Article

Exact Vacuum Solution of Bianchi Type-V Space-Time in *f*(*R*) Theory of Gravity

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Abstract

In this paper, we find the exact vacuum solution of Bianchi type-V space-time in the *f*(*R*) theory of gravity. The physical behavior of the models is discussed and the function of the Ricci scalar is evaluated.

Keywords: *f*(*R*) theory of gravity, Bianchi type-V, space-time, Ricci scalar.

§ 1. Introduction

Observation of type Ia Supernovae (SNa Ia) [1,2] for accelerating expansion of universe in the recent era is the outstanding result. In addition to this work cosmic acceleration have confirmed on cosmic microwave background (CMB) from Wilkinson Microwave Anisotropy Probe (WMAP) [3,4]. The negative pressure created by the component of dark energy which is responsible for transition from decelerating to accelerating phase of universe [5]. From the history, universe occupied 2/3 of dark energy, 1/3 of dark matter and other component known as baryon matter. Many authors have study dark energy in general theory of relativity and obtained their solution in different context. A number of theoretical models have been suggested in the literature in order to give explanation about the nature of the dark energy and the accelerated expansion of the universe.

The generalized version of Einstein's theory of relativity is considered as most suitable for tackling the problem of dark matter & dark energy, also occurrence of singularity and mainly accelerating expansion of universe. The modified theory of gravity, specially $f(R)$ theory gives answers such type of problems. Among various modified theory, $f(R)$ theory is most appropriate in the view of cosmological important of $f(R)$ model. The problem regarding dark matter & dark energy also unification of early-time inflation and late-time acceleration addressed in $f(R)$ theory of gravity.

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Nojiri and Odintsov [6-9] proved that the $f(R)$ theory of gravity provides very natural unification of the early time inflation and late time acceleration. Carrol et. al. [10] explained the presence of late time comic acceleration of the universe in $f(R)$ gravity. Bertolami et. al. [11] have proposed a generalization of $f(R)$ modified theory of gravity, by including in the theory an explicit coupling of an arbitrary function of the Ricci scalar R with the matter Lagrangian density. M. Farasat Shamir [12] studied some Bianchi type cosmological models in $f(R)$ gravity. Also, M. Sharif and M. Farasat Shamir [13,14] exhibited vacuum as well as non-vacuum solutions of Bianchi Types-I and V Space-times in $f(R)$ theory of gravity. M. Sharif and H. Rizwana Kausar [15] discussed non-vacuum solutions of Bianchi type-VIo universe in $f(R)$ gravity. M. Sharif and H. Rizwana Kausar [16] explained anisotropic fluid and Bianchi type-III model in $f(R)$ gravity. K. S. Adhav [17] obtained Bianchi type-III string cosmological model in $f(R)$ gravity. Along with this plane symmetric solutions in *f(R)* gravity and plane symmetric vacuum Bianchi type-III cosmology in *f(R)* gravity discussed by M. Sharif et. al. [18,19]. Multamaki and Vilja [20, 21] investigated static spherically symmetric vacuum and non-vacuum solutions by taking fluid respectively. Capozziello et. al. [22] study spherically symmetric solutions in *f(R)* theory of gravity by noether symmetries approch. Azadi A. et. al. [23] study cylindrical solutions in metric *f(R)* gravity and D.Momeni [24] explained constant curvature solutions in cylindrically symmetric metric *f(R)* gravity. Aktas et. al. [25] have studied anisotropic models in $f(R)$ gravity. M. Sharif and M. Farasat Shamir [26] calculated energy distribution in *f(R)* Gravity. Lukas Hollenstein and Francisco S. N. Lobo [27] discussed exact solutions of *f(R)* gravity coupled to nonlinear electrodynamics. Antonio De Felice [28] gives revive of *f(R)* theories. Moreover, Valerio Faraoni [29] discussed *f(R)* gravity (successes and challenges). Kazuharu Bamba et. al. [30] studied thermodynamics in *f(R)* gravity in the palatini formalism. Reddy et. al. [31] obtained the vacuum solution of Bianchi type-I and V models in *f(R)* theory of gravity under the assumption of special form of deceleration parameter.

