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Plane Symmetric Universe with Λ in *f*(*R*,*T*) Gravity

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Abstract

In the present study, Plane symmetric cosmological models in $f(R, T)$ theory of gravity with a term Λ have been studied. The gravitational field equations in the metric formalism, which follow from the covariant divergence of the stress-energy tensor, are obtained. The field equations correspond to a specific choice of $f(R,T) = f_1(R) + f_2(T)$ with the individual superior functions $f_1(R) = \lambda R$ and $f_2(R) = \lambda T$. The exact solutions of the field equations are obtained by considering a constant deceleration parameter which leads to two different aspects of the volumetric expansion, namely a power law and an exponential volumetric expansion. Some physical and geometric properties of the models along with physical acceptability of the solutions have also been discussed in detail.

Keywords: Plane symmetric universe, variable cosmological constant, modified theory, gravity, dark energy, constant deceleration parameter.

1. Introduction

Recent observations show that universe is apparently going through accelerated expansion (Perlmutter et al. 1999; Riess et al. 2007; Komatsu et al.2011). The available data indicates that the universe is spatially flat and is dominated by 76% dark energy and 24% by different matter (20% dark matter and 4% regular matter). So dark energy has become necessary in modern cosmology and there has been a substantial interest in cosmological models with dark energy. In recent years, modifications of Einstein's theory are attracting a lot of attention in order to elucidate the late time acceleration and dark energy.

Among the varied modifications of Einstein's theory, $f(R)$ gravity (Akbar and Cai 2006) and $f(R,T)$ gravity (Harko et al.2011) theories are attracting more attention throughout the last decade as a result of these theories are purported to give natural gravitational alternatives to dark

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energy. Among the assorted modifications, $f(R)$ theory of gravity is treated as best suited because of its cosmological importance.

It is been prompt that cosmic acceleration is achieved by substitution the Einstein-Hilbert action of Einstein's theory of relativity with a general function of Ricci scalar, $f(R)$.Nojiri and Odintsov (2006) developed the final theme for modified $f(R)$ gravity reconstruction from any realistic FRW cosmology. They evidenced that the modified $f(R)$ gravity representing a realistic different to general relativity theory is additional consistent in dark epoch. Further, Nojiri et. al. (2006) developed the final programme of the unification of matter-dominated era with acceleration epoch for scalar tensor theory or dark fluid. Katore and Shaikh (2015) studied Bianchi Type-III Cosmological Models with Bulk Viscosity in $f(R)$ Theory.

Recently, Harko et al. (2011) developed a generalized $f(R,T)$ gravity where the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and of the trace T of the energy-momentum tensor. The justification of selecting T as associate degree argument for the Lagrangian is from exotic imperfect fluids or quantum effects. The corresponding field equations are derived in metric formalism for many explicit cases. Harko et al. (2011) have argued that owing to the coupling of matter and geometry, this gravity model depends on a supply term, representing the variation of the matter-stress-energy tensor with relevancy the metric. The $f(R,T)$ gravity models can explain the late time cosmic accelerated expansion of the Universe.

The exact solutions of the field equations in respect of LRS Bianchi type-I space time filled with perfect fluid in the framework of $f(R,T)$ gravity are derived by K.S.Adhav(2012). Katore and Shaikh (2012) studied Kantowaski-Sachs Dark Energy model in $f(R,T)$ gravity. A five dimensional Kaluza Klein cosmological model in $f(R,T)$ gravity with a negative constant deceleration parameter have been investigated by Reddy et al. (2012). Shamir et al.(2012) obtained exact solution of Bianchi type-I and type-V cosmological models in $f(R,T)$ gravity. A new class of Bianchi cosmological models in $f(R,T)$ gravity by using a special law of variation for the average scale factor have been investigated by Chaubey and Shukla (2013).

Kiran and Reddy (2013), Reddy et al.(2014a, 2014b), have studied the Bianchi type cosmological models in the presence of cosmic strings and bulk viscosity in $f(R,T)$ gravity. Samanta (2013) derived the exact solutions of the field equations in respect of Kantowski-Sachs universe filled with perfect fluid in the framework of $f(R,T)$ theory of gravity .Naidu et. al.(2013) have discussed spatially homogeneous and anisotropic Bianchi type-V space-time in the presence of bulk viscous fluid with one dimensional cosmic strings in $f(R,T)$ gravity. Shri Ram and Priyanka (2013) have derived algorithms for generating new solutions of the field equations with a perfect fluid for a five dimension Kaluza-Klein space-time within the framework of $f(R,T)$ gravity theory. Yadav et. al. (2014) have considered $f(R,T)$ gravity model with an arbitrary coupling between matter and geometry in LRS Bianchi-I space-time.

