Article

Bianchi Type - IX Minimally Interacting Holographic Dark Energy Model with Linearly Varying Deceleration Parameter in Scalar-tensor Theory of Gravitation

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Abstract

In this paper, we investigate spatially homogeneous and anisotropic Bianchi type-IX space-time filled with two minimally interacting fields, matter and holographic dark energy components in the scalar-tensor theory of gravitation formulated by Saez and Ballester (1986).To obtain exact solution of the field equations we have used: (i) The fact that scalar expansion is proportional to shear scalar of the space-time; and (ii) Linearly varying deceleration parameter proposed by Akarsu and Dereli (I2012). Some physical and kinematical properties of the model are, also, discussed.

Keywords: Bianchi type-IX, scalar-tensor theory, holographic dark energy, linearly varying, deceleration parameter.

1. Introduction

It is well known that the experiments conducted by Riess (1998) and Perlmutter (1999) have confirmed that the present day universe is in a state of accelerated expansion. It is said that the agent causing this cosmic acceleration is dark energy and it violates the strong energy condition. However, nothing is known certain about this till today. The dynamical cosmological term Λ is the favorite choice of research workers working in Lamda-dark energy as a possible candidate for dark energy. But this has coincidence problem. Hence, quintessence, phantom, tachyon, quintom, Chapligin gas, holographic dark energy have been proposed as possible candidates for dark energy.

In recent years several modified theories of gravitation have been proposed as alternatives to Einstein's theory of gravitation to explain dark energy. Among the various modifications of Einstein's gravity, f(R) gravity proposed by Akbar and Cai (2006), f(R,T)gravity formulated by Harko et al (2011) and scalar-tensor theories of gravitation constructed by Brans and Dicke

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(1961) and Saez and Ballester (1986) are ,recently, attracting the attention of several researchers and various dark energy models in these theories have been discussed.

Here we are interested in a dark energy candidate based on sound thermodynamic consideration and which is receiving growing attention in literature, namely, 'holographic dark energy.' This is based on the holographic principle which states that the number of degrees of freedom of a physical system should scale with its bounding area rather than with its volume and it should be constrained by an infrared cutoff (Hooft, 1995).

Investigation of dark energy cosmological models in modern cosmology is playing a vital role. In particular, the studies of minimally interacting and interacting holographic dark energy models are attracting the attention of several research workers. Cohen et al (1999),Horova and Minic (2000), Thomas (2002), Hsu (2004),Li (2004),Setare (2007), Sheyki (2009), Setare and Vagenas (2009) and Das and Mamon (2014) have discussed several aspects of holographic dark energy models both in general relativity and in Brans-Dicke theory of gravitation.

Recently, Sarkar and Mahanta (2013),Sarkar (2014a,2014b,2014c),Adhav et al.(2015),Kiran et al. (2014a,2014b),Umadevi and Ramesh(2015) have investigated minimally interacting and interacting holographic dark energy Bianchi models in general relativity and in scalar-tensor theories of gravitation. Very recently, Kiran et al.(2015) and Reddy et al.(2015) have discussed Bianchi type minimally interacting holographic dark energy models using linearly varying deceleration parameter proposed by Akarsu and Dereli (2012).

In spite of the fact that the present day universe is homogeneous and isotropic and is better described by Friedman-Robertson –walker (FRW) model, it is said that Bianchi models are useful to study the anisotropies present in the early stages of evolution of the universe .The above investigations and the discussion motivated us to investigate, in this paper, Bianchi type-IX minimally interacting holographic dark energy model in Saez-Ballester scalar-tensor theory of gravitation using a linearly varying deceleration parameter proposed by Akarsu and Dereli(2012). Bianchi type-IX models are important because of the fact that they describe homogeneous but anisotropic expanding (contracting) cosmologies. They also provide models where the effects of anisotropy can be estimated and compared with FRW models (Thorne 1967) Also these models play an important role in understanding the realistic picture of the universe immediately after the big bang.