Motivating by above research work, this paper is devoted to study the exact vacuum solution of Bianchi type-V space-time in *f(R)* theory of gravity. The paper is organized as follows: In section-2, we give a brief introduction about field equations in *f(R)* theory of gravity. Section-3 is used to find exact vacuum solution of Bianchi type-V space-time and section-4 has discussed physical behavior of the model, in the last section we summarize and conclude the results.

§ 2. Field equations in $f(R)$ theory of gravity

Due to the cosmological important of $f(R)$ model, the modified $f(R)$ theory of gravity plays a vital role to study of universe. The $f(R)$ theory of gravity studied by using two methods first one known as metric and second is palatini formalism. In the present paper we propose to solve the vacuum field equations of $f(R)$ theory of gravity in Bianchi type-V space-time using the metric approach. The field equations in $f(R)$ theory of gravity are given by

$$F(R)R_{ij} - \frac{1}{2} f(R)g_{ij} - \nabla_i \nabla_j F(R) + g_{ij} \Box F(R) = kT_{ij}, \tag{1}$$

Where

$$F(R) \equiv \frac{df(R)}{dR}, \qquad \Box \equiv \nabla^i \nabla_i \tag{2}$$

with ∇_i the covariant derivative and T_{ij} is called the standard matter energy momentum tensor obtained from Lagrangian L_m and k is noted as coupling constant usually in our paper taken as $k = \frac{8\pi G}{c^4}$.

These are the fourth order partial differential equations in the metric tensor. The fourth order is due to the last two terms on the left hand side of the equations. It is interesting to note that if we consider $f(R) = R$, these equations of $f(R)$ theory of gravity reduce to the field equations of Einstein's general theory of relativity.

After contraction of the field equation (1), we get

$$F(R)R - \frac{5}{2} f(R) + 4 \Box F(R) = kT. \tag{3}$$

In vacuum, this field equation (3) reduces to

$$f(R) = \frac{2}{5}[F(R)R + 4 \Box F(R)] \tag{4}$$

This yields a relationship between $f(R)$ and $F(R)$ which can be used to simplify the field equations and to evaluate $f(R)$.

§ 3. Exact vacuum solutions of the Bianchi type -V space-time

The most significant generalisation of FRW space time is a Bianchi type-V space time. To elaborate the vacuum solution of Bianchi type-V space time in *f*(R) gravity and create an important result to study the nature of the universe. We consider spatially homogeneous and anisotropic Bianchi type-V space-time described by the line element is given by

$$ds^2 = dt^2 - A^2(t)dx^2 - e^{2mx}[B^2(t)dy^2 + C^2(t)(dz^2)] \tag{5}$$

where A, B and C are cosmic scale factors and m is an arbitrary constant. The corresponding Ricci scalar is

$$R = -2[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{3m^2}{A^2}], \tag{6}$$

where dot denotes the derivative with respect to t.

We define the average scale factor a as

$$a = (ABC)^{\frac{1}{3}} \tag{7}$$

and the volume scale factor is defined as

$$V = a^3 = ABC \tag{8}$$

The generalized mean Hubble parameter H is defined by

$$H = \frac{1}{3}\sum_{i=1}^{3} H_i \ , \tag{9}$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$, $H_3 = \frac{\dot{C}}{C}$ are the directional Hubble parameters in the directions of x, y and z axes respectively. Using equations (7), (8) and (9), we obtain

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}\sum_{i=1}^{3} H_i = \frac{\dot{a}}{a}. \tag{10}$$

