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Early Decelerating and Late-Time Accelerating Anisotropic Cosmological Model in Scale-Covariant Theory of Gravitation

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Abstract

In this paper we study an anisotropic Bianchi type-V cosmological model filled with a perfect fluid within the framework of scale-covariant theory of gravitation. Exact solutions of the field equations are obtained by using the hybrid expansion law for the average scale factor that yields power-law and exponential-law cosmologies in its special cases. The model obtained presents a cosmological scenario which describes an early deceleration and late-time acceleration. The model approaches isotropy and tends to a de-sitter universe at late times. We observe that the universe expands forever with the dominance of dark energy. The physical and kinematical properties of the model are discussed.

Keywords: Cosmology, decelerating, accelerating, Bianchi-V, Hybrid expansion law.

1 Introduction

The present day observational evidences have led cosmologists to believe that the universe in its present state is in the phase of accelerated expansion. The limitations of Einstein's general relativity in providing satisfactory explanation of this phase in its evolution have led cosmologists to adopt various other hypotheses and study their implications in this context. These hypotheses include those assuming (1) time-dependence to the gravitational constant G or the cosmological constant Λ (ii) other geometries or physical fields with the universe and(iii) modified or alternative theories of gravity. Such theories are expected to bring out a number of aspects of mathematical or physical interest associated with them.

Canuto et al.[1, 2] formulated the self-covariant theory of gravitation by associating the mathematical operation of scale transformation with the physics of using different dynamical systems to measure space-time distances. Corresponding to each dynamical system of units there is a gauge condition which determines the otherwise arbitrary gauge function. For gravitational units, they have chosen the gauge condition so that the standard Einstein's equations

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are recovered. This theory is a variable alternative to general relativity which allows a natural interpretation of the possible variation of the gravitational constant G (Wesson [3], Will [4]). Beesham [5] has discussed power asymptotic singularities in this theory with special attention to Friedmann model and the Kasner model, and generalized the corresponding general relativistic results. Reddy et al [6] presented an LRS Bianchi type-I cosmological model with constant negative deceleration parameter in scale-covariant theory of gravitation. Shri Ram et al.[7] investigated an anisotropic cosmological model Bianchi type-V a perfect fluid by applying the law of variation for Hubble's parameter proposed by Berman [8], Berman and Gomide [9]. Singh and Sharma [10] obtained Bianchi type-II string cosmological model with magnetic field in this theory. Shri Ram and Chandel[11] investigated a Bianchi type-V inflationary cosmological model in scale-covariant theory of gravity.

The astronomical observations of the luminosity of type Ia supernovae (Bahcall et al.[12]; Riess et al.[13]; Permutter et al.[14]), cosmic background radiation (Spergel et al.[15]), the galaxy power spectrum (Tegmark et al.[16]) have confirmed that the expansion of the universe is accelerating. Many authors have suggested a number of ideas to explain the current accelerating universe. After estimating various energy components of the universe, the cause of accelerated expansion of the universe has been attributed to some exotic energy, called dark energy (DE), which behaves like a vacuum field energy with repulsive character arising from the negative pressure. Some cosmologists attributed the observed acceleration to a possible breakdown of our understanding of the laws of gravitation, thus they attempted to modify Friedmann equations. Bamba et al.[17] reviewed different DE cosmologies with early deceleration and late-time acceleration. For an universe decelerating in the past and accelerating at present time, the deceleration parameter must show signature flipping. Cunha and Lima [18] favours recent acceleration and past time deceleration with high degree of statistical confidence level by analyzing three SNe type Ia samples. Adhav et al.[19] investigated early decelerating and late-time accelerating anisotropic Bianchi types cosmological models with dynamical equation of state parameter.

Akarsu et al.[20] proposed the following form for the average scale factor $a(t)$ of the universe:

$$a(t) = bt^{\alpha}e^{\beta t} \tag{1.1}$$

where $b > 0$ and $\alpha \geq 0$, $\beta \geq 0$ are constants. They called $a(t)$ in (1) as hybrid expansion law, which is the combination of power-law ($\beta = 0$) and exponential law ($\alpha = 0$) cosmologies respectively. In fact, power-law and exponential law cosmologies can be used only to describe epoch based evolution of the universe because of the constancy of deceleration parameter. Akarsu et al.[21] studied the evolution of the universe with hybrid expansion law in FRW model. Further, Kumar[22] studied the dynamics of the universe within the framework of a Bianchi type-V spacetime filled with perfect fluid composed of non-interacting matter and dynamical DE using the hybrid expansion law (1). Using the law (1), Shri Ram and Chandel [23] discussed the dynamics of magnetized string cosmological model of Bianchi type-V in f(R, T) gravity theory using hybrid expansion law. Recently, Chandel and Shri Ram [24] studied an early decelerating and late-time accelerating Bianchi type-V cosmological model with perfect fluid and heat conduction by using (1).