The plan of this work is the following: In Sec.2, we obtain the field equations of Saez-Ballester theory in the presence of minimally interacting dark matter and holographic dark energy in Bianchi type-IX space time. By solving the field equations using a linearly varying deceleration parameter given by Akarsu and Dereli (2012), we present the holographic dark energy model in Sect.3. In Sect.4, we compute the physical and kinematical parameters of the model and discuss their physical consequences. The last section contains some concluding remarks.

2. Metric & field equations

Spatially homogeneous and anisotropic Bianchi type-IX metric is

$$ds^2 = dt^2 - A^2dx^2 - B^2dy^2 - (B^2\sin^2 y + A^2\cos^2 y)dz^2 + 2A^2\cos y dxdz \tag{1}$$

where A and B are functions of the cosmic time t only.

Saez-Ballester (1986) field equations for the combined scalar and tensor fields are

$$R_{ij} - \frac{1}{2}g_{ij}R - w\phi^n\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) = -\left(T_{ij} + \overline{T}_{ij}\right) \tag{2}$$

and the scalar field ϕ satisfies the equation

$$2\phi^n\phi^i_{;i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \tag{3}$$

where R_{ij} is the Ricci tensor, R is the Ricci scalar, w and n are arbitrary dimensionless constants and $8\pi G = c = 1$ in the relativistic units.

The energy momentum tensor for matter and the holographic dark energy are defined as

$$T_{ij} = \rho_m u_i u_j \quad \text{and} \quad \overline{T}_{ij} = \left(\rho_\lambda + p_\lambda\right)u_i u_j - g_{ij}p_\lambda \tag{4}$$

where ρ_m, ρ_λ are the energy densities of matter and the holographic dark energy and p_λ is the pressure of the holographic dark energy.

Also, the energy conservation equation is

$$T^{ij}_{;j} + \overline{T}^{ij}_{;j} = 0 \tag{5}$$

In a commoving coordinate system, the field equations (2) and (3) for the metric (1) with the help of Eqns.(4) can be, explicitly, written as

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2} - \frac{3}{4}\frac{A^2}{B^4} - \frac{w}{2}\phi^n\dot{\phi}^2 = -p_\lambda \tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{1}{4}\frac{A^2}{B^4} - \frac{w}{2}\phi^n\dot{\phi}^2 = -p_\lambda \tag{7}$$

$$2\frac{\dot{A}}{A}\frac{\dot{B}}{B}+\frac{\dot{B}^2}{B^2}-\frac{1}{4}\frac{A^2}{B^4}+\frac{1}{B^2}+\frac{w}{2}\phi^n\dot{\phi}^2=\rho_m+\rho_\lambda \tag{8}$$

$$\ddot{\phi}+(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B})\dot{\phi}+\frac{n}{2}\frac{\dot{\phi}^2}{\phi}=0 \tag{9}$$

where an overhead dot denotes differentiation with respect to t.

Now using barotropic equation of state(EoS),defined by

$$p_\lambda=\omega\rho_\lambda \tag{10}$$

where ω is EoS parameter, we can write the conservation equation(5) for the matter and dark energy as

$$\dot{\rho}_m+\rho_m(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B})+\dot{\rho}_\lambda+(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B})(1+\omega)\rho_\lambda=0 \tag{11}$$

Here we are considering the minimally interacting matter and holographic dark energy components. Hence both the components conserve separately so that we have (Sarkar 2014a, 2014b)

$$\dot{\rho}_m+\rho_m(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B})=0 \tag{12}$$

$$\dot{\rho}_\lambda+(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B})(1+\omega)\rho_\lambda=0 \tag{13}$$

For the space-time given by Eq.(1), the following are the formulae for physical and kinematical parameters which are useful to solve the field equations of this theory.

The average scale factor is defined as

$$a(t)=(AB^2)^{\frac{1}{3}} \tag{14}$$

The spatial volume is given by

$$V=a^3=AB^2 \tag{15}$$

The average Hubble parameter is given by

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) \tag{16}$$

The expressions for scalar expansion θ and shear scalar σ^2 are given by

$$\theta = \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) \tag{17}$$

$$\sigma^2 = \frac{2}{3}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2 \tag{18}$$

The average anisotropy parameter is

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \tag{19}$$

3. Solutions & the Model

Equations (6)-(9) are a system of three independent equations in five unknowns, $A, B, \rho_\lambda, p_\lambda$ and ϕ (since ρ_m can be determined from Eq.(12)). Hence to find determinate solution of the field equations we use the following physically viable conditions:

(i) The shear scalar σ^2 is proportional to scalar expansion θ so that we can take(Collins et al. 1983)

$$A = B^m \tag{20}$$

where $m \neq 1$ is a constant and takes care of the anisotropy of the space-time.