Putting value of $f(R)$ in the vacuum field equations (1), we obtain

$$\frac{F(R)R_{ij} - \nabla_i \nabla_j F(R)}{g_{ij}} = \frac{1}{4}[F(R)R - \Box F(R)]. \tag{11}$$

Since the metric (5) depends only on t, one can view equation (11) as the set of differential equations for $F(R), A,$ B and C. It follows from equation (11) that the combination

$$A_i = \frac{F(R)R_{ii} - \nabla_i \nabla_i F(R)}{g_{ii}}, \tag{12}$$

is independent of the index i and hence $A_i - A_j = 0$ for all i and j. Consequently $A_0 - A_1 = 0$ gives

$$-\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} - \frac{2m^2}{A^2} + \frac{\dot{A}\dot{F}}{AF} - \frac{\ddot{F}}{F} = 0. \tag{13}$$

Also, $A_0 - A_2 = 0$, $A_0 - A_3 = 0$ gives respectively

$$-\frac{\ddot{A}}{A}-\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}-\frac{2m^2}{A^2}+\frac{\dot{B}\dot{F}}{BF}-\frac{\ddot{F}}{F}=0\,, \tag{14}$$

$$-\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}-\frac{2m^2}{A^2}+\frac{\dot{C}\dot{F}}{CF}-\frac{\ddot{F}}{F}=0\,. \tag{15}$$

The 01-component can be written by using equation (1) in the following form

$$\frac{2\dot{A}}{A}-\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=0\ . \tag{16}$$

Subtracting equation (14), (15) and (15) from equation (13), (14) and (13), we get respectively

$$-\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}+\frac{2\dot{C}}{C}(\frac{\dot{A}}{A}-\frac{\dot{B}}{B})+\frac{\dot{F}}{F}(\frac{\dot{A}}{A}-\frac{\dot{B}}{B})=0\,, \tag{17}$$

$$-\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}-\frac{\dot{C}^2}{C^2}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}-\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{F}}{F}(\frac{\dot{B}}{B}-\frac{\dot{C}}{C})=0\,, \tag{18}$$

$$-\frac{\ddot{A}}{A}-\frac{\ddot{C}}{C}-\frac{\dot{C}^2}{C^2}+\frac{\dot{A}\dot{B}}{AB}-\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{F}}{F}(\frac{\dot{A}}{A}-\frac{\dot{C}}{C})=0\,. \tag{19}$$

After integration the above equations imply that

$$\frac{A}{B}=d_1\exp[c_1\int\frac{dt}{a^3F}], \tag{20}$$

$$\frac{B}{C}=d_2\exp[c_2\int\frac{dt}{a^3F}], \tag{21}$$

$$\frac{C}{A}=d_3\exp[c_3\int\frac{dt}{a^3F}] \tag{22}$$

where c_1, c_2, c_3 and d_1, d_2, d_3 are constants of integration which satisfied the relation

$$c_1+c_2+c_3=0 \ \text{ and } \ d_1\,d_2\,d_3=1.$$

Sharif and Shamir [13] have established a result in the context of $f(R)$ gravity which show that

$$F\ \alpha\ a^m\,. \tag{23}$$

Thus, we have

$$F = l\,a^{m}, \tag{24}$$

where l is the constant of proportionality, m is any integer (here taken as -2).

From equation (16), we get

$$A^2 = BC \tag{25}$$

We consider variation law of Hubble's parameter proposed by Berman [32] that yields constant deceleration parameter models of the universe defined by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = const. \tag{26}$$

On integrating (26), we have

$$a = (ct+d)^{\frac{1}{1+q}} \tag{27}$$

where $c \neq 0$ and d are the constant of integration. This equation implies that the condition for accelerated expansion of the universe is $1+q>0$.