In this paper, we investigate a Bianchi type-V early decelerating and late-time accelerating cosmological model filled with perfect fluid in scale-covariant theory of gravitation by applying the hybrid expansion law (1) for average scale factor.

2 The metric and field equations

In scale-covariant theory of gravitation, Einstein's field equations are valid in the gravitational units whereas the physical quantities are measured in atomic units. The metric tensors in two system of coordinates are related by the conformal transformation

$$\bar{g}_{ij} = \phi^2 g_{ij} \tag{2.1}$$

where the gauge function ϕ is a function of coordinates. Here a bar denotes gravitational units and unbar denotes atomic units. By the use of, (2) Canuto et al.[1] transformed the standard Einstein's field equations into

$$R_{ij} - \frac{1}{2}Rg_{ij} + f_{ij}(\phi) = -8\pi G(\phi)T_{ij} + \Lambda(\phi)g_{ij} \tag{2.2}$$

where

$$\phi^2 f_{ij} = 2\phi\phi_{;ij} - 4\phi_{,i}\phi_{,j} - g_{ij}(\phi\phi^{;l}_{;l} - \phi^{,l}\phi_{,l}). \tag{2.3}$$

A comma denotes ordinary derivative and a semicolon denotes covariant derivative. All symbols have their usual meaning.

For a perfect fluid the energy-momentum has the form

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij} \tag{2.4}$$

where ρ is the energy-density of matter, p the pressure and u^i is the four-velocity vector of the fluid satisfying $u^i u_i = 1$.

The spatially homogeneous and anisotropic Bianch type-V space-time is given by the line-element

$$ds^2 = dt^2 - A^2(t)dx^2 - e^{2mx}\left[(B^2(t)dy^2 + C^2(t)dz^2\right] \tag{2.5}$$

where A, B, C are cosmic scale functions and m is a constant of parameter. Bianchi type-V models are of particular interest since they are sufficiently complex because Einstein tensor has off-diagonal terms, while at the same time they are simple generalizations of negative curvature FRW models. Such a type of model plays significant role in the description of the universe at the early stages of its evolution.

For the metric (6), the field equations (3), (4) and (5), in comoving coordinates, yield the following equations:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{m^2}{A^2} - 2\frac{\dot{A}}{A}\frac{\dot{\phi}}{\phi} + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \frac{\ddot{\phi}}{\phi} - \frac{\dot{\phi}^2}{\phi^2} = -8\pi Gp, \tag{2.6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{m^2}{A^2} - 2\frac{\dot{B}}{B}\frac{\dot{\phi}}{\phi} + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \frac{\ddot{\phi}}{\phi} - \frac{\dot{\phi}^2}{\phi^2} = -8\pi Gp, \tag{2.7}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}}{A}\frac{\dot{B}}{B}-\frac{m^2}{A^2}-2\frac{\dot{C}}{C}\frac{\dot{\phi}}{\phi}+\frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)+\frac{\ddot{\phi}}{\phi}-\frac{\dot{\phi}^2}{\phi^2}=-8\pi Gp, \tag{2.8}$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B}+\frac{\dot{A}}{A}\frac{\dot{C}}{C}+\frac{\dot{B}}{B}\frac{\dot{C}}{C}-\frac{3m^2}{A^2}+\frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)-\frac{\ddot{\phi}}{\phi}+3\frac{\dot{\phi}^2}{\phi^2}=8\pi G\rho, \tag{2.9}$$

$$2\frac{\dot{A}}{A}-\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=0. \tag{2.10}$$

Here and in what follows an overdot denotes ordinary derivative with respect time t. The energy conservation equation which is a consequence of the field equations is givens rise to

$$\dot{\rho}+(\rho+p)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)+\rho\left(\frac{\dot{G}}{G}+\frac{\dot{\phi}}{\phi}\right)+3p\frac{\dot{\phi}}{\phi}=0. \tag{2.11}$$

Eq.(11), on integration, yields

$$A^2=BC \tag{2.12}$$

where the constant of integration in observed into B or C.