(ii) A generalized linearly varying deceleration parameter(Akarsu and Dereli2012)

$$q = -\frac{\ddot{a}}{\dot{a}^2} = -kt + l - 1 \tag{21}$$

where $k \geq 0$ and $l \geq 0$ are constants and $k = 0$ reduces to the law of Berman (1983) which yields models with constant deceleration parameter. Umadevi and Ramesh(2015) have studied Bianchi type-III holographic dark energy model in Brans-Dicke theory with constant deceleration parameter while Reddy et al(2016) discussed Kantowski –Sachs holographic dark energy model the deceleration parameter given by Eq.(21). Here we are concerned with linearly varying deceleration parameter given by Eq. (21) when $k > 0$ and $l \geq 0$.

In this particular case we have

$$a(t) = a_0 \exp[\frac{2}{\sqrt{l^2 - 2c_1 k}} \arctan h(\frac{kt - l}{\sqrt{l^2 - 2c_1 k}})] \tag{22}$$

where a_0, c_1 are constants of integration. For convenience, in the following we consider the solution for $k > 0, l > 0$ and choose the integration constant $c_1 = 0$. The reason for considering the solution only for $k > 0, l > 0$ is not only for simplicity but also for compatibility with the observed universe. The condition $k > 0$ means that we are dealing with increasing acceleration ($\dot{q} = -k < 0$). With this, the equation (22) reduces to

$$a(t) = a_0 \exp[\frac{2}{l} \arctan h(\frac{kt}{l} - 1)] \tag{23}$$

Using Eqs.(23)and(20) in Eq.(14) we obtain the metric potential as(choosing a_0 =1)

$$A = \exp[\frac{6m}{l(m+2)} \arctan h(\frac{kt}{l} - 1)] \tag{24}$$

$$B = \exp[\frac{6}{l(m+2)} \arctan h(\frac{kt}{l} - 1)]$$

Using Eq.(24), the metric(1) can be written as

$$ds^2 = dt^2 - \exp[\frac{12m}{l(m+2)} \arctan h(\frac{kt}{l} - 1)]dx^2 - \exp[\frac{12}{l(m+2)} \arctan h(\frac{kt}{l} - 1)]dy^2$$

$$-\{\exp[\frac{12}{l(m+2)} \arctan h(\frac{kt}{l} - 1)]\sin^2 y + \exp[\frac{12m}{l(m+2)} \arctan h(\frac{kt}{l} - 1)]\cos^2 y\}dz^2$$

$$2\exp[\frac{12m}{l(m+2)} \arctan h(\frac{kt}{l} - 1)]\cos y dx dz \tag{25}$$

and the Saez-Ballester scalar field in the model is given by

$$\phi^{\frac{n+2}{2}} = \frac{\phi_0 (n+2)}{12k} \left(k^2 t^2 - 2lkt\right) \exp[-\frac{6}{l} \arctan h(\frac{kt}{l} - 1)] \tag{26}$$

where ϕ_0 is a constant of integration which can be taken as unity.

4. Discussion

Equation (25) along with Eq. (26) represents Bianchi type-IX minimally interacting holographic dark energy model in Saez-Ballester scalar-tensor theory of gravitation with the following physical and kinematical parameters which are very important in the physical discussion of the model.

