Article

Heuristic Assessment of Cosmic Structure & Energy Density

Alex Vary*

Abstract

The cosmos can be partitioned into hierarchical domains embracing all classes of discrete objects from electrons and planetoids to galaxies and galactic superclusters. Heuristic analysis of these hierarchical domains leads to the estimation of a mass flux peculiar to the objects in each domain. At each level of the cosmic hierarchy the mass flux is found to be associated with a particular constant. This leads to a cosmic mass flux expressed by the relation $mF_c = A$, where m is mass, F_c is the cumulative flux (passage per unit area per unit time) of objects with masses equal to or greater than m, and the quantity A which is mass flux (the mass flow per unit area per unit time). It is found that the quantity A equals the product of the mean density of matter in the universe, ρ_o, and the velocity of light, c, that is, $A = \rho_o c$. Apparently, the $mF_c = A = \rho_o c$ relation represents the cumulative mass flux of all classes of objects in the cosmic hierarchy from the least to most massive, i.e., from electrons to galactic superclusters. The mass flux parameter A is apparently a constant which accounts for the kinetic energy content of the universe due to all cosmic objects from electrons to galactic superclusters. It is found that there is close agreement between cosmic kinetic energy density, as derived from A, and the cosmic microwave background thermal energy density. The cumulative kinetic energy density of all classes of objects in the cosmos apparently equals their total mass conversion energy density. Other implications of the cosmic parameter A are discussed.

Key Words: Cosmic hierarchy, energy density, momentum density, kinetic temperature, mass flux, neutrinos, fermions, hadrons, meteoroids, galaxies, superclusters, microwave background.

Introduction

The hierarchical nature of the cosmos has been described previously (Refs. 1, 2) and various models of the cosmos and have been used in defining hierarchical boundary conditions and partitions (Refs. 1, 3). In this paper the cosmos is partitioned into a systematic series of hierarchical domains, each containing a class of discrete self-similar units of matter, i.e., objects, from electrons and planetoids to galaxies and galactic superclusters.

Consideration of the random and regular motion of objects in each hierarchical domain leads to a spatial mass flux for each domain. Evaluation of these fluxes depends on the fact that the mean spatial density of objects in any given level of the cosmic hierarchy is so much greater than in the next higher level that objects in the lower levels may be treated as if they existed in empty space. This latter condition is essential in deriving the cumulative mass flux constant described in this paper. This approach leads to some insights concerning attributes of the cosmos that have remained unexplored despite the availability of relevant data.

* Correspondence: Alex Vary, PhD, Retired NASA Scientist & Independent Researcher. Email: axelvary@wowway.com

A cosmic mass flux constant is proposed and formulated in terms of two fundamental parameters, i.e., the mean spatial density of matter in the cosmos and the velocity of light. The mass flux constant accounts for the kinetic energy content of the universe due to all cosmic objects from electrons to galactic superclusters. It is shown that there is close agreement between cosmic kinetic energy density and cosmic background microwave thermal energy density. Also, the cumulative kinetic energy density of all classes of objects in the cosmos apparently equals their total mass conversion energy density.

Approach

Since there are more free fermions, hadrons (neutrinos, electrons, protons), atoms, and molecules than condensed objects such as planets and galaxies, their spatial flux can be calculated independently of the more massive objects in higher hierarchical domains. Objects like meteoroids, asteroids, comets, satellites, planets, and stars constitute the next higher levels of the cosmic hierarchy. Although these latter objects may move stochastically or under the influence of gravitation, their cumulative spatial flux can be calculated as though they moved independently in empty space.

Progressing from the solar neighborhood to intergalactic space, there is a clear *hierarchical succession of increasing rarity* of more massive objects relative to the abundance of objects in the lower hierarchical levels. There are enormous volumes of space separating concentrated luminous objects such as galaxies and galactic clusters. Taking these massive, diffuse objects as single point entities, i.e., simple centers of mass, their mean spatial density is insignificant compared with that of less massive lower hierarchical level objects such as protons and intergalactic dust.

Recognition of a hierarchical succession of increasing rarity permits the calculation of cumulative flux at any given level of the cosmic hierarchy while ignoring the flux contributed by higher levels (more massive objects). A *cosmic cumulative mass flux* is calculated by exploiting the hierarchical nature of the cosmos and calculating a cumulative flux for matter at each level of the hierarchy in terms of number of self-similar objects flowing through an arbitrary unit area per unit time. It will be seen that mass flux due to lower hierarchical levels always dominates that due to higher levels.