Singh and Bishi (2014) have studied the FRW metric for variable G and Λ in $f(R,T)$ gravity with the modified Chaplygin gas equation of state. Evolution of Bianchi type-V cosmological model is studied in presence of perfect fluid and variable cosmological constant in $f(R,T)$ theory of gravity have been investigated by Ahmed and Pradhan (2014). Sharif and Zubair (2014) discuss the Bianchi type I model with perfect fluid as matter content in $f(R,T)$ gravity, where R the Ricci is scalar and T is the trace of the energy-momentum tensor. Reddy et. al (2014) studied a spatially homogeneous Bianchi type-VI_0 space-time in the frame work of $f(R,T)$ gravity ,when the source for energy momentum tensor is a bulk viscous fluid containing one dimensional cosmic strings. The spatially homogeneous and totally anisotropic Bianchi type-II cosmological solutions of massive strings have been investigated by Sharma and Singh (2014) in the presence of the magnetic field in the framework of $f(R,T)$ gravity.

Singh and Singh (2014) present the cosmological viability of reconstruction of an alternative gravitational theory, namely, the modified $f(R,T)$ gravity, where R is the Ricci scalar curvature and T the trace of stress energy momentum tensor. The spatially homogeneous and totally anisotropic Bianchi Type-II space-time dark energy model with EoS parameter is considered by Singh and Sharma (2014) in the presence of a perfect fluid source in the framework of $f(R,T)$ gravity. Spatially homogeneous and anisotropic Bianchi type-II, -VIII and -IX cosmological models filled with perfect fluid in the framework of the $f(R,T)$ gravity and also in general relativity have been studied by Rao et. al.(2014). Mahanta (2014) have constructed Locally Rotationally Symmetric Bianchi type I (LRSBI) cosmological models in the $f(R,T)$ theory of gravity when the source of gravitation is the bulk viscous fluid. Sharif and Yousaf (2014) study the factors affecting the stability of locally isotropic spherical self-gravitating systems in $f(R,T)$ gravity.

The spatially homogeneous and totally anisotropic Bianchi type-III string cosmological models have been investigated by Rani et. al. (2014) in the presence as well as absence of the magnetic field within the framework of $f(R,T)$ gravity . The exact solutions of Bianchi type V space-time in $f(R,T)$ theory of gravity has been investigated Shamir (2014). Sharif and Zubair (2014, 2014a) reconstructed the $f(R,T)$ gravity with Ricci and modified Ricci dark energy. An axially symmetric space-time is considered in the presence of a perfect fluid source in the framework of $f(R,T)$ gravity by Sahoo et. al.(2014).Mishra and Shaoo (2014) investigated Bianchi type VI_h perfect fluid cosmological model in $f(R,T)$ theory.

Sahoo and Mishra (2014) studied Kaluza-Klein dark energy model in the form of wet dark fluid in $f(R,T)$ gravity. Sahoo et. al. (2014) constructed Kaluza-Klein cosmological models in $f(R,T)$ gravity with $\Lambda(T)$. Biswal et. al. (2015) discussed Kaluza-Klein cosmological model in $f(R,T)$ gravity with domain walls. Singh et. al. (2015) obtained Scalar field and Cosmological constant in $f(R,T)$ gravity for Bianchi type-I Universe. Non-static cosmological model in $f(R,T)$ gravity have been investigated by Mishra et. al. (2015) . Chirde and Shekh (2015) discussed Dark Energy Cosmological Model in a Modified Theory of Gravity.

Motivated by the above research works, in the present work, classes of cosmological models described by plane symmetric space-time for a perfect fluid distribution in the framework of $f(R,T)$ gravity have been investigated. With a specific choice of $f(R,T)= f_1(R)+ f_2(T)$, with the individual superior functions $f_1(R)=\lambda R$ and $f_2(T)=\lambda T$.

2. Gravitational field equations of $f(R,T)$ modified gravity theory

The $f(R,T)$ theory of gravity is the generalization or modification of General Relativity (GR).In this theory the modified gravity action if given by

$$s=\frac{1}{16\pi}\int f(R,T)\sqrt{-g}\,d^4x+\int L_m\sqrt{-g}\,d^4x\,, \tag{1}$$

where $f(R,T)$ is an arbitrary function of the Ricci scalar R and the trace T of the stress energy tensor T_{ij} of the matter, L_m is the matter Lagrangian density. If $f(R,T)$ is replaced by $f(R)$, we get the action for $f(R)$ gravity and replacement of $f(R,T)$ by R leads to the action of general relativity.

Varying the action S with respect to metric tensor g_{ij} , the field equations of $f(R,T)$ gravity are obtained as

$$f_R(R,T)R_{ij}-\frac{1}{2}f(R,T)g_{ij}+f_R(R,T)\left(g_{ij}\nabla^i\nabla_i-\nabla_i\nabla_j\right)=8\pi T_{ij}-f_T(R,T)T_{ij}\text{ - }f_T(R,T)\theta_{ij}\,, \tag{2}$$

Where

$$\theta_{ij}=-2T_{ij}+g_{ij}L_m-2g^{\alpha\beta}\frac{\partial^2 L_m}{\partial g_{ij}\partial g^{\alpha\beta}}\,. \tag{3}$$

Here $f_R = \frac{\delta f(R,T)}{\delta R}$, $f_T = \frac{\delta f(R,T)}{\delta T}$, ∇_i is the covariant derivative and T_{ij} is the standard matter energy momentum tensor derived from the Lagrangian L_m.

