Article

Existence of [*z*-(*1*/ √*2*)(t_1+t_2)]-type and [(t_1+t_2)*z*√*2*]-type Plane Waves in V_4 for Bimetric Relativity

Jyotsna. K. Jumale*

Dept. of Phys., R. S. Bidkar Arts, Comm. & Science College, Hinganghat, India

Abstract
The biometric relativity of Rosen (1973,74) admits[*z*-(*1*/ √*2*)(t_1+t_2)]-type and[(t_1+t_2)*z*√*2*]-type plane gravitational waves in four dimensional space-times V_4 having two time axes where in the latter case the spacetime can be reduced to conformal one.

Keywords: Gravitational wave, biometric relativity, two time axes.

1. Introduction

For biometric theory of Rosen (1973,74), Karade (1994) has obtained the plane wave solutions of the field equations $N_i^j = 0$ and established the existence of $(z-t)$-type and (t/z)-type plane gravitational waves in four dimensional space-time V_4. Reformulating Karade's (1994) definition of plane wave, in the paper referred it to [1], we have obtained the plane wave solutions of the field equations $N_i^j = 0$in BR theory of Rosen(1973,74) where at each point of the space-time there are two metrics.

$$ds^2 = g_{ij} dx^i dx^j \qquad \text{and} \qquad d\sigma^2 = f_{ij} dx^i dx^j \tag{1.1}$$

are given by g_{ij} which satisfied

$$Q\rho_i^j + P\sigma_i^j = 0 \tag{1.2}$$

which further breaks in

$$\overline{\overline{w}}_2\rho_i^j + \overline{w}_2\sigma_i^j = 0 = \overline{\overline{\phi}}_2\rho_i^j + \overline{\phi}_2\sigma_i^j, \qquad \overline{\overline{w}}_3\rho_i^j + \overline{w}_3\sigma_i^j = 0 = \overline{\overline{\phi}}_3\rho_i^j + \overline{\phi}_3\sigma_i^j, \tag{1.3}$$

where $w_2 = t_2 + \phi_2 z$, $\qquad w_3 = t_2 + \phi_3 t_1$

* Correspondence Author: J. K. Jumale, Department of Physics, R. S. Bidkar College, Hinganghat - 442 301 Wardha, India. E-mail: jyotsnajumale@yahoo.com

$$\phi_2 = \frac{Z,_2}{Z,_4}, \qquad \phi_3 = \frac{Z,_3}{Z,_4},$$

$$M_2 = \overline{w}_2 - \overline{\phi}_2 z, \qquad M_3 = \overline{w}_3 - \overline{\phi}_3 t_1,$$

$$N_2 = \overline{\overline{w}}_2 - \overline{\overline{\phi}}_2 z, \qquad N_3 = \overline{\overline{w}}_3 - \overline{\overline{\phi}}_3 t_1,$$

$$\rho_i^j = [(\phi_2^2 - \phi_3^2) - 1] g^{hj} \overline{g}_{hi} \quad \text{and} \quad \sigma_i^j = \frac{d}{dZ}\{[1 - (\phi_2^2 - \phi_3^2)] g^{hj} \overline{g}_{hi}\}.$$

In the present paper, we have studied these solutions (1.2) in detail for $[z - \frac{1}{\sqrt{2}}(t_1 + t_2)]$-type and $[(t_1 + t_2)/z\sqrt{2}]$-type plane waves in four dimensional space-times V_4 having two time axes for BR theory of Rosen(1973,74).

2. $[z - \frac{1}{\sqrt{2}}(t_1 + t_2)]$-type plane wave in V_4

Let $Z = [z - \frac{1}{\sqrt{2}}(t_1 + t_2)] \quad \Rightarrow Z_{,2} = 1, \quad Z_{,3} = -\frac{1}{\sqrt{2}}, \quad Z_{,4} = -\frac{1}{\sqrt{2}}.$

Then $\phi_2 = \frac{Z,_2}{Z,_4} = -\sqrt{2}, \qquad \phi_3 = \frac{Z,_3}{Z,_4} = 1.$

Also $w_2 = t_2 + \phi_2 z = -Z\sqrt{2} - t_1, \qquad w_3 = t_2 + \phi_3 t_1 = -Z\sqrt{2} + z\sqrt{2}$

