Exploration

Special Theory of Relativity with Invariant Length and Time Intervals

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Abstract

In an earlier article, we proposed a new Position Space Doubly Special Relativity Theory that makes use of quantum fluctuations to achieve an invariant scale-length. The invariant scale-length is fixed to a special characteristic length, ℓ_p. Herein, we make improvements to this theory by suggesting a new set of Lorentz transformations that allow any two observers to agree on the numerical value of any length and time interval. These new set of transformations leave the philosophy and the phenomenology of the Special Relativity Theory intact.

Keywords: Doubly Special Relativity , Lorentz transformation, Special Theory of Relativity.

1 Introduction

Professor Giovanni Amelino-Camelia (2002*a*,*b*) of the University *of* Rome in Italy set-forth a fecund revision of Einstein (1905)'s seemingly sacrosanct Special Theory of Relativity (STR) by adding to it – *via* momentum space $\Gamma(x, y, z, p_x, p_y, p_z)$; an absolute and universal minimum scale-length [$\ell_p = (G\hbar/c^3)^{\frac{1}{2}} = 5.11 \times 10^{-36}\,\text{m}$, where $\hbar = 1.06 \times 10^{-35}\,\text{Js}$, is Planck's normalised constant and $G = 6.667 \times 10^{-11}\,\text{kg}^{-1}\text{m}^3\text{s}^{-2}$ is Newton's universal constant of gravitation]. The proposal by Amelino-Camelia is popularly known as the Doubly Special Relativity (DSR) theory (Amelino-Camelia 2002*a*,*b*). So, to the already well established absolute universal constant – the speed of light $c = 2.99792458 \times 10^8\,\text{ms}^{-1}$ in vacuum, Giovanni Amelino-Camelia added a second, ℓ_p; thus his theory contains not one, but two absolute universal constants (c, ℓ_p). Because the theory has two universal absolute constants, Giovanni Amelino-Camelia dubbed it "*Doubly Special*" hence the name *Doubly Special Relativity*. Some call such theories Deformed Special Relativity while others call them Extended STR (*e.g.* Pavlopoulos 1967). However, the current popular name is Doubly Special Relativity.

The motivation for the formulation of DSR theories is well founded and is based on the subtle observations that: the Planck energy (or Planck scale, ℓ_p) is expected to play not only a fundamental role in any theory of Quantum Gravity (QG) but a pivotal role in setting the absolute universal scale-length at which quantum gravity effects cannot be neglected, leading to new physically observable phenomena. If Einstein's bare STR is to hold up exactly to this scale,

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then, inevitably – due to the Lorentz-FitzGerald contraction, different observers observing the same physical phenomenon will not agree as to where a the phenomenon they are observing has entered the QG regime as determined by the the scale-length ℓ_p because the numerical values of the system under probe will be different for the different observers.

From the foregoing, it follows that, QG would lead to a clear contradiction of the sacred Principle *of* Relativity that beholds that the Laws *of* Nature are the same for all inertial observers, hence, all inertial observers should agree on the existence of a physical phenomenon if of cause, they both are observing it. One can not – as would happen if quantum gravity occurs; claim a system has entered the QG regime while another observer observing the same physical system measures something to the contrary. In STR, they can disagree on the numerical values of their respective measurements; this disagreement in the numerical values of their measurements is (in the STR) resolved by the Lorentz transformations. If QG is to occur, contrary to all the dictates of binary logic, they will disagree on the existence of the physical phenomenon itself. This is where the need for a fundamental, absolute and universal scale-length comes in.