If the matter is regarded as a perfect fluid the stress energy tensor of the matter Lagrangian is given by $T_{ij} = (\rho + p)u_i u_j - p g_{ij}$. Here $u^i = (0,0,0,1)$ is the velocity vector in comoving coordinates which satisfies the condition $u^i u_i = 1$ and $u^i \nabla_j u_i = 0$.Here ρ, p are energy density and pressure of the fluid respectively.

For perfect fluid, the matter Lagrangian density can have two choices either $L_m = -p$ or $L_m = \rho$ which has widely been studied in literature (Bertolami et al. 2007; Sotiriou and Faraoni 2008; Bisabr 2013).Here the matter Lagrangian as $L_m = -p$ is assumed.Now θ_{ij} in equation (3) can be reduced to

$$\theta_{ij} = -2T_{ij} - g_{ij} p . \tag{4}$$

It is mentioned here that these field equation depends on the physical nature of the matter field. There are three classes of these models

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R) f_3(T) \end{cases} . \tag{5}$$

Reddy et al. (2012), Naidu et al. (2013), Samanta (2013), Sahoo et al.(2014) , Singh and Singh (2014) and Singh and Bishi (2014) have studied the cosmological models, assuming $f(R,T) = R + 2f(T)$.

Considering in the form $f(R,T) = f_1(R) + f_2(T)$ with $f_1(R) = \lambda R$ and $f_2(T) = \lambda T$, where λ is an arbitrary constant.

The field equation (2), for the specific choice of $f(R,T) = \lambda(R+T)$, reduces to

$$G_{ij} = \left(\frac{8\pi + \lambda}{\lambda}\right)T_{ij} + \left(p + \frac{1}{2}T\right)g_{ij} . \tag{6}$$

Choosing a tiny low negative worth for the impulsive λ to draw a much better analogy with the same old Einstein field equations and that we shall keep this selection of λ throughout.

The Einstein field equation with cosmological constant as

$$G_{ij} - \Lambda g_{ij} = -8\pi T_{ij}. \tag{7}$$

Comparing equations (6) and (7), we yield

$$\Lambda = \Lambda(T) = p + \frac{1}{2}T, \tag{8}$$

and

$$8\pi = \frac{8\pi + \lambda}{\lambda}. \tag{9}$$

The dependence of the cosmological constant Λ on the trace of the energy momentum tensor T has been proposed before by Poplawski (2006) where the cosmological constant in the gravitational Lagrangian is considered as a function of the trace of the energy-momentum tensor. Since we have considered the perfect fluid as the source, according to Poplawski (2006), the trace of energy-momentum tensor is function of isotropic pressure and energy density i.e.

$$T = -3p + \rho \tag{10}$$

3. Metric and Field equations

The universe is spherically symmetric and the matter distribution is isotropic and homogeneous. But during the early stages of evaluation, it is unlikely that it could have had such a smoothed-out picture. Hence we consider a plane symmetric model, which provides an opportunity for the study of inhomogeneity. Plane symmetric metric can be written in the form

$$ds^2 = dt^2 - A^2(dx^2 + dy^2) - B^2 dz^2, \tag{11}$$

where the metric potentials A and B are the functions of time t only.

Katore et. al. (2011) investigated Plane symmetric cosmological models with perfect fluid and dark energy. Katore and Shaikh (2012) studied Plane symmetric dark energy model in Brans-Dicke theory of gravitation. Statefinder Diagnostic for Modified Chaplygin Gas in Plane Symmetric Universe has been discussed by Katore and Shaikh (2012). Katore and Shaikh (2014) have investigated Plane Symmetric cosmological model in the presence of cosmic string and Bulk Viscosity in Saez-Ballester scalar tensor theory of gravitation. Very recently , Katore et. al. (2015) obtained Plane Symmetric Inflationary Universe with Massless Scalar Field and Time Varying Lambda.