Boundary Equations

Radius Boundaries - A plot of radius versus mass, *log* r versus *log* m, for cosmic objects appears in Fig. 1. Data for Fig. 1 appear in Table I (Refs. 2, 4). All cosmic objects from superclusters to subatomic particles lie within the upper and lower bounds defined by r_d and r_s (see table of Symbols and Constants),

$$r_d = [\,4a_oR/(m_pm_c)^{½}\,]^{½}\, m^{½} \approx 10^4 m^{½} \tag{1}$$

$$r_s = (2\gamma/c^2)m \approx 1.5\cdot 10^{-28} m \tag{2}$$

where, r_d is an upper bound while r_s is a lower bound on corresponding masses m. Equation (1) is derived in Appendix A. Equation (2) is the Schwarzchild limit, i.e., event horizon radius for black hole masses from 10^{10} to 10^{60} g (Ref. 5, 6).

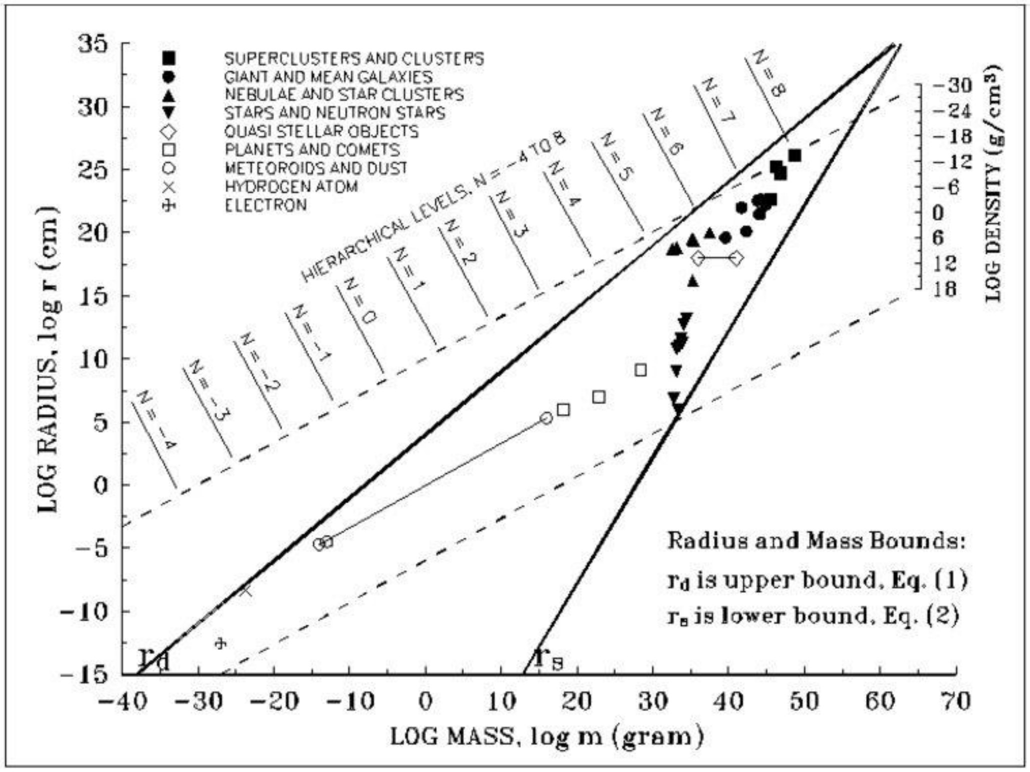

Figure 1 - Hierarchical structure of the cosmos from electrons to galaxy superclusters.

Density Boundaries - A density isomer field also appears in Fig. 1. The density isomers are defined in terms of radii versus mass, *log* r_p versus *log* m,

$$r_p = 10^{4+P} m^{1/3} \quad (3)$$

where, P assumes the series of integer values, e.g., ... -2, -1, 0, 1, 2, ..., where $[(4/3)\pi]^{1/3}$ is subsumed in the coefficient of $m^{1/3}$, and where P = -4 is assigned to density isomer $\rho = 1$ g/cm^3. The isomers for $\rho = 10^{-30}$ and $\rho = 10^{18}$ g/cm^3 also bound all known cosmic objects.

Hierarchical Boundaries - A hierarchy should be describable by some functional relation among its successive levels. One such function was suggested in Ref. 1. It was based on the progression of values of discrete powers of the quantity $e^2/(\gamma m_p m_e)$. The resultant quantities were used in a plot of m/r versus m to define mass bounds of successive hierarchical levels. The approach described below is similar except that the hierarchy is partitioned differently, but the partitioning scheme takes a similar account of the hierarchical features of the cosmos

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Table I - Radius and Mass Data for Cosmic Objects