For the metric (6), the kinematical parameters are given as

$$Averagescalefactor: a=A, \tag{2.13}$$

$$Spatialvolume: V=a^3, \tag{2.14}$$

$$Expansionscalar: \theta=\frac{3\dot{A}}{A}, \tag{2.15}$$

$$Shearscalar: \sigma^2=\frac{1}{2}\left(\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)^2, \tag{2.16}$$

$$Hubbleparameter: H=\frac{\dot{A}}{A}. \tag{2.17}$$

An important observational interest in cosmology is the deceleration parameter q defined as

$$q=-\frac{\ddot{a}a}{\dot{a}^2}. \tag{2.18}$$

The sign of q indicates whether the model inflates or not. The positive value of q corresponds to a standard decelerating model whereas the negative sign indicates accelerated expansion.

From Eqs. (7)-(10), we obtain the energy density and pressure in terms of H, q and σ as

$$8\pi G\rho=3H^2-\sigma^2-\frac{3m^2}{A^2}-\frac{\ddot{\phi}}{\phi}+3\left(\frac{\dot{\phi}}{\phi}\right)^2+3H\frac{\dot{\phi}}{\phi}, \tag{2.19}$$

$$8\pi Gp=H^2(2q-1)-\sigma^2+\frac{m^2}{A^2}-\frac{\ddot{\phi}}{\phi}+\left(\frac{\dot{\phi}}{\phi}\right)^2-H\frac{\dot{\phi}}{\phi}. \tag{2.20}$$

In the next section, we obtain exact solutions for the scale factors $A(t)$, $B(t)$ and $C(t)$ by using the hybrid expansion law for the average scale factor $a(t)$.

3 Solution of field equations

Eqs.(7)-(10) are highly non-linear in nature. For the complete determination of the model, we need additional assumptions relating field variables. Combining Eqs. (7)-(10) and integrating the results, we obtain solutions of these equations in quadrature form as [7]:

$$A(t) = a, \tag{2.21}$$

$$B(t) = Da \exp\left(X \int \frac{dt}{a^3\phi^2}\right), \tag{2.22}$$

$$C(t) = D^{-1} a \exp\left(-X \int \frac{dt}{a^3\phi^2}\right), \tag{2.23}$$

where X and D are constants. Clearly we can obtain the explicit solutions for $A(t)$, $B(t)$ and $C(t)$ in terms of cosmic time t if the average scale factor $a(t)$ and the gauge function $\phi(t)$ are known. Here we assume that the gauge function $\phi(t)$ is inversely proportional to $a(t)$

$$\phi = \frac{k}{a(t)} \tag{2.24}$$

where k is an arbitrary constant. Then Eqs.(23) and (24) reduce to

$$B(t) = Da \exp\left(\frac{X}{k^2} \int \frac{dt}{a}\right), \tag{2.25}$$

$$C(t) = D^{-1} a \exp\left(\frac{-X}{k^2} \int \frac{dt}{a}\right). \tag{2.26}$$

We now use the hybrid expansion law (1) for the average scale factor. Substituting (1) into Eqs. (22), (26) and (27) and integrating the results, we obtain

$$A(t) = bt^\alpha e^{\beta t}, \tag{2.27}$$

$$B(t) = Dbt^\alpha e^{\beta t} \exp\left(\frac{-X}{k^2 D} \beta^{\alpha-1} \gamma[(1-\alpha), \beta t]\right), \tag{2.28}$$

$$C(t) = D^{-1} bt^\alpha e^{\beta t} \exp\left(\frac{X}{k^2 D} \beta^{\alpha-1} \gamma[(1-\alpha), \beta t]\right), \tag{2.29}$$

where γ denotes the lower incomplete gamma function. Clearly we must have $0 < \alpha \leq 1$. The solution for the gauge function $\phi(t)$ is given by

$$\phi = \frac{k^2}{b^2 t^{2\alpha} e^{2\beta t}}. \tag{2.30}$$

For the cosmological model corresponding to the scale factors in Eqs.(28)-(30), the kinematical parameters are obtained as the expansion scalar(θ), shear scalar(σ), mean Hubble parameter(H) and the deceleration parameter(q) are obtained as