In a co-moving coordinate system the field equations (8), for the metric (11), can be explicitly written as

$$\frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{B}}{B} = \left(\frac{8\pi + \lambda}{\lambda}\right)p - \Lambda, \tag{12}$$

$$2\frac{\ddot{A}}{A}+\frac{\dot{A}^2}{A^2}=\left(\frac{8\pi+\lambda}{\lambda}\right)p-\Lambda\,, \tag{13}$$

$$\frac{\dot{A}^2}{A^2}+2\frac{\dot{A}\dot{B}}{AB}=-\left(\frac{8\pi+\lambda}{\lambda}\right)\rho-\Lambda\,, \tag{14}$$

where an overhead dot hereafter, denote ordinary differentiation with respect to cosmic time t only. The trace in our model is given by equation (10) i.e. $T=-3p+\rho$, so that the effective cosmological constant in equation (8) reduces to

$$\Lambda=\frac{1}{2}(\rho-p). \tag{15}$$

The spatial volume is given by

$$V=a^3=A^2B\,, \tag{16}$$

where a is the mean scale factor.
Using equations (12) and (13), we get

$$\frac{d}{dt}\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)+\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)\frac{\dot{V}}{V}=0\,, \tag{17}$$

which on integration yields

$$\frac{A}{B}=c_2\exp\left(c_1\int\frac{dt}{V}\right)\,, \tag{18}$$

where c_1 and c_2 are integration constants.
Using equation (16), the values of scale factors A and B can be written explicitly as

$$A=V^{\frac{1}{3}}c_2^{\frac{1}{3}}\exp\left(\frac{c_1}{3}\int\frac{dt}{V}\right) \tag{19}$$

and

$$B=V^{\frac{1}{3}}c_2^{\frac{-2}{3}}\exp\left(\frac{-2c_1}{3}\int\frac{dt}{V}\right). \tag{20}$$

4. Solution of field equations

The anisotropy of the expansion can be parameterized after defining the directional Hubble parameters and the mean Hubble parameter of the expansion. The directional Hubble parameters, which determine the universe expansion rates in the directions of the x, y, z axes, defined as

$$H_x=H_y=\frac{\dot{A}}{A}\,, H_z=\frac{\dot{B}}{B} \tag{21}$$

and in terms of the average scale factor $a=(A^2B)^{\frac{1}{3}}$, the Hubble parameter H , which determines the volume expansion rate of the universe , may be generalized to anisotropic cosmological models:

$$H=\frac{1}{3}\frac{\dot{V}}{V}=\frac{1}{3}\left(2\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right). \tag{22}$$

The physical quantities of observational interest are the expansion scalar θ, the average anisotropy parameter A_m and the shear scalar σ^2. These are defined as

$$\theta=u^i_{;i}=\left(2\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right), \tag{23}$$

$$A_m=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2, \tag{24}$$

$$\sigma^2=\frac{3}{2}A_mH^2. \tag{25}$$

Now we have a set of three equations with five unknown functions A,B,p,ρ,Λ.To get a determinate solution of field equations, we need extra conditions. One will introduce a lot of conditions either by associate assumption comparable to some physical scenario of associate capricious mathematical supposition. However, these procedures have some drawbacks. Physical scenario could result in differential equations which are able to be tough to integrate and mathematical supposition could ends up in non physical scenario.

Therefore, we assume two volumetric expansion laws

$$V=e^{3kt}, \tag{26}$$

$$V=t^n. \tag{27}$$

where c_1, c_2, k *and* n are positive constant. The exponential law model exhibit acceleration volumetric expansion for $n>1$. In power law model, for $0<n<1$ the universe decelerates and for $n>1$ the universe accelerates.

4.1 Model for Power Law Expansion

Here, a power law volumetric expansion is given by

$$V = A^2 B = t^n \ , \tag{28}$$

which covers all possible expansion histories throughout the evolution of the universe, where n is a positive constant.

The scale factor can be obtained by using equation (28) in equations (19) and (20) as

$$A = t^{\frac{n}{3}} c_2^{\frac{1}{3}} \exp\left(\frac{c_1}{3(1-n)} t^{1-n} \right) \tag{29}$$

and

$$B = t^{\frac{n}{3}} c_2^{\frac{-2}{3}} \exp\left(\frac{-2c_1}{3(1-n)} t^{1-n} \right). \tag{30}$$

At $t = 0$, both the scale factors vanish, start evolving with time and finally as $t \to \infty$ they diverge to infinity. This is consistent with the big bang model. As scale factors diverge to infinity at large time there will be Big rip at least as far in the future.

The Hubble parameter is obtained as

$$H = \frac{n}{3t} . \tag{31}$$

The deceleration parameter yields as

$$q = \frac{d}{dt}\left(\frac{1}{H} \right) - 1 = \frac{3}{n} - 1 . \tag{32}$$

The deceleration parameter q is a very important factor for understanding cosmic evolution. The positive sign of q corresponds to standard decelerating model where as the negative sign of q indicates inflation. Though the current observations of SN Ia (Permutter et al (1999), Riess et al (1998)) and CMBR favors accelerating models i.e. $q < 0$. Pertmutter et al. (1999) and Riess et al. (1998) have shown that the decelerating parameter of the universe is in the range $-1 \le q \le 0$ and the present day universe is undergoing accelerated expansion. But both do not altogether rule out the decelerating ones which are also consistent with these observations (Vishwakarma 2000). The deceleration parameter is always negative for $n > 3$ indicating accelerating universe and attained its fastest rate of expansion $q = -1$ for large n which resembles with the investigations of Adhav (2012).