The spatial volume of the universe is

$$V = \exp[\frac{2}{l}\arctan h(\frac{kt}{l}-1)] \tag{27}$$

The average Hubble parameter is

$$H = \frac{\dot{a}}{a} = \frac{2}{t\left(2l-kt\right)} \tag{28}$$

The scalar expansion is

$$\theta = 3H = \frac{6}{t\left(2l-kt\right)} \tag{29}$$

The shear scalar is

$$\sigma^2 = \frac{12\left(m-1\right)^2}{\left(m+2\right)^2 t^2 \left(2l-kt\right)^2} \tag{30}$$

The average anisotropy parameter is

$$A_h = \frac{2\left(m-1\right)^2}{\left(m+2\right)^2} \tag{31}$$

The matter energy density from Eqs.(12), (24) and (26)can be obtained as

$$\rho_m = \rho_0 \exp[-\frac{6}{l}\arctan h(\frac{kt}{l}-1)] \tag{32}$$

From Eqs.(6), (24) and (26)we get the holographic pressure as

$$\begin{aligned} p_{\lambda}= & \frac{24k(m+2)(l-kt)-108}{(m+2)^2(2kl-k^2t^2)^2}+\exp[-\frac{12}{l(m+2)}\arctan h(\frac{kt}{l}-1)] \\ & -\frac{3}{4}\exp[\frac{12(m-2)}{l(m+2)}\arctan h(\frac{kt}{l}-1)]+\frac{w}{2}\exp[-\frac{6}{l}\arctan h(\frac{kt}{l}-1)] \end{aligned} \tag{33}$$

From Eqs. (8), (24), (26) and (32) the holographic energy densityis determined as

$$\rho_{\lambda}=\frac{36(2m+1)}{(m+2)^2(2klt-k^2t^2)^2}+\exp[-\frac{12}{l(m+2)}\arctan h(\frac{kt}{l}-1)]-\frac{1}{4}\exp[\frac{12(m-2)}{l(m+2)}\arctan h(\frac{kt}{l}-1)]-\frac{w}{2}\exp[-\frac{6}{l}\arctan h(\frac{kt}{l}-1)]$$

$$-\rho_0\exp[-\frac{6}{l}\arctan h(\frac{kt}{l}-1)] \quad (34)$$

From Eqs.(10), (33) and(34)we observe that the EoS parameter is a function of cosmic time t.

The overall density parameter is given by

$$\Omega=\Omega_m+\Omega_\lambda \quad (35)$$

Where

$$\Omega_m=\frac{\rho_m}{H^2}\ \text{and}\,\Omega_\lambda=\frac{\rho_\lambda}{H^2} \quad (36)$$

Here ρ_m and ρ_λ are given by Eqs.(32) and(34) respectively.

The cosmic jerk parameter is dimensionless third order derivative of the scale factor with respect to the cosmic time defined as

$$j(t)=\frac{1}{H^3}\frac{\dddot{a}}{a}=q+2q^2-\frac{\dot{q}}{H} \quad (37)$$

where q is the deceleration parameter defined as

$$q=\frac{d}{dt}\left(\frac{1}{H}\right)-1=-kt+l-1 \quad (38)$$

Hence from equations (36), (47), and (48) we get

$$j(t)=\frac{3}{2}(kt-l+1)^2+\frac{l^2-1}{2} \quad (39)$$

It may be observed from the above results as t increases, the spatial volume increases showing the spatial expansion of the universe. Also at the initial epoch i.e. at t=0 $H,\theta,\sigma^2,\rho_\lambda$ and p_λ diverge and as $t\to\infty$, they all become finite. However, the scalar field φ does not diverge at $t=0$. It is also observed that the EoS parameter is function of cosmic time t only .It may also be observed that when m=1, $A_h=0$ so that the universe becomes isotropic

5. Conclusions

Here we have considered a minimally interacting holographic dark energy Bianchi type-IX cosmological model in Saez-Ballester (1986) scalar-tensor theory of gravitation. To obtain the cosmological model we have used linearly varying deceleration parameter proposed by Akarsu and Dereli (2012).

The physical behavior of the model is discussed and it is observed that the model is in good agreement with the scenario of early inflation and late time acceleration. It is observed that in our model the universe has finite life time. It starts with a big bang and ends at a finite time. It is also observed that holographic energy density, holographic pressure and scale factor diverge in finite time which is nothing but the big rip behavior first suggested by Caldwell et al, (2006). This model will help to have a deep insight into the behavior of the anisotropic holographic dark energy universe in Saez-Ballester scalar-tensor cosmology.

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