$$\theta = 3\left(\frac{\alpha}{t} + \beta\right), \tag{2.31}$$

$$\sigma = \frac{X}{kbt^{\alpha}e^{\beta t}}, \tag{2.32}$$

$$\theta = \left(\frac{\alpha}{t} + \beta\right), \tag{2.33}$$

$$q = \frac{\alpha}{(\alpha + \beta t)^2} - 1. \tag{2.34}$$

With the help of Eqs.(28)-(35), we obtain the expressions for the energy density and pressure as follows:

$$8\pi G\rho = \frac{2(\alpha + \beta t)^2 - \alpha}{t^2} - \frac{3m^2k^4 + 2X^2}{b^2k^4t^{2\alpha}e^{2\beta t}}, \tag{2.35}$$

$$8\pi Gp = \frac{\alpha - 2(\alpha + \beta t)^2}{t^2} + \frac{m^2k^4 - 2X^2}{b^2k^4t^{2\alpha}e^{2\beta t}}. \tag{2.36}$$

Now, to find the energy density and pressure in the forms independent of G, we recall that in most of the variable G cosmologies, G is a decreasing function of time. The possibility of increasing G has also been studied by Levit [25]. Beesham [26] discussed the possibility of creation field with G proportional to some power of time. Sistero [27] presented exact solution for zero- curvature Robertson-Walker models with G proportional to some power of the average scale factor. Here we obtain a physically realistic model of the universe assuming that

$$G = la^2 = lb^2t^{2\alpha}e^{2\beta t} \tag{2.37}$$

where l is a constant. Substituting Eqs.(28) into Eqs (36) and (37), we obtain

$$8\pi G\rho = \frac{2(\alpha + \beta t)^2 - \alpha}{lb^2t^{\alpha+2}e^{2\beta t}} - \frac{3m^2k^4 + 2X^2}{lb^4k^4t^{4\alpha}e^{4\beta t}}, \tag{2.38}$$

$$8\pi Gp = \frac{\alpha - 2(\alpha + \beta t)^2}{lb^2t^{\alpha+2}e^{2\beta t}} + \frac{m^2k^4 - 2X^2}{lb^4k^4t^{4\alpha}e^{4\beta t}}. \tag{2.39}$$

4 Results and discussions

We observe that for this model, the spatial volume V tends to zero as $t \to 0$. The energy density, pressure, expansion scalar, shear scalar and the mean Hubble parameter all assume infinite values at this epoch. Thus the model evolves from a big-bang type singularity at $t = 0$. The spatial volume tends to infinite as $t \to \infty$. The energy density, pressure and mean Hubble parameter decrease with the passage of time and eventually tend to zero as $t \to \infty$. We also find that $H = \beta$ and $\theta = 3\beta$ for large time. Also the deceleration parameter q obtained in Eq.(35) tend to -1 as $t \to \infty$. Thus, the present universe asymptotically tend to the de sitter universe and expands forever with the dominance of DE. Since $lim_{t\to\infty}\frac{\sigma}{\theta} = 0$, the anisotropic model becomes isotropic at late time. From Eq.(38), gravitational constant G varies with time.

Initially G is zero and increases with the increases of time and ultimately attains infinity value. The gauge function ϕ is infinite at $t = 0$ and tend to zero as $t \to \infty$.

From Eq.(35), we see that the universe with hybrid expansion law evolves with time-dependent deceleration parameter and the transition from deceleration to acceleration takes place at time

$$t = \frac{\sqrt{\alpha} - \alpha}{\beta}. \tag{2.40}$$

Thus, the model is early decelerating and late-time accelerating. The cosmological models in power-law and exponential law cosmologies are special cases of our model which can be obtained by putting $\beta = 0$ and $\alpha = 0$ respectively.

5 Conclusions

In this paper, we have studied a spatially homogeneous and anisotropic Bianchi type-V cosmological model filled with perfect fluid within the framework of scale-covariant theory of gravitation. Exact solution of the field equations have been obtained by using the hybrid expansion law for the average scale factor. The derived model exhibits transition from deceleration phase to acceleration phase which is an essential feature of dynamical evolution of universe. The gravitational constant G, being zero at $t = 0$, assumes infinite value which is consistent with observations. The universe is anisotropic at the early stages of its evolution and becomes isotropic at late time. At late time the dark energy dominates leading to accelerated expansion of the universe. For large values of t, the model results in a de-Sitter universe.

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