The mean anisotropic parameter becomes

$$A_m = \frac{2c_1^2}{n^2 t^{2n-2}} . \tag{33}$$

The expansion scalar and shear scalar are found to be

$$\theta = \frac{n}{t} , \tag{34}$$

$$\sigma^2 = \frac{c_1^2}{3t^{2n}} . \tag{35}$$

It is observed that the Hubble parameter H , expansion scalar θ, shear scalar σ are very large near $t \sim 0$ and finally tends to zero as $t \to \infty$. The rate of expansion of the universe decreases with time.

The Pressure can be obtained as

$$p = \frac{(\alpha+1)n^2 - 2(\alpha+1)n - n^2 + n}{3\alpha(\alpha+1)t^2} + \frac{c_1^2}{3\alpha t^{2n}} . \tag{36}$$

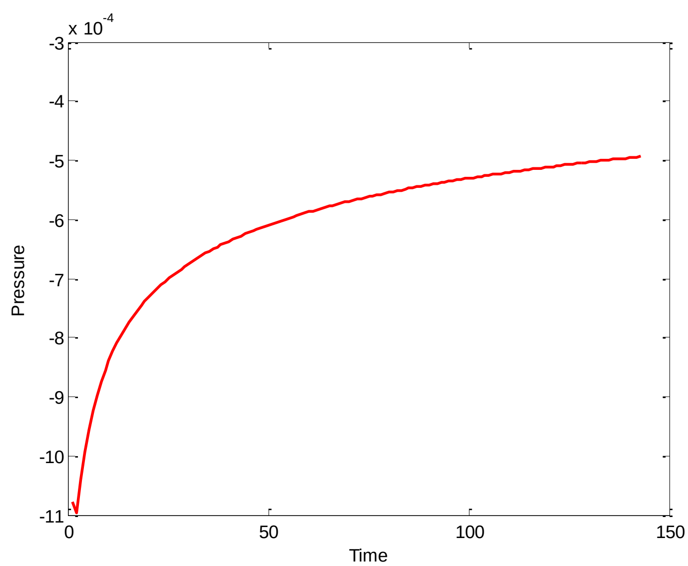

Figure 1. Pressure vs time.

From the figure No.1, we observe that pressure is increasing function of time. It starts from a large negative value and approaches to a small negative value near zero. From the invention of the accelerated expansion of the universe, it's typically assumed that this cosmic acceleration is because of some quite energy matter with negative pressure referred to as 'dark energy' that resembles with Ahmed and Pradhan (2014).

The Energy Density is given by

$$\rho = \frac{n^2 - (\alpha+1)n^2 - n}{3\alpha(\alpha+1)t^2} + \frac{c_1^2}{3\alpha t^{2n}}. \quad (37)$$

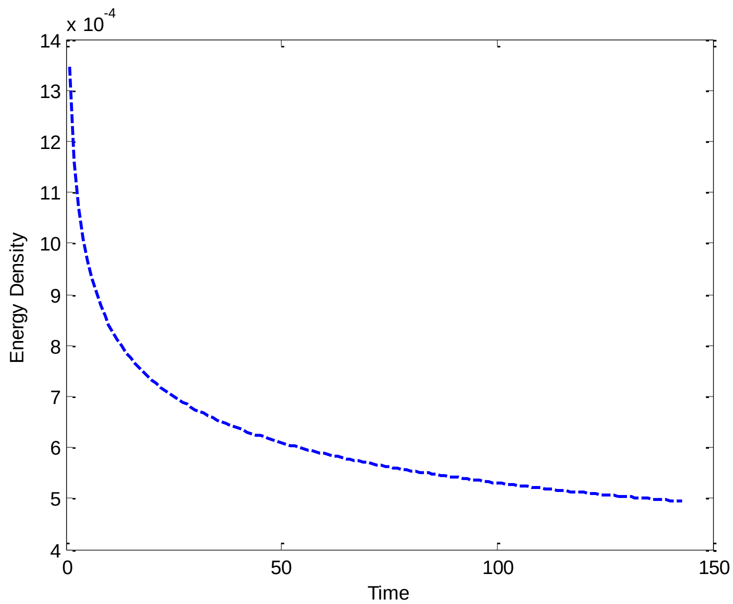

Figure 2. Energy Density vs time.

Figure No.2 shows the variation of energy density ρ versus time *t*. Here it is observed that ρ is a positive decreasing function of time and it approaches some constant value as $t \to \infty$.

The cosmological constant

$$\Lambda = \frac{-n(n-1)}{3(\alpha+1)t^2}. \quad (38)$$

Here $\alpha = \dfrac{8\pi + \lambda}{\lambda}$.

