m = 0.5$ and $\rho_0 = \phi_0 = 1$ in interacting two fluid model.

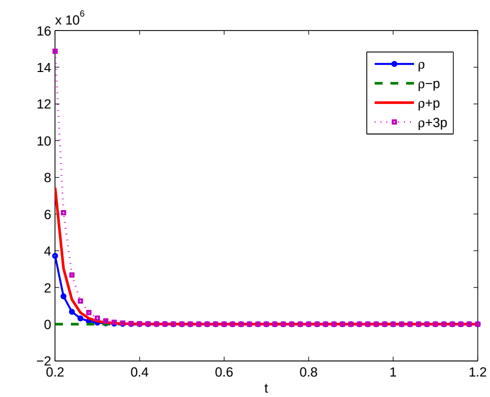

Figure 11: The plot of energy conditions versus t for ω=2, $\omega_m = m = 0.5$ and $\rho_0 = \phi_0 = 1$ in interacting two fluid model.

The behavior of EoS parameter for dark energy(ω_{de}) in terms of cosmic time t is shown in Fig. 12. It is observed that the EoS parameter of dark energy(ω_{de}) is a decreasing function of time and it is crossing -1 near the past i.e., from quintessence region($\omega_{de} > -1$) to phantom region($\omega_{de} < -1$) it is well-known Quintom dark energy scenario Xin [45]. So the model obtained and presented here is represent not only the early stages of evolution but also the present universe.

The expressions for the matter-energy density Ω_m and dark-energy density Ω_{de} are given by

$$\Omega_m = \frac{\rho_0 k_2^2}{3k_1^2 c_4^2 (c_4 t + c_5)^{(4(1+\omega_m - \frac{3\sigma}{4})k_1/k_2)-2}} \tag{4.9}$$

$$\Omega_{de} = \frac{16k(k+1)(k-1)^2}{k_1^2} + \frac{\omega \phi_0^2 k_2^2}{2c_4^2 k^2 (c_4 t + c_5)^{(8k_1/k_2)-2}} - \frac{\rho_0 k_2^2}{3k_1^2 c_4^2 (c_4 t + c_5)^{(4(1+\omega_m - \frac{3\sigma}{4})k_1/k_2)-2}} \tag{4.10}$$

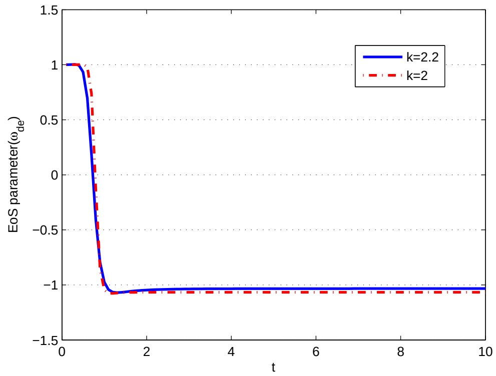

Figure 12: The plot of EoS parameter(ω_{de}) versus t for ω=2, $\omega_m = m = 0.5$, $\rho_0 = \phi_0 = 1$ and $\sigma = 0.4$ in interacting two fluid model.

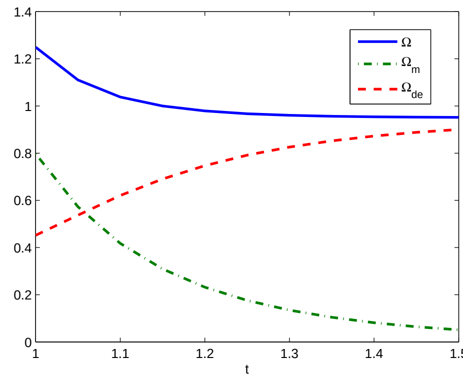

Figure 13: The plot of density parameters versus t in interacting two fluid model.

where as the total density parameter(Ω) is given by

$$\begin{aligned}\Omega &= \Omega_m + \Omega_{de} \\ &= \frac{16k(k+1)(k-1)^2}{k_1^2} + \frac{\omega\phi_0^2 k_2^2}{2c_4^2 k^2 (c_4 t + c_5)^{(8k_1/k_2)-2}}. \end{aligned} \tag{4.11}$$

The expression for total density parameter(Ω) in interacting case (4.11) is same as the (3.18). Therefore, we observe that in interacting case the density parameter(Ω) has the same properties as in non-interacting case. From Fig. 13 we observed that ordinary matter density and dark energy density parameters also have same properties as in non-interacting case.