We are of the strong view that the issue of a special scale-length (of time) must be solved in position-space (as has been done *e.g.* by Deriglazov 2004, Deriglazov & Rizzuti 2005) and not in momentum-space. Noting that most DSR theories are formulated in momentum space (see *e.g.* Aloisio et al. 2004, Gao & Wu 2003, Jacob et al. 2010), in the reading Nyambuya (2012), we proposed a new *Position Space Doubly Special Relativity* (PS-DSR) theory. In this new PS-DSR, the usual Lorentz transformations, namely:

$$\begin{array}{rlr} \Delta x' = & \Gamma\left(\Delta x - V\Delta t\right) & (\mathbf{a}) \\ \Delta y' = & \Delta y & (\mathbf{b}) \\ \Delta z' = & \Delta z & (\mathbf{c}) \\ ic\Delta t' = & i\Gamma\left(c\Delta t - V\Delta x/c\right) & (\mathbf{d}) \end{array}, \tag{1.1}$$

where modified so that they are now given by:

$$\begin{array}{rlr} \Delta x' = & \Gamma_\phi\left(\Delta x - \phi_x V\Delta t\right) & (\mathbf{a}) \\ \Delta y' = & \Delta y & (\mathbf{b}) \\ \Delta z' = & \Delta z & (\mathbf{c}) \\ ic\Delta t' = & i\Gamma_\phi\left(c\Delta t - \phi_t V\Delta x/c\right) & (\mathbf{d}) \end{array}, \tag{1.2}$$

where the functions ϕ_x and ϕ_t are defined as:

$$\phi_x = \sqrt{1 - \left(\frac{\ell_p}{\delta x}\right)^2}, \text{ and } \phi_t = \sqrt{1 - \left(\frac{\ell_p}{\delta t}\right)^2}, \tag{1.3}$$

respectively, and:

$$\Gamma_\phi = \frac{1}{\sqrt{1 - \phi^2 v^2/c^2}}, \tag{1.4}$$

where ℓ_p is the invariant scale-length and $(\delta x, \delta t)$ give the size of the quantum fluctuations associated with x and t points on the spacetime grid respectively.

According to these ideas set-forth in Nyambuya (2012), a point x^μ has associated with it a corresponding quantum fluctuation of magnitude δx^μ. That is to say, the point x^μ is uncertain by an amount δx^{μ}; in the usual terminology, we can write this as $x^{\mu} \pm \delta x^{\mu}$. These quantum fluctuations are assumed to be truly random and intrinsic property of spacetime. Other than that, these quantum fluctuations have been assumed to be absolute in nature, the meaning of which is that every observer will measure the same numerical value of δx^{μ} irrespective of their state of motion. It should be said here without fear that, the insertion of the ϕ-functions (in Nyambuya 2012) has been done purely by the *sleight of hand*. Other than our intuition, there is no rigour to the insertion of these functions into the Lorentz transformation expect that we noted that such an insertion allows us the seemingly elusive opportunity of have an invariant scale-length at the position level of the Lorentz transformation. In the present reading, we make amendments to this earlier theory. We believe and feel that the present amendments are better as they are much easier and straight forward to understand.

2 New Modification

In our new modification, we propose the following Lorentz coordinate transformations:

$$\begin{array}{rll} \Delta x' = & \Gamma\left(\phi\Delta x - V\Delta t\right) & (\mathbf{a}) \\ \Delta y' = & \Delta y & (\mathbf{b}) \\ \Delta z' = & \Delta z & (\mathbf{c}) \\ ic\Delta t' = & i\Gamma\left(\phi c\Delta t - V\Delta x/c\right) & (\mathbf{d}) \end{array} . \tag{2.1}$$

Obviously, these transformations and the resulting theory coincides with Einstein's STR whenever ($\phi = 1$). For the same observer O′ observing O, but now moving in the opposite direction ($V \mapsto -V$), the transformations (2.1), will be given by:

$$\begin{array}{rll} \Delta x'_* = & \Gamma\left(\phi\Delta x + V\Delta t\right) & (\mathbf{a}) \\ \Delta y'_* = & \Delta y & (\mathbf{b}) \\ \Delta z'_* = & \Delta z & (\mathbf{c}) \\ ic\Delta t'_* = & i\Gamma\left(\phi c\Delta t + V\Delta x/c\right) & (\mathbf{d}) \end{array} . \tag{2.2}$$