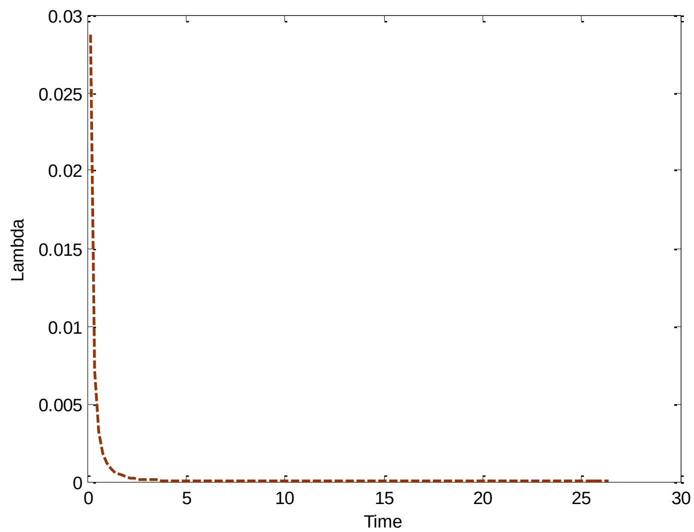

Figure 3. Cosmological Constant vs time.

From Figure 3 , the cosmological constant is decreasing function of time and it approaches a small positive value at late time (i.e. at present epoch).The existence of a positive cosmological constant Λ with the magnitude $\Lambda(G\hbar/c^3)\approx 10^{-123}$ have been suggested by (Schmidt, et al(1998), Garnavich, et al(1998), Perlmutter, et al(1999),Riess et. al.(2004)). These observations on magnitude and red-shift of type Ia supernova suggest that our universe may be an accelerating one with induced cosmological density through the cosmological Λ-term. Thus the nature of Λ in our derived model is supported by recent observations which resembles with the results of Pradhan et. al. (2011).

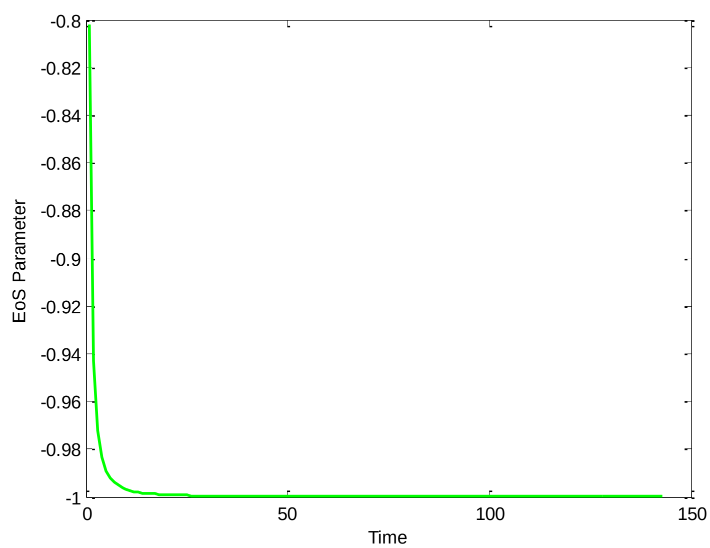

Figure 4. EoS Parameter vs time.

Here $\omega \approx -1$ for sufficiently large time t as shown in figure no.4. Therefore, the late time dynamics of EoS parameter ω represents the vacuum fluid dominated Universe which resembles with the observations of Kumar and Yadav(2011).

4.2. Model for Exponential Expansion

The exponential expansion of volume factor

$$V = A^2 B = e^{3kt} \ , \tag{39}$$

usually leads to a de Sitter kind of universe.

The scale factor can be obtained by using equation (26) in equations (19) and (20) as

$$A = c_2^{\frac{1}{3}} \exp\left(kt - \frac{c_1}{9k} e^{-3kt} \right) \tag{40}$$

and

$$B = c_2^{\frac{-2}{3}} \exp\left(kt + \frac{2c_1}{9k} e^{-3kt} \right). \tag{41}$$

It is clear that, the scale factor admit constant values at time $t = 0$, afterwards they evolve with time without any type of singularity and finally diverge to infinity. This is consistent with big bang scenario which resembles with Katore and Shaikh (2015).

The Hubble parameter is obtained as

$$H = k \, . \tag{42}$$

The deceleration parameter yields as

$$q = \frac{d}{dt}\left(\frac{1}{H} \right) - 1 = -1 \, . \tag{43}$$

For this model $q = -1$ and $\frac{dH}{dt} = 0$, which suggests the best price of the Edwin Powell Hubble parameter and the quickest rate growth of the universe. Thus, this model could represent the inflationary era within the early universe and also the terribly late times of the universe. The exponential model favors the accelerated growth with representing the Delaware Sitter model that resembles with the investigations of Sharif and Zubair (2014).