Using equations (20),(21),(22), (24), (25) and (27), we obtained the scale factor as

$$A = (ct+d)^{\frac{1}{1+q}} \tag{28}$$

$$B = (d_2 d_3)(ct+d)^{\frac{1}{1+q}} \exp\left[\frac{(c_2+c_3)(1+q)}{l(n-2+q)}(ct+d)^{\frac{n-2+q}{1+q}}\right] \tag{39}$$

$$C = (d_2 d_2)^{-1}(ct+d)^{\frac{1}{1+q}} \exp\left[\frac{-(c_2+c_2)(1+q)}{l(n-2+q)}(ct+d)^{\frac{n-2+q}{1+q}}\right] \tag{30}$$

The directional Hubble parameters in the directions of x, y and z axes are as

$$H_x = \frac{c}{1+q}(ct+d)^{-1}, \tag{31}$$

$$H_y = \frac{c_2 + c_3}{l}(ct+d)^{\frac{n-3}{1+q}} + \frac{c}{1+q}(ct+d)^{-1}, \tag{32}$$

And

$$H_z = \frac{-c_1 - c_2}{l}(ct+d)^{\frac{n-3}{1+q}} + \frac{c}{1+q}(ct+d)^{-1} \tag{33}$$

The mean Hubble parameter found to be

$$H = \frac{c_3 - c_1}{3l}(ct+d)^{\frac{n-3}{1+q}} + \frac{c}{1+q}(ct+d)^{-1}, \tag{34}$$

The volume scale factor is calculated as

$$V = (ct+d)^{\frac{3}{1+q}}. \tag{35}$$

The expansion scalar θ and shear scalar σ^2 are given by

$$\theta = \frac{c_3 - c_1}{l}(ct+d)^{\frac{n-3}{1+q}} + \frac{3c}{1+q}(ct+d)^{-1}, \tag{36}$$

$$\sigma^2 = \frac{(c_3 - c_1)^2 + 3(c_1 + c_2)(c_2 + c_3)}{3l^2}(ct+d)^{\frac{2(n-3)}{1+q}}. \tag{37}$$

The mean anisotropy parameter $\overline{A}$ measure of deviation from isotropic expansion and suggested that model is isotropic if $\overline{A} = 0$, otherwise the model is anisotropic. The average anisotropic parameter for the present model can be expressed as

$$\overline{A} = \frac{6[(c_3 - c_1)^2 + 3(c_1 + c_2)(c_2 + c_3)](1+q)}{[3lc\,(ct+d)^{\frac{-(n-3)}{1+q}-1} + (1+q)(c_3 - c_1)]^2}, \tag{38}$$

From Eq. (38), we observe that at late time when $t \to \infty$, $\overline{A} \to 0$. Thus, our model has transition from initial anisotropy to isotropy at late time which is in good harmony with current observations. Thus, we observed that the isotropy of the universe can be achieved in our model.

From equation (6), the scalar R for Bianchi type-V model we find

$$R=-2\left[\begin{array}{l}\frac{(c_1-c_3)(1+m)\,c}{l(1+q)}(ct+d)^{\frac{-(m+q+4)}{1+q}}+\frac{c_1^2-c_1c_3+c_3^2}{l^2}(ct+d)^{\frac{-2(m+3)}{1+q}}\\ \quad+\frac{[1+(1-c)q]c}{(1+q)^2}(ct+d)^{-2}\end{array}\right] \tag{39}$$

This equation clearly indicates that $f(R)$ cannot be explicitly written in terms of R. However, by inserting this value of R, $f(R)$ can be written as a function of t, which is true as R depends upon t.

$$f(R)=\left[\begin{array}{l}\frac{(c_3-c_1)(2+5m)\,c}{2(1+q)}(ct+d)^{\frac{-q-4}{1+q}}+\frac{cl}{(1+q)^2}(ct+d)^{\frac{m-2(1+q)}{1+q}}\\ {[3cm(1-\frac{1}{2}m-\frac{3}{2}q)+q(c-1)-1]}\end{array}\right] \tag{40}$$