RADIUS log r (cm)	MASS log m (g)	DENSITY log ρ (g/cm^3)	COSMIC OBJECT	REF (PAGE)
26.0	48.8	-30.0	Superclusters of galaxies	2 (1208)
24.7	46.8	-27.9	Clusters of galaxies	4 (292)
25.2	46.3	-29.9	Mean galactic cluster	4 (292)
22.6	45.5	-22.9	Compact groups of galaxies	2 (1208)
22.2	44.9	-22.3	Giant elliptical galaxies	2 (1208)
22.6	44.2	-24.2	Mean bright galaxy	4 (288)
21.5	44.1	-21.0	Mean Seyfert galaxy	4 (290)
22.5	43.8	-24.3	Spiral galaxies	4 (287)
20.1	42.3	-18.6	Dwarf elliptical galaxies	2 (1208)
22.0	41.7	-24.9	Mean local galaxy	4 (287)
19.6	39.6	-19.8	Mean global cluster	4 (281)
18	36-41	-16.1	Quasars	2 (1210)
20.2	37.5	-23.7	Large bright diffuse nebulae	3 (260)
19.4	35.4	-23.4	Interstellar clouds	4 (262)
19.5	35.2	-23.9	Mean open star cluster	4 (278)
18.8	33.2	-23.8	Mean dark nebula	4 (260)
18.7	32.6	-24.1	Small bright diffuse nebulae	4 (260)
16.2	35.3	-13.9	Protostars	2 (1208)
13.2	34.5	-5.72	Supergiant stars	2 (1208)
12.7	34.1	-4.62	Mean classical Cephied	4 (216)
11.3	33.9	-0.62	Mean double stars	4 (226)
11.6	33.7	-1.72	Mean cluster variables	4 (216)
11.0	33.6	-0.0221	Main sequence stars	4 (235)
10.8	33.1	-0.0779	Mean near star	4 (231)
8.99	33.1	5.51	Mean white dwarf	4 (226)
6.00	33.3	14.67	Critical neutron-pulsar	5 (456)
6.69	32.8	12.11	Neutron-pulsar	2 (1208)
9.11	28.4	0.448	Mean solar planet	4 (140)
7.02	22.9	1.217	Mean asteroid	4 (152)
6.00	18.2	-0.422	Median comet	4 (154)
5.3	16	-0.522	Upper range for meteoroids, zodiacal dust, space particles,	4 (156)
-4.7	-14	-0.522	Lower range for meteoroids	4 (156)
-4.5	-13	-0.122	Mean interstellar grain	4 (264)

A hierarchical succession is readily perceived among large cosmic objects, e.g., stars, galaxies, clusters, and superclusters of galaxies. On this basis, an intuitive partition function is given by,

$$r_N = 10^{4+9N} m^{-1} \tag{4}$$

where N = ... -2, -1, 0, 1, 2 . . . and where greater values of N designate higher hierarchical levels. Partitions for objects bounded by r_d and r_s are indicated by the short parallel line segments plotted as *log* r_N versus *log* m in Fig. 1.

The hierarchy of galactic clusters is bounded by $r_{N=7}$, and $r_{N=8}$. The next lower hierarchic levels consist of: individual galaxies in the $r_{N=6}$ to $r_{N=7}$ range, protostars and nebulae in the $r_{N=5}$ to $r_{N=6}$ range, main sequence stars in the $r_{N=4}$ to $r_{N=5}$ range, and planets and planetoids in the $r_{N=3}$ to $r_{N=4}$ range. In Fig. 1 quasars appear in the midst of nebulae, protostars, stellar clusters, and galaxies (Ref.

2). For N <3 a hierarchical structure is not obvious but this is unessential for determining fluxes for the objects involved, e.g., comets, meteoroids, dust, protons, hydrogen, etc.

Mass Flux Relations

Near Earth Flux - It has been found experimentally that in the vicinity of the earth the cumulative micrometeoroid and meteoroid flux may be represented by a general relation of the form, $F_c = Am^B$, where F_c is cumulative particulate flux (rate per unit area) of meteoroids, solar and interplanetary dust, etc. The relation also applies to particulate fluxes associated with other portions of solar space.

Solar Space Flux - Classes of particles in solar space for which the $F_c = Am^B$ relation holds range from zodiacal dust to asteroids, i.e., for masses from 10^{-12} to 10^{25} g, where the exponent B takes values from -0.7 to -1.3 and A takes values from $\approx 10^{-15}$ to $\approx 10^{-20}$ g/cm^{2}s, see Table II (Refs. 4, 7-20). These variations of A and B are likely because of uncertainties in the data, measurement methods, or peculiarities of local spaces. These are minor discrepancies relative to the overall scale to be considered next.

The data suggest that the average particulate flux in solar space may be represented by,

$$F_c = Am^B = 10^{-18}m^{-1} \tag{5}$$

It is assumed that Eq. (5) characterizes not only solar space, and the vicinity of all similar stars, but the cosmos. Before attempting to demonstrate this, it is useful to consider the hierarchical nature of cosmic matter and how this nature supports the calculation of F_c on a cosmic scale.