We have inserted the subscript-$*$ as a way of labelling and distinguishing the transformations (2.1) from the transformations (2.2). Clearly ($\Delta x'_* \neq \Delta x'$) and ($\Delta t'_* \neq \Delta t'$). With this in mind, let us now define the length ($\Delta\ell$) and time ($\Delta\tau$) intervals:

$$\begin{aligned} (\Delta\ell)^2 = & \quad \Delta x'\Delta x'_*, \\ (\Delta\tau)^2 = & \quad \Delta t'\Delta t'_*. \end{aligned} \tag{2.3}$$

Now, if we are to define the function ϕ so that it is given by:

$$\phi^2 = 1 - \frac{V^2}{c^2} + \frac{V^2}{v^2} = \phi^2(V, v), \tag{2.4}$$

where:

$$v = \frac{\Delta x}{\Delta t}, \tag{2.5}$$

then, we find that:

$$\begin{aligned} (\Delta\ell)^2 = & \quad \Delta x'\Delta x'_* & = (\Delta x)^2 \\ (\Delta\tau)^2 = & \quad \Delta t'\Delta t'_* & = (\Delta t)^2 \end{aligned} . \tag{2.6}$$

What this means is that $\Delta\ell$ and $\Delta\tau$ are invariants and are common to both observers. In Einstein's bare STR *i.e.* in the case ($\phi = 1$), we will always have:

$$\begin{aligned} (\Delta\ell)^2 = & \quad \Delta x'\Delta x'_* & \neq (\Delta x)^2 \\ (\Delta\tau)^2 = & \quad \Delta t'\Delta t'_* & \neq (\Delta t)^2 \end{aligned} . \tag{2.7}$$

Clearly, our new proposal seems to be a much neater way of resolving the issue of a universal scale length in the STR, for with this proposal – *in-principle* – there is a way for the two observers (O′ and O) to agree (numerically) not only on the length measurements but time as-well. This is not possible in the STR.

Though 'quickly' fixable, one apparent problem with the present proposal is that $\phi(V, v)$ 'blows' up when ($v = 0$) and this is a serious problem! If not resolvable, the theory can as-well be cast into the dustbins of history. However, this '*prima facie*' problem can be whisked-away using not only Heisenberg (1927)'s uncertainty principle but our own proposal of a fundamental unit of length presented in the reading Nyambuya (2010).

From Heisenberg (1927)'s uncertainty principle ($\delta p\delta x \sim \hbar$), v can not be zero, therefore, ϕ will always be a finite quantity. According to the number theoretic and physical arguments presented in the reading Nyambuya (2010), an upper bound speed such as the speed of light c directly points the fact that there must exist a minimum finite time and hence length interval. This implies that ($v \neq 0$). In this way, the problem of $\phi(V, v)$ 'blowing' up when ($v = 0$) does not exist hence, our theory is completely free from this '*prima facie*' singularity.

It is expected with any modification that, it would naturally lead to new predications and alterations of the predicted phenomenology by the current theory. On that note, it is important to note that the present modification does not alter the philosophy nor the phenomenology of Einstein (1905) theory. The one-way length and time intervals remain relative, while the product of these intervals for the two different directions are for the two different observers.

3 General Discussion

While the theory proposed herein solves (*in-principle*) the problem which DSR theories try to solve, it would not be correct to call it a DSR theory as in this theory there exists no special length (and time) scale which observers will agree numerically, because, in our present theory, observers will always agree on the numerical value of any length and time interval. The 'stationery' observer simple has to measure in both directions the length ($\Delta x', \Delta x'_*$) and time ($\Delta t', \Delta t'_*$) intervals for a given physical phenomenon in the 'moving' frame. The product $[(\Delta x')(\Delta x'_*); (\Delta t')(\Delta t'_*)]$ of these two way directional measurements of length and time intervals is an invariant quantity, the meaning of which is that the two observers will agree on the numerical values of these quantities as measured in their respective frames of reference. In the end, the 'stationery' and 'moving' observers will agree on any length and time interval. This is very much unlike Einstein (1905)'s bare STR where *in-principle*, observers will never agree on the numerical value of any length and time interval as measured in their respective frames of reference.

4 Conclusion

Assuming the correctness (acceptability) of what has been presented herein, we hereby make the following conclusions:

1. Without having to drastically modify the Lorentz transformations – as demonstrated herein; it