§ 4. Physical behavior of the model

The spatial volume V is zero at $t\to 0$ and it expand as t increases and become infinitely large as shown in the figure (1). From figure (2), it observed that the expansion scalar starts expansion with a finite value at $t\to 0$, and as time increases it decreases to a constant value and remain constant as $t\to\infty$. Also in figure (3) represent shear scalar vs time, it cleared that the shear scalar have finite value at $t\to 0$, and as time increases it decreases to a constant value and remain constant as time increased. The behavior of anisotropic parameter vs time shows in figure (4), the anisotropy parameter $\overline{A}$ is very large at the initial moment but decreases with time and vanish at $t\to\infty$. Thus the model shows an isotropic state at the later time of its evolution.

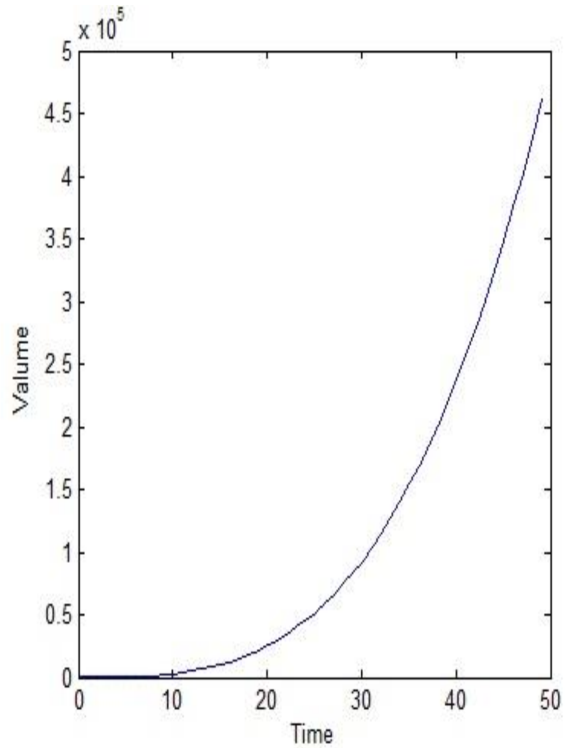

Fig (1) Volume vs. Time

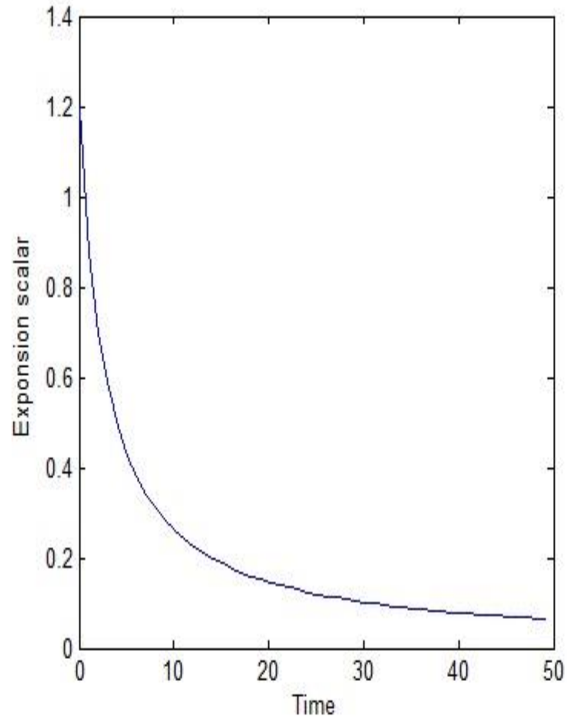

Fig (2) Expansion scalar vs. Time

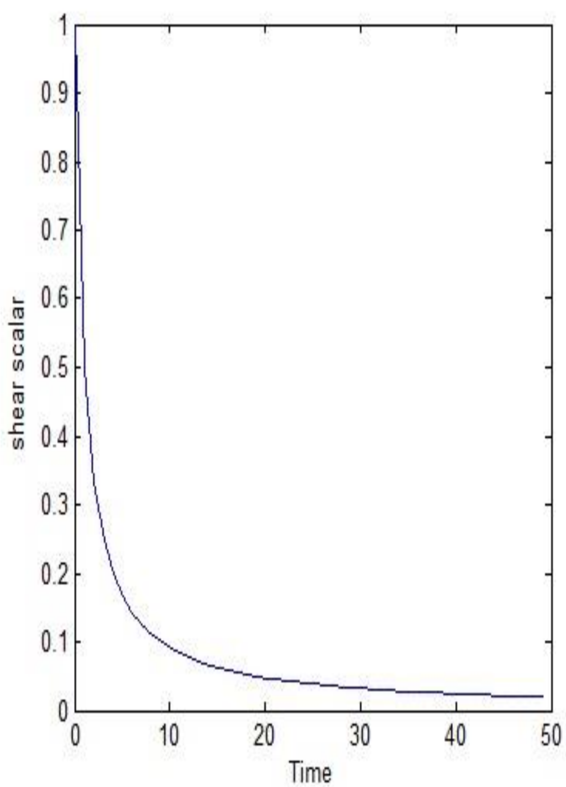

Fig (3) Shear scalar vs. Time

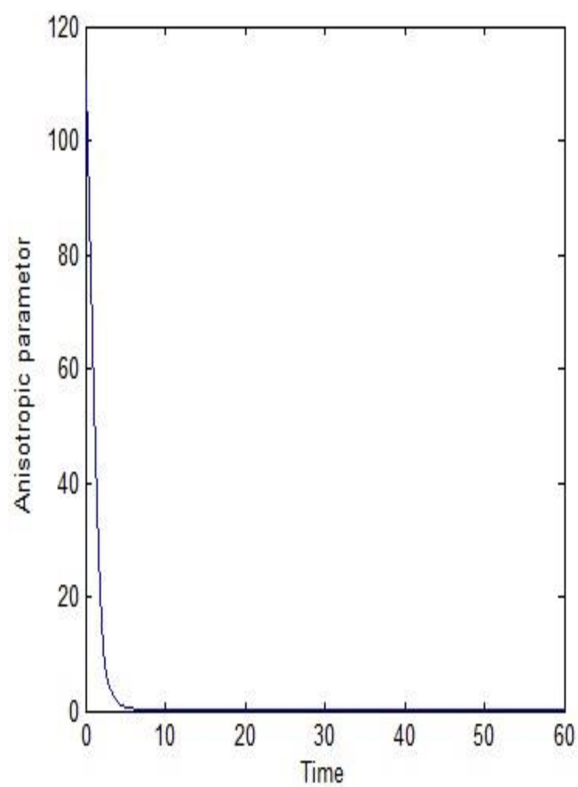

Fig (4) Anisotropic parameter vs. Time

§ 5. Concluding Remark

In this paper, we have investigated the exact vacuum solution of the Bianchi type-V space-time in $f(R)$ theory of gravity and discussed the dynamics of the universe expansion by using some physical parameter. Also the function $f(R)$ is evaluated.

The physical behavior of presented model has observed as

(i) It is interesting to note that our model does not have initial singularity.

(ii) The volume scale factor of the universe increases exponentially with increases in time and becomes infinitely very large. This indicates that the universe starts its expansion with zero volume from infinite past.

(iii) It is observed that the expansion scalar starts expansion with a finite value initially and as time increases it decreases to a constant value and remain constant.

(iv) The anisotropy parameter $\overline{A}$ is very large at the initial moment but decreases with time and vanishes at $t \to \infty$. Thus the model shows an isotropic state at the later time of its evolution.

(v) The model has an acceleration phase for $1+q>0$. This shows that there is a transition from decelerated phase to accelerated phase.

Thus, the presented model has no initial singularity, initially anisotropic & achieved isotropy at later time also expanding and accelerating.

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