The mean anisotropic parameter becomes

$$A_m = \frac{2c_1^2}{9k^2} e^{-6kt}. \tag{44}$$

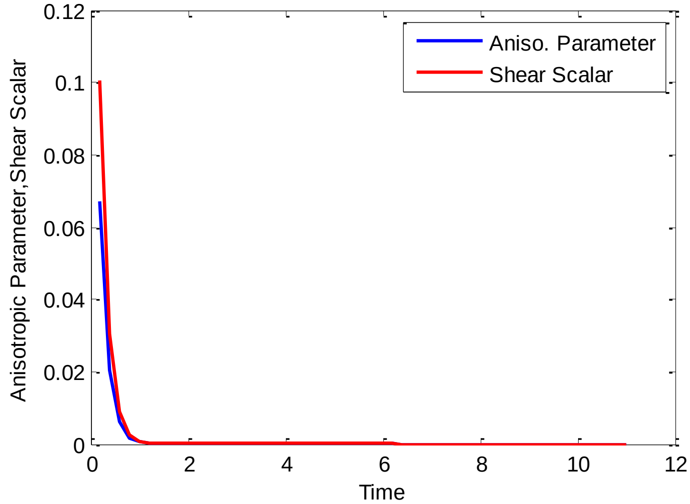

Figure 5. Anisotropic Parameter, Shear Scalar vs time.

The expansion scalar and shear scalar are found to be

$$\theta = 3k \, , \tag{45}$$

$$\sigma^2 = \frac{c_1^2}{3} e^{-6kt} \, . \tag{46}$$

From (44), it is observed that the anisotropy parameter measures a constant value at $t = 0$ while it vanishes at infinite time of the universe. This indicates that the universe expands isotropically at later times. The shear scalar $\sigma \to 0, \; as \;\; t \to \infty$. The shear scalar is finite at $t = 0$ .The behavior of anisotropic parameter and shear scalar with respect to time is shown in figure no. 5.The spatial volume is finite at $t = 0$. It expands exponentially as t increases and becomes infinitely large as $t = \infty$.The expansion scalar for these scale factors exhibits the constant value. This shows uniform exponential expansion from $t = 0$ to $t = \infty$ i.e. universe expands homogeneously.

The Pressure is given by

$$p = \frac{(\alpha+1)c_1^2 e^{-6kt} + 9\alpha k^2}{3\alpha(\alpha+1)}, \tag{47}$$

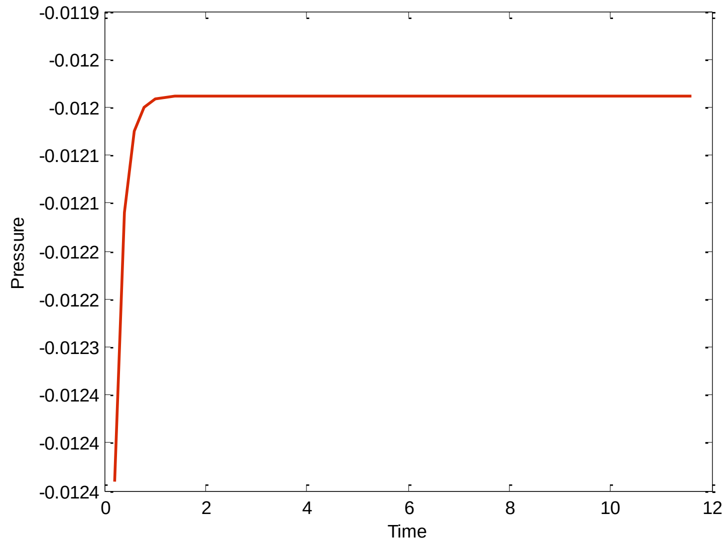

Figure 6. Pressure vs time.

The Energy Density is given by

$$\rho = \frac{(\alpha+1)c_1^2 e^{-6kt} - 9\alpha k^2}{3\alpha(\alpha+1)} \tag{48}$$

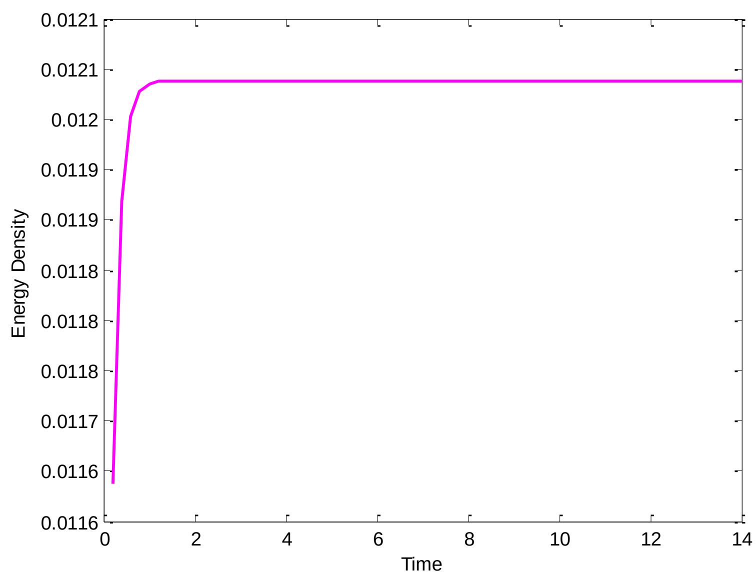

Figure 7. Energy Density vs. time.