Now using (4.6), (3.7) and (3.8) in (2.10) and (2.11), the deviation parameters $\delta(t)$ and $\gamma(t)$ are obtained as

$$\delta(t) = \frac{4mk_1(k-1)c_4^2}{48k(k+1)(k-1)^2c_4^2 + \omega\phi_0^2k_2^2(c_4t+c_5)^{2-(8k_1/k_2)} - \rho_o k_2^2 (c_4t+c_5)^{2-(4k_1(1+\omega_m-\frac{3\sigma}{4})/k_2)}} \tag{4.12}$$

$$\gamma(t) = \frac{12mkk_1(1-k)c_4^2}{48k(k+1)(k-1)^2c_4^2 + \omega\phi_0^2k_2^2(c_4t+c_5)^{2-(8k_1/k_2)} - \rho_o k_2^2 (c_4t+c_5)^{2-(4k_1(1+\omega_m-\frac{3\sigma}{4})/k_2)}} \tag{4.13}$$

The anisotropy measure of anisotropic fluid(dark energy) is given by
$\frac{\delta-\gamma}{\omega_{de}} =$

$$\frac{8mk_1(k^2-1)c_4^2}{\omega\phi_0^2k_2^2(c_4t+c_5)^{2-\frac{8k_1}{k_2}} - (8(k-1)^2(9k^2+2k+1) - 2k_2(k^2-1))c_4^2 - \omega_m\rho_0k_2^2(c_4t+c_5)^{2-\frac{4k_1(1+\omega_m-\frac{3\sigma}{4})}{k_2}}} \tag{4.14}$$

The deviation parameters and anisotropic parameter of dark energy have the same properties as in non-interacting two fluid model. Here also the anisotropic parameter of dark energy doesn't vanish throughout the evolution.

The expressions for deceleration parameter(q) and jerk parameter(j) are same as in the case of non-interacting case.

5 Physical stability of solutions:

For the stability of corresponding solutions in both non-interacting and interacting models, we should check whether our models are physically acceptable or not. For this,

i. The velocity of sound should be less than velocity of light i.e. within the range $0 \leq v_s = \frac{dp_{de}}{d\rho_{de}} \leq 1$. The sound speed in non-interacting and interacting two fluid cosmological models are obtained as

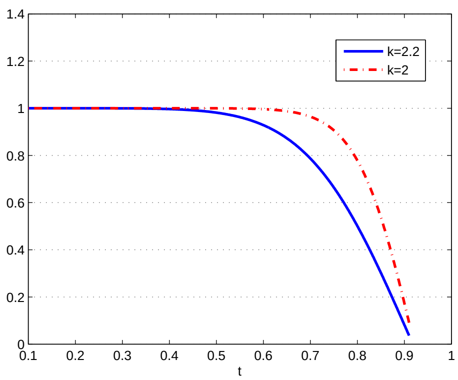

Figure 14: The plot of sound speed v_s versus t in non-interacting two fluid model.

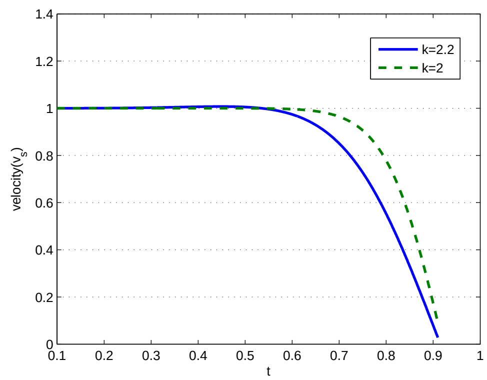

Figure 15: The plot of sound speed v_s versus t in interacting two fluid model.

$$\frac{dp_{de}}{d\rho_{de}} = \frac{\frac{(4(k-1)^2(9k^2+2k+1)-k_2(k^2-1))c_4^2}{k_2(c_4t+c_5)^3} - \frac{k_1\omega\phi_0^2}{(c_4t+c_5)^{(8k_1+k_2)/k_2}} + \frac{k_1\omega_m\rho_o(1+\omega_m)}{(c_4t+c_5)^{(4k_1(1+\omega_m)+k_2)/k_2}}}{\frac{-24k(k+1)(k-1)^2c_4^2}{k_2(c_4t+c_5)^3} - \frac{k_1\omega\phi_0^2}{(c_4t+c_5)^{(8k_1+k_2)/k_2}} + \frac{k_1\rho_o}{(c_4t+c_5)^{(4k_1(1+\omega_m)+k_2)/k_2}}} \tag{5.1}$$