Hierarchical Flux - A heuristic equation for computing cumulative fluxes on a cosmic scale for any hierarchical level, N, from electrons and protons through galaxies, galactic clusters, and superclusters, is,

$$F_N = 10^{-6(3+N)} \tag{6}$$

where N assumes only integral vales, i.e., ... -2, -1, 0, 1, 2 ... etc. Note that relative to the hierarchical scale at the top of Fig. 2, Eq. (6) gives the same cumulative flux, F_c, for integral values of N as the curve for Eq. (5) which is co-plotted with data from Tables II, III, and IV.
It is apparent from Eq. (6) that successive integral values of N correspond to huge decrements in flux. Equation (6) invokes the observation that the mean spatial density of objects in any given level of the cosmic hierarchy is so much greater than in the next higher level that objects in the lower levels may be treated as if they existed in empty space. Accordingly, when calculating cumulative flux, contributions of higher levels of hierarchy may be ignored. Therefore, it is possible to calculate a non-cumulative flux F as defined below and equate this to cumulative flux F_c at any lower hierarchical level.

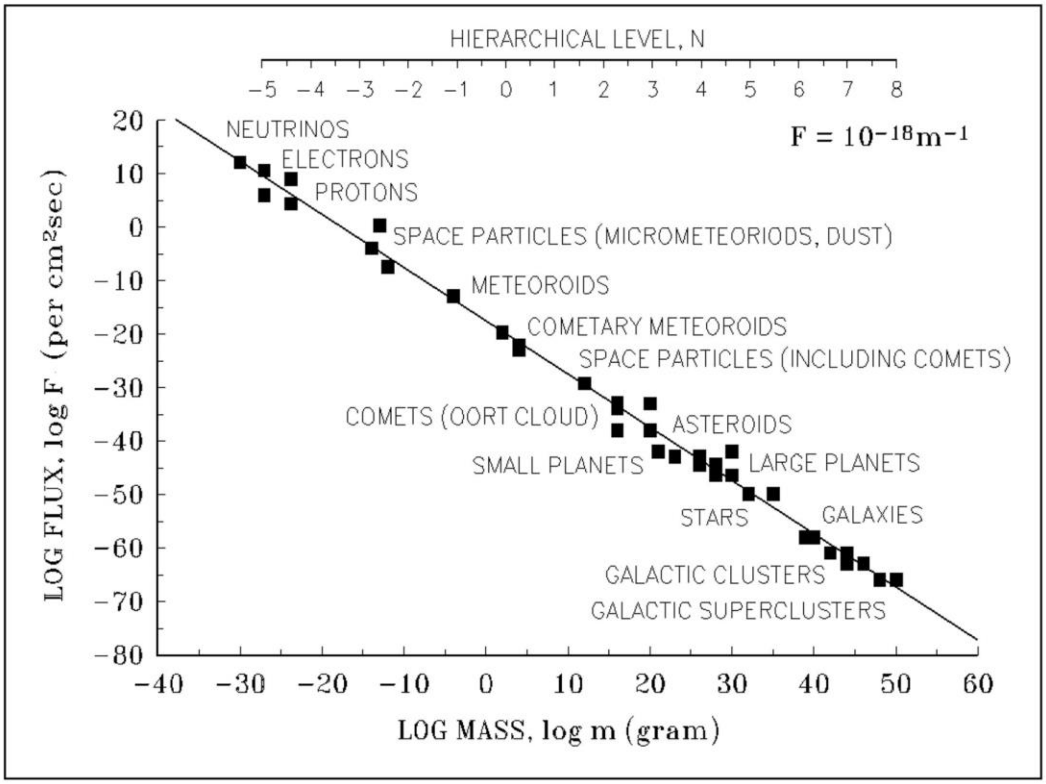

Figure 2 - Cosmic Particulate Flux from Neutrinos to Galaxies

Cumulative Flux - The method of computing cumulative fluxes for objects associated with the hierarchical levels N = -5 through N = 8 is described in Appendix B. Flux estimates for protons and electrons (N = -5 to -4) are presented in Table III (Refs. 21-25). Estimated fluxes for all cosmic objects from protons to superclusters are presented in Table IV (Refs. 4, 26, 27).

Figure 2 shows a plot of cumulative flux for all known cosmic objects within the mass range from m = 10^{-30} g to m = 10^{50} g. Corresponding hierarchical levels N = -5 through 8 are indicated along the top of the figure. Figure 2 includes neutrinos, electrons, protons, micrometeoroids, meteoroids, asteroids, planets, stars, galaxies, galactic clusters, and superclusters based on data given in Tables II, III, IV. The Eq. (5) conforms with the empirical data.

Cosmic Flux - While it may vary locally, the quantity A is apparently a cosmic parameter with empirically determined value between $10^{-18.6}$ and $10^{-17.7}$ or a mean value of $\approx 10^{-18}$ g/cm^{2}s over the entire mass range between 10^{-30} to 10^{50} g, Table IV. Cumulative mass flux $mF_c = A$ seems to have roughly the same value at each level of the cosmic hierarchy. To further support the notion that A is a cosmic parameter, the following argument is offered.