$\Rightarrow \overline{w}_2 = -\sqrt{2}, \qquad \overline{w}_3 = -\sqrt{2}.$

Hence $M_2 = \overline{w}_2 - \overline{\phi}_2 z = -\sqrt{2}, \qquad M_3 = \overline{w}_3 - \overline{\phi}_3 t_1 = -\sqrt{2}$

$\Rightarrow P = -\sqrt{2}, \qquad \because M_2 = M_3 = P$

and $N_2 = \overline{\overline{w}}_2 - \overline{\overline{\phi}}_2 z = 0, \qquad N_3 = \overline{\overline{w}}_3 - \overline{\overline{\phi}}_3 t_1 = 0$

$\Rightarrow Q = 0 \qquad \because N_2 = N_3 = Q$

$\Rightarrow \sigma_i^j = 0.$

With the above values, the L.H.S. of the field equations (1.2) become zero and hence the equation is identically satisfied. Therefore, it implies that $[z-\frac{1}{\sqrt{2}}(t_1+t_2)]$-type plane gravitational wave exists in four dimensional space-times for biometric relativity.

3. $[(t_1+t_2)/z\sqrt{2}]$-type plane wave in V_4

Let $Z=[(t_1+t_2)/z\sqrt{2}]$ $\qquad \Rightarrow Z_{,2}=-(t_1+t_2)/z^2\sqrt{2}$,

$$Z_{,3}=\frac{1}{z\sqrt{2}}, \qquad Z_{,4}=\frac{1}{z\sqrt{2}}$$

Then $\phi_2=\frac{Z_{,2}}{Z_{,4}}=-Z\sqrt{2}$, $\qquad \phi_3=\frac{Z_{,3}}{Z_{,4}}=1$.

Also $w_2=t_2+\phi_2 z=-t_1$, $\qquad w_3=t_2+\phi_3 t_1=zZ\sqrt{2}$,

$$\Rightarrow \overline{w}_2=0, \qquad \overline{w}_3=z\sqrt{2}.$$

Hence $M_2=\overline{w}_2-\overline{\phi}_2 z=z\sqrt{2}$, $\qquad M_3=\overline{w}_3-\overline{\phi}_3 t_1=z\sqrt{2}$

$$\Rightarrow P=z\sqrt{2}, \qquad \because M_2=M_3=P$$

and $N_2=\overline{\overline{w}}_2-\overline{\overline{\phi}}_2 z=0$, $\qquad N_3=\overline{\overline{w}}_3-\overline{\overline{\phi}}_3 t_1=0$

$$\Rightarrow Q=0 \qquad \because N_2=N_3=Q$$

$$\Rightarrow \sigma_i^j=0$$

and the field equation (1.2) reduces to

$$\{[1-(\phi_2^2-\phi_3^2)]g^{hj}\overline{g}_{hi}=c_i^j \qquad \text{i.e.,} \qquad 2[1-Z^2]g^{hj}\overline{g}_{hi}=c_i^j$$

where c_i^j are constants.

If we choose δ_i^j in particular, we get

$$[1-Z^2]g^{hj}\bar{g}_{hi}=\delta_i^j \qquad \text{i.e., } [1-Z^2]\bar{g}_{ki}=g_{ki}$$

and then $g_{ki}=D_{ki}[\frac{1+Z}{1-Z}]^{1/2}$ where D_{ki} are constants.

Noting (1.1), the space-times V_4 admitting $[(t_1+t_2)/z\sqrt{2}]$-type plane gravitational waves becomes

$$ds^2=\left[\sqrt{\frac{z\sqrt{2}+(t_1+t_2)}{z\sqrt{2}-(t_1+t_2)}}\right]D_{ij}dx^i dx^j$$

which is reducible to a conformal space-times. Hence the space-times V_4 having two time axes admitting $[(t_1+t_2)/z\sqrt{2}]$-type plane gravitational wave in bimetric relativity is reducible to a conformal space-times.

4. Conclusion

The biometric relativity admits $[z-\frac{1}{\sqrt{2}}(t_1+t_2)]$-type and $[(t_1+t_2)/z\sqrt{2}]$-type plane gravitational waves in four dimensional space-times V_4 having two time axes where in the later case the space-times can be reduced to conformal one.

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