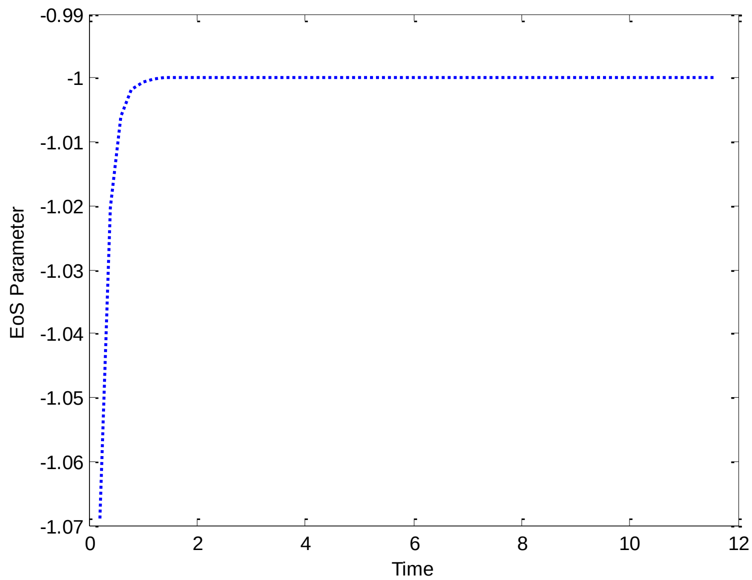

Figure 8. EoS Parameter vs time.

The plots of Pressure and Energy Density are shown in figure no. 6 and no.7 respectively. Pressure increases from a large negative value in the beginning of cosmic time and approaches to a constant value which resembles with the investigations of Sahoo et. al.(2014).The behaviour of energy density differ from the behaviour of energy density of Sahoo et. al.(2014). The study of the EoS parameter of the dark energy is very significant. We cannot determine the geometry of the universe without the knowledge of the EoS of dark energy. The SN Ia data suggests that $-1.67 < \omega < -0.62$ (Knop et. al (2003)) while the limit imposed on ω by a combination of SN Ia data (with CMB anisotropy) and galaxy clustering statistics is $-1.33 < \omega < -0.79$ (Tegmark et. al. (2004)) .When $\omega = -1$, the universe passes through Λ CDM epoch. If $\omega < -1$, then we live in the phantom-dominated universe and for $\omega > -1$, the quintessence dark era occurs. It is evident from Figure no. 8 that $\omega \to -1$ as $t \to \infty$ which indicate that the non-singular model corresponds to a vacuum fluid dominated universe which resembles the investigations of Shamir (2014).

The Cosmological constant

$$\Lambda = \frac{-3k^2}{(\alpha+1)}. \tag{49}$$

5. Conclusion

To deal with the problems of late time acceleration of the universe, Harko et al. (2011) proposed a new theory known as $f(R,T)$ theory of gravity by modifying general theory of relativity. For this purpose, we take $f(R,T)=f_1(R)+f_2(T)$, with the individual superior functions $f_1(R)=\lambda R$ and $f_2(T)=\lambda T$ and investigate the exact solutions of plane symmetric cosmological model. The exact solutions of the field equations are obtained by considering a constant deceleration parameter that leads two different aspects of the volumetric expansion namely a power law and an exponential volumetric expansion. The physical and geometrical aspects of the models are investigated. It has been found that Λ may be a decreasing performs of your time and it converges to a tiny low positive worth at late time. The character of decaying vacuum energy density Λ in our derived model is supported by recent cosmological observations.

These observations on magnitude associated red-shift of type Ia supernova recommend that our universe is also an fast one with evoked cosmological density through the cosmological Λ-term.
In the Exponential volumetric expansion, the universe starts with zero volume, at the initial epoch and expands exponentially approaching to infinite volume. The universe expands homogeneously. The deceleration parameter appears with negative sign which implies accelerating expansion of the universe. The shear scalar $\sigma \to 0,\ as\ t \to \infty$. The shear scalar is finite at $t=0$. The anisotropy parameter measures a constant value at initial epoch while it vanishes at infinite time of the universe. Hence, the present model is isotropic at late time which is consistent to the current observations.

In the Power law expansion, at $t=0$ both the scale factors vanish, start evolving with time and finally as $t \to \infty$ they diverge to infinity. This is consistent with the big bang model. It is observed that A_m decreases with time and tends to zero as $t \to \infty$. Thus, the observed isotropy of the universe can be achieved in our model at present epoch.
Thus, the solutions incontestable during this paper could also be helpful for higher understanding of the characteristic of Plane symmetric cosmological models within the evolution of the universe inside the framework of $f(R,T)$ gravity theory.

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