Table II - Cumulative Flux for Meteoroids to Asteroids

MASS RANGE log m (g)	FLUX CONSTANTS log A (g/cm^2s)	B	BASIS OF DATA	REFERENCE
-14 to 16	-17.5	-1.0	Average, all particles	4, p. 156
-13 to 25	-18.6	-0.95	Average, all particles	7 - 20
-13 to 0	-16.3	-0.70	Solar corona	7
-13 to 2	-17.1	-1.34	Cometary meteors	8
-12 to 0	-16.0	-1.12	Photo/radar meteors	9
-10 to -7	-21.0	-1.7	Micrometeoroids	10
-10 to -5	-11.9	-1.0	Micrometeoroids	11
-10 to 0	-16.7	-0.70	Solar corona	7
-10 to 0	-18.2	-1.35	Photographic meteors	12
-9 to 0	-16.4	-1.0	Photographic meteors	13
-5 to 0	-17.6	-1.0	Meteorites	14
-4 to 4	-18.0	-1.25	Meteors	15
-4 to 6	-14.6	-1.0	Meteorites	16
0 to 11	-19.2	-0.77	Stone meteorites	17
0 to 11	-20.7	-0.77	Stone meteorites	17
0 to 11	-21.0	-0.76	Iron meteorites	17
0 to 11	-21.5	-0.76	Iron meteorites	17
0 to 14	-18.2	-1.0	Stone meteorites	8
0 to 14	-21.5	-0.7	Cometary meteors	18
0 to 18	-21.0	-0.7	Iron meteorites	8
4 to 12	-18.5	-0.9	Meteorites	18
4 to 13	-20.1	-0.81	Lunar crater counts	19
13 to 20	-23.6	-0.54	Lunar crater counts	19
15 to 25	-20.0	-0.76	Asteroid counts	17, 20

If the mass content of the universe is dominated by an intergalactic distribution of protons with a spatial density of $\rho_o \approx 10^{-29}$ g/cm^3, then a universal proton flux can be determined as described in Appendix C. The resultant cosmic proton mass flux would be $m_pF = 2\rho_p c \approx 10^{-18}$ g/cm^2s if, as argued in Appendix C, m_pF is the product of proton spatial mass density, ρ_p, and the velocity of light, c.

Table III - Estimated Proton and Electron Flux

PROTON FLUX log F (/cm^{2}s)	ELECTRON FLUX log F (/cm^{2}s)	BASIS OF ESTIMATE	REFERENCE
8.87 to 8.91	10.50 to 10.54	Solar atmosphere at 1AU[a]	21, 22
8.61 to 8.73	10.24 to 10.36	Interplanetary medium[b]	23
4.63 to 5.63	6.20 to 7.72	Interplanetary medium[c]	24
4.33 to 7.18	5.96 to 8.82	Interstellar medium[d]	22
5.13 to 6.13	6.76 to 7.76	Intergalactic medium[e]	4, 25

a. For electron and proton density 550/cm^3 at 10^5 °K to 600/cm^3 at 10^5 °K.
b. For proton density 300/cm^3 at 10^5 °K to 400/cm^3 at 10^4 °K.
c. For atomic hydrogen density 0.1/cm^3 at 10^4 °K to 1.0/cm^3 at 10^4 °K.
d. For protons and hydrogen density 0.64/cm^3 at 10^5 °K to 13/cm^3 at 10^oK.
e. For proton density 0.01/cm^3 at 10^5 °K to 0.01/cm^3 at 10^9 °K.

Table IV - Estimated Mean Flux from Electrons to Galaxies

MASS log m (g)	FLUX log F (/cm^{2}s)	MASS FLUX log A (g/cm^{2}s)	BASIS OF ESTIMATE	REFERENCE
-27.04	8.44	-18.6	Cosmic electrons, average	Table III
-23.78	6.81	-17.0	Cosmic protons, average	Table III
-23.78	5.78	-18.0	Cosmic protons	Appendix C
-12(min)	-7.5	-19.5	Space particles	4
16(max)	-32.8	-16.8	Space particles	4
16 to 20	-38	-20.0	Oort comet cloud	4, 26
16 to 20	-33	-15.0	Whipple comet belt	4, 27
21 to 30	-42	-16.5	Asteroid counts	4
23 to 26	-43	-18.5	Satellites of solar planets	4
26 to 28	-44.5	-17.5	Four minor solar planets	4
28 to 30	-46.5	-17.5	Five major solar planets	4
32 to 35	-50	-16.5	Main sequence stars	4
39 to 40	-58	-18.5	Globular star clusters	4
42 to 44	-61	-18.0	Individual galaxies	4
44 to 46	-63	-18.0	Galactic cluster	4
48 to 50	-67	-18.0	Galactic superclusters	Appendix B
		-17.7	(mean value of A) = mF	

Discussion

Cosmic Background - One point of this paper is that is important to recognize and exploit the hierarchical nature of the universe. This allows the calculation of a cumulative flux for matter at each level of the cosmic hierarchy independently of the sparser flux of objects at higher levels of the hierarchy. As evidence for high order clustering, i.e., superclustering, of galaxies has accumulated, it has also become evident that objects at lower the hierarchical levels pervade the space between and within objects of the higher levels (Refs. 2, 28-31).