and

$$\frac{dp_{de}}{d\rho_{de}} = \frac{\frac{(4(k-1)^2(9k^2+2k+1)-k_2(k^2-1))c_4^2}{k_2(c_4t+c_5)^3} - \frac{k_1\omega\phi_0^2}{(c_4t+c_5)^{(8k_1+k_2)/k_2}} + \frac{k_1\omega_m\rho_o(1+\omega_m-\frac{3\sigma}{4})}{(c_4t+c_5)^{(4k_1(1+\omega_m-\frac{3\sigma}{4})/k_2}}}{\frac{-24k(k+1)(k-1)^2c_4^2}{k_2(c_4t+c_5)^3} - \frac{k_1\omega\phi_0^2}{(c_4t+c_5)^{(8k_1+k_2)/k_2}} + \frac{k_1\rho_o}{(c_4t+c_5)^{(4k_1(1+\omega_m-\frac{3\sigma}{4})+k_2)/k_2}}}. \tag{5.2}$$

From Fig. 14 and 15 we observe that in both non-interacting and interacting cases $v_s \leq 1$.

ii. The weak and dominant energy conditions are given by (i) $\rho \geq 0$, (ii) $\rho + p \geq 0$ and (iii) $\rho - p \geq 0$. Strong energy conditions are given by $\rho + 3p \geq 0$ are should be satisfy.

From Fig. 4 and 11 we observed that weak, dominant and strong energy conditions are satisfied in both non-interacting and interacting cases.

Therefore, on the basis of above discussions and analysis, our corresponding solutions are physically stable.

6 Conclusions:

In this paper we have studied the two fluid scenario in a scalar tensor theory proposed by Saez and Ballester for the spatially homogeneous five dimensional Kaluza-Klien space-time. It is found that in both non-interacting and interacting cases EoS parameter of dark energy (ω_{de}) is a decreasing function of time and started its evolution from quintessence region and crosses the PDL ($\omega_{de} = -1$) and finally approaches to -1 (i.e. cosmological constant). The rapidity of its decrease at the early stage depends on the type of the universe. Thus, we find that the EoS parameter for non-interacting and interacting two fluid models change from $\omega_{de} > -1$(quintessence) to $\omega_{de} < -1$(phantom), which is consistent with recent observations.

The anisotropy measure of the dark energy is dynamical and found to be finite for both earlier and later times of the universe. The isotropic distribution dark energy can be recovered by choosing"α" to be zero, where "α" parameterizes the deviation parameters. In both non-interacting and interacting cases the total density parameter (Ω) approaches to unity for sufficiently large time which is consistent with the current observations. Also we have obtain the cosmic jerk parameter in our derived models is also found to be in good agreement with the recent data of astrophysical observations.

Our proposed solutions are physically stable. Thus, the solutions obtained in this paper may be useful for better understanding of the characteristic of dark energy in the evolution of universe within the framework of Kaluza-Klien.

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References

[1] D. Saez, V. J. Ballester: Physics Letters A, 113, 467 (1986).

[2] A. G. Riess et al., Astron. J. 116, 1009 (1998); S. Perlmutter et al., Astrophys. J. 517, 565 (1999).

[3] C. L. Bennett et al., Astrophys. J. Suppl. 148, 1 (2003); D.N. Spergel et al., Astrophys. J. Suppl. 148, 175 (2003).

[4] M. Tegmark et al., Phys. Rev. D 69, 103501 (2004); K. Abazajian et al., astro-ph/0410239; K. Abazajian et al., Astron. J. 128, 502 (2004); K. Abazajian et al., Astron. J. 126, 2081 (2003); E. Hawkins et al., Mon. Not. Roy. Astron. Soc. 346, 78 (2003); L. Verde et al., Mon. Not. Roy. Astron. Soc. 335, 432 (2002).

[5] T. Koivisto, D. F. Mota: JCAP 0806, 018 (2008).

[6] T. Koivisto, D. F. Mota: Astrophys. J. 679, 1 (2008).

[7] C. J. A. P. Martins: Philos. Trans. R. Soc. Lond. A 360, 2681 (2002).

[8] P. J. Steinhardt, L. M. Wang, I. Zlatev: Phys. Rev. D 59, 123504 (1999).

[9] R. R. Caldwell: Phys. Lett. B 545, 23 (2002).

[10] B. Feng, X.L. Wang, X. Zhang: Commun. Theor. Phys. 44, 948 (2005).

[11] M. R. Setare: Phys. Lett. B 641, 130 (2006).

[12] Y. F. Cai, E. N. Saridakis, M. R. Setare and J. Q. Xia: Phys. Rep. 493, 1 (2010).