Matter in the form of electrons, protons, nucleons, molecules, grains, dust, and the like probably fills all cosmic space somewhat hierarchically with a cumulative spatial density of about 10^{-29} g/cm^3. Thus, objects at lower levels of the cosmic hierarchy apparently form background fields for the successively higher levels, ultimately represented by condensed luminous matter. There is probably a copious intergalactic distribution of objects ranging from protons and dust to asteroid- and planet-sized objects. These intergalactic objects may be stellar debris or they may have accumulated prior to star formation. A consideration of elemental abundances supports the premise that even the synthesis of heavy elements preceded star formation (Ref. 32). Moreover, molecules, grains, comets, and similar objects may predate either planets or stars (Ref 33-35).

Background Luminosity - The hierarchical evidence and cumulative flux data herein support the idea that increasingly larger scale luminous objects thin out exponentially in cosmic space relative to the background spatial density of lower order objects. The Olbers paradox vanishes simply because not every line of sight terminates on a luminous surface. In an infinite universe containing a minority of stars there is no need to explain why the sky is not filled with blazing illumination. What we see, in fact, is only the extremely low temperature kinetic glow of the cosmic background radiation.

Energy Density - Apparently, the quantity A is a universal parameter formed by the product of the mean cosmic spatial density of matter and the velocity of light,

$$A = mF_c \approx m_pF \approx \rho_o c \approx 10^{-18} \text{ g/cm}^2\text{s} \qquad (7)$$

This notion is based on the derivation given in Appendix C and the observation that the mass content of the universe is dominated by protons, and that the combined spatial density, ρ_o, of all luminous and dark matter in the universe is probably $\approx 10^{-29}$ g/cm^3. The quantity A is by dimensional analysis equivalent to momentum density, since [g/cm^{2}s] = [(g·cm/s)/cm^3)]. Accordingly, a kinetic energy density can be derived from A by,

$$\varepsilon = Av = \rho_o cv \qquad (8)$$

where, v is the mean velocity for a particular class of objects in the cosmic hierarchy. For example, for interstellar protons v equals approximately 10^6 cm/s (Refs. 4, 6). Based on $mF_c = 10^{-18}$, $\varepsilon = 10^{-12}$ erg/cm^3 for protons. This kinetic energy density is virtually identical to the peak microwave isotropic background which equals roughly $6\cdot10^{-13}$ erg/cm^3 (Ref. 4). The mean random velocity of galaxies is 10^7 cm/s and this gives $\varepsilon = 10^{-11}$ erg/cm^3. This is comparable to the cosmic background radiation density which when integrated over all wavelengths is $=2\cdot10^{-11}$ erg/cm^3. (Ref. 3, p. 249).

Using the logic of Appendix C, one can also form the product $\varepsilon' = Ac = \rho_o c^2$ arguing that the cumulative kinetic energy density of the cosmos equals the mass conversion energy of all cosmic matter. This leads to the inference that while half the total energy content of the universe resides in matter the other half emerges as motion. Assuming that this balance is being maintained, what needs to be determined is whether and how kinetic (gravitational, electromagnetic, thermal) energies and mass energies continuously transform from one to the other.

Neutrino Flux - Equation (5), can be used for estimating a cosmic neutrino flux by extrapolation. Based on a theoretical neutrino mass of $\approx10^{-30}$ g, the cosmic neutrino flux is $\approx10^{12}$ /cm^{2}s, Fig. 2. It is not surprising that this neutrino flux is greater than the theoretical solar neutrino flux of $\approx6\cdot10^{10}$ /cm^{2}s and, indeed, measured solar neutrino flux (Refs. 36, 37). Because actual neutrino mass may be considerably less than 10^{-30} g, cosmic neutrino flux may be even greater than 10^{12} /cm^{2}s, if Eq. (5) applies. Consequently, neutrinos, rather than protons or baryonic matter, should dominate the mass content and kinetic energy density of the cosmos, see Appendix C. The question arising from this idea is what effect if any does this copious neutrino flux have on cosmic physics and the relation between cosmic mass and kinetic energy density.

Implicit Premises - A fundamental premise herein is that local aspects of the cosmos pervade the entire cosmos. That is, for example, particulate fluxes in found in solar, interstellar, and intergalactic space should not be unique to the local intergalactic system. Present conditions at distances of, say, 1, 10, or 100 billion light-years are assumed to be indistinguishable from our local conditions. This follows from the cosmological principle that, in the current epoch, no cosmological location is different from any other and that contemporary observers throughout the cosmos will agree on its general properties. Another premise is that the entire cosmos is everywhere the same age, nonrelativistic, and spatially flat. No assumptions are made about early stages of evolution of the universe or how current conditions arose (Ref. 38).