[13] M. R. Setare, E. N. Saridakis: Phys. Lett. B 668, 177 (2008).

[14] M. R. Setare, E. N. Saridakis: Int. J. Mod. Phys. D 18, 549 (2009).

[15] M. R. Setare: Eur. Phys. J. C 50, 991 (2007).

[16] M. R. Setare, E. N. Saridakis: J. Cosmol. Astropart. Phys. 0903, 002 (2009).

[17] A. Melchiorri, L. Mersimi, C. J. Odman and M. Trodden: Phys. Rev. D 68, 043509 (2003).

[18] A. Pradhan, H. Amirhashchi, J. Rekha: Astrophys. Space Sci. 334, 249 (2011).

[19] T. Singh and R. Chaubey: Astrophys. Space Sci. 319, 149 (2009).

[20] R. Chaubey: Astrophys. Space Sci. 321, 241 (2009).

[21] A. Pradhan, J. Rekha, J. Kanti, K. R. Kumar: Astrophys. Space Sci. 337, 401 (2012).

[22] V. U. M. Rao, G. Sreedevi Kumari, D. Neelima: Astrophys. Space Sci. 337, 449 (2012).

[23] A. K. Yadav, B. Saha: Astrophys. Space Sci. 337, 759 (2012).

[24] V. U. M. Rao, K. V. S. Sireesha, D. Neelima: ISRN Astronomy and Astrophysics 2013, ID 924834 (2012).

[25] M. Sharif, M. Zubair: Astrophys. Space Sci. 330, 399 (2010).

[26] D. R. K. Reddy, R. Santhi Kumar: Int. J. Theor. Phys. 52, 1362 (2012).

[27] D. R. K. Reddy, S. Anitha, S. Umadevi: Astrophys. Space Sci. 350, 799 (2014).

[28] R. Venketeswarlu: Prespacetime Journal, 4, 801 (2013).

[29] N. M. Liang et al.: Chin. Phys. Lett. 26, 069501 (2009).

[30] L. P. Chimento et al.: Phys. Rev. D 67, 083513 (2003).

[31] L. P. Chimento, D. Pavon: Phys. Rev. D 73, 063511 (2006).

[32] B. Saha et al.: Astrophys. Space Sci. 342, 257 (2012).

[33] H. Amirharshchi et al.: Chi. Phys. Lett. 28, 039801 (2011).

[34] H. Amirhashchi, A. Pradhan, H. Zainuddin: Research in Astron. Astrophys. 13, 129 (2013).

[35] A. Pradhan: Indian J. Phys. 88, 215 (2014).

[36] H. Amirhashchi et al.: Int. J. Theor. Phys. 50, 3529 (2011).

[37] H. Amirhashchi, D. S. Chouhan, A. Pradhan: EJTP 11, 109 (2014).

[38] V. U. M. Rao, T. Vinutha, M. Vijaya Santhi: Astrophys. Space Sci. 312, 189 (2007).

[39] V. U. M. Rao, M. Vijaya Santhi, T. Vinutha: Astrophys. Space Sci. 314, 73 (2008).

[40] V. U. M. Rao, M. Vijaya Santhi, T. Vinutha: Astrophys. Space Sci. 317, 27 (2008).

[41] R. L. Naidu, B. Satyanarayana, D. R. K. Reddy: Astrophys. Space Sci. 338, 351 (2012).

[42] R. L. Naidu, B. Satyanarayana, D. R. K. Reddy: Astrophys. Space Sci. 338, 333 (2012).

[43] O. Akarsu, C. B. Kilinc: Gen. Relativ. Gravit. 42, 119 (2010).

[44] O. Akarsu, C. B. Kilinc: Gen. Relativ. Gravit. 42, 763 (2010).

[45] Zhang Xin: Commun. Theor. Phys. 44, 762 (2005).

[46] G. F. R. Ellis: General Relativity and Cosmology. Academic Press, New York (1971).

[47] S. W. Hawking, G. F. R. Ellis: The Large-Scale Structure of Spacetime. Cambridge University Press, Cambridge(1973).

[48] R. D. Blandford, M. Amin, E. A. Baltz, K. Mandel, P. J. Marshall: arXiv astroph/ 0408279v1 (2004).

[49] D. Pavon, B. Wang: Gen. Relativ. Gravit. 41, 1 (2009).

[50] L. Amendola et al.: Phys. Rev. D 75, 083506 (2007).

[51] Z. K. Guo et al.: Phys. Rev. D 76, 023508 (2007).