Conclusion

The cosmos can be partitioned into hierarchical domains embracing all classes of discrete objects from electrons and planetoids to galaxies and galactic superclusters. Heuristic analysis of these hierarchical domains leads to the estimation of a mass flux peculiar to the objects in each domain. At each level of the cosmic hierarchy the mass flux is found to be a particular constant expressed by the relation $mF_c = A$, where m is mass, F_c is the cumulative flux of objects with masses equal to or greater than m, and A is mass flux. It was found that $mF_c = A = 10^{-18}$ g/cm^{2}s represents the cumulative mass flux of all classes of objects in the cosmic hierarchy from the least to most massive, i.e., from electrons to galactic superclusters. It is proposed that the quantity A equals the product of the mean density of matter in the universe, $\rho_o = 10^{-29}$ g/cm^3, and the velocity of light, c, that is, A =

$\rho_o c \approx 10^{-18}$ g/cm^{2}s. Combining A with the ambient velocity of interstellar protons, v_p, the product Av_p corresponds to the isotropic peak microwave background energy density, combining A with the random velocity of galaxies, v_g, the product Av_g corresponds to the cosmic background radiation energy density integrated over all wavelengths, and $Ac = \rho_o c^2 = 9{\cdot}10^{-8}$ erg/cm^3 is the kinetic energy density of cosmic matter.

Appendix A: Upper Bound Domain Radii

There is a association between the radius versus mass ratios of the hydrogen atom and the universe (Refs. 1, 3), i.e., $a_o/m_p^{1/2} = R/m_c^{1/2} \approx 4{\cdot}10^3$, where a_o is the radius of the first Bohr orbit, m_p is the proton rest mass, R is the radius of the universe, and m_c is the mass of a galactic supercluster (see Symbols and Constants). This assumes that galactic superclusters constitute an ultimate level in the cosmic hierarchy (Ref. 30). Therefore, let the domain dimension of the hydrogen atom be $2a_o$ and the domain dimension of a galactic supercluster be 2R and construct the relation,

$$r_d/m^{1/2} = [(2R/m_c^{1/2})(2a_o/m_p^{1/2})]^{1/2} \qquad (1A)$$

giving the equation,

$$r_d = [4a_oR/(m_pm_c)^{1/2}]^{1/2}m^{1/2} \approx 10^4m^{1/2} \qquad (2A)$$

which forms an upper bound on the radii of all cosmic objects. It is significant that at planetary and successively higher levels, cosmic hierarchical domains increase in size by roughly equal increments in the domain dimension with size ratios of roughly 10^4.

Appendix B: Flux Computation

Cumulative flux data are available for the hierarchical levels N = -3 through N = 3, in Solar space. Corresponding objects range from micrometeoroids through asteroids, objects with masses from m = 10^{-12} to 10^{25} g. The data are given in Table II which shows *log* A, B, and the mass range for the particular object class.

For fluxes associated with hierarchical level N = -5 to -4, the representative mass is 10^{-24} g and the associated object is the proton. Protons pervade interplanetary, interstellar, and intergalactic space. The flux computation can be based on kinetic temperature and on spatial density. Equating proton flux to the product of proton spatial density and ambient velocity gives $F_c = 2\rho_p v_p/m_p$. Estimates of proton and associated electron fluxes appear in Table III.

Hierarchical levels N = 3 to N = 5 correspond to the mass range from 10^{18} g to 10^{30} g which includes comets, asteroids, satellites, and planets. Since these objects are orbital and periodic their fluxes can be based on the frequency **f** with which they pass through a plane centered on the sun and a domain radius $r \approx 6 \cdot 10^{18}$ cm, the mean interstellar distance in the solar neighborhood, i.e., $F = 2\mathbf{nf}/\pi r^2$.

Hierarchical levels N = 5 to N = 6 correspond to the mass range from 10^{30} g to 10^{36} g which includes main sequence stars. For these objects the flux is based on the frequency **f** with which a mean stellar mass, 10^{33} g, passes through a plane centered on the galactic nucleus and a domain radius equal to the mean distance between galaxies. In this case **f** in $F = 2\mathbf{nf}/\pi r^2$ may be taken as the reciprocal of the galactic rotation period. An alternative calculation that yields a similar result is based on the mean spatial density and the stochastic velocity of main sequence stars, $F = 2\rho_s v_s/m_s$.

The hierarchical level N = 7 and the associated mass 10^{46} g corresponds to a mean galaxy with a random velocity of $\approx 10^7$ cm/s. The flux may be computed as $F = 2\rho_g v_g/m_g$, as was done for stars. For galactic clusters F is based on the mean red shift recessional velocity $\approx 1.8 \cdot 10^9$ cm/s and mean spatial density of clusters (Ref. 4). Since roughly ten thousand, 10^4, clusters comprise a galactic supercluster, F for superclusters should be diminished accordingly. Results of these flux estimates appear in Tables II, III, and IV.

Appendix C: Flux Due to Universal Expansion

Take any arbitrary spherical surface as a reference. This surface can be treated as coincident with the horizon of the universe as viewed by any observer. The net flux through this surface will be zero because the universe is by definition self-contained. For a set of **n** objects with mean mass m, distributed in the volume $(4/3)\pi r^3$ the spatial density $\rho = (3\mathbf{n}m)/(4\pi r^3)$. Taking r and **n** as functions of time, the net bidirectional flux is,

$$F = (d\mathbf{n}/dt)/(4\pi r^2) = r(d\rho/dt)/(3m) + 3\mathbf{n}(dr/dt)/(4\pi r^3) \quad (1C)$$

were, dr/dt and dρ/dt are positive and non-zero, and,

$$3\mathbf{n}(dr/dt)/(4\pi r^3) = r(d\rho/dt)/(3m) \quad (2C)$$

Let r = R, the radius of the universe, and dR/dt = c, the velocity of light. R = cT = c/H, and dT/dt = 1, where T is the age of the universe and H is the Hubble constant for the current epoch. Then, the flux through area $4\pi R^2$ is,

$$F = 2(3\mathbf{n}c)/(4\pi R^3) \quad (3C)$$

The previous equation leads to a *virtual* flux for **n** universally pervasive objects such as protons. For protons with mass m_p and mean spatial density $\rho_p = (3m_p\mathbf{n}_p)(4\pi R^3) \approx 10^{-29}$,

$$m_pF = 2\rho_pc = A_p = 6 \cdot 10^{-19} \approx 10^{-18} \quad (4C)$$

The cosmological principle that there is no singular or unique location in the universe allows taking $A_p \approx 10^{-18}$ as applicable to **any** location in the universe. Therefore, this derivation suggests that, on a universal scale, the cumulative mass flux $A = mF_c = \rho_o c$ since baryonic particles in the form of protons may dominate cosmic space and, hence, the cumulative flux.

This derivation utilized the fact that the universe appears to be expanding and that extrapolated to its "boundary" at time -T the expansion velocity is c. There is no evidence that the expansion velocity has changed. Since this conclusion should be reached by any arbitrary observer at any epoch, an infinite self-contained universe should have the property described by Eq. (4C).

The previous derivation may be moot if neutrinos rather that protons dominate cosmic space. The mean spatial mass density of neutrinos probably exceeds 10^{-29} g/cm^3, according to Eq. (5) given a neutron mass approaching 10^{-30} g. Since neutrinos already appear to travel at velocities near that of light, $c_n \approx c$, we have $A = mF_c = m_n F = \rho_n c_n \approx \rho_o c$.

Symbols and Constants

a_o	radius of first Bohr orbit, $h^2/(4\pi^2 m_e e^2) = 0.529\cdot 10^{-8}$ cm
A	mass flux (momentum density) constant, Eq. (5), g/cm^{2}s
B	numerical exponent, Eq. (5)
c	velocity of light, $2.998\cdot 10^{10}$ cm/s
e	electron charge, $4.803\cdot 10^{-10}$ ESU (electrostatic units)
F	flux, object or particle rate, /cm^{2}s
F_c	cumulative flux of masses >m, Eq. (5), /cm^{2}s
h	Planck constant, $6.626\cdot 10^{-27}$ erg
H	Hubble constant, $2.0\cdot 10^{-18}$ /s
m	mass or characteristic mass, g
m_c	mass of mean galactic supercluster, $\approx 10^{49}$g
m_e	rest mass of electron, $9.109\cdot 10^{-28}$ g
m_p	rest mass of proton, $1.673\cdot 10^{-24}$ g
n	number of discrete objects in set
N	hierarchical level, integers ... -2, -1, 0, 1, 2 ...
P	density increment index, integers ... -2, -1, 0, 1, 2 ...
r	physical radius of particle or cosmic object, cm
r_d	upper bound on domain radius, Eq. (1), cm
r_N	hierarchical partition dimension, Eq. (4), cm
r_P	isodensity line dimension, Eq. (3), cm
r_s	lower bound on radius, Schwarzchild limit, Eq. (2), cm
R	radius of the universe, $1.5\cdot 10^{28}$ cm
t	time, s
T	Hubble time (age of the universe), $5.1\cdot 10^{17}$ s

v velocity, cm/s
α fine structure constant, $2\pi e^2/hc = 7.29\cdot10^{-3}$
ε energy density, erg/cm^3
ε' energy density of the universe, $Ac = \rho_o c^2 \approx 9\cdot10^{-8}$ erg/cm^3
γ gravitational constant, $6.670\cdot10^{-8}$ dyn·cm^2/g^2
ρ density, g/cm^3
ρ_s spatial density, g/cm^3
ρ_o universal spatial density of matter, $2\cdot10^{-29}$ to $1\cdot10^{-28}$ g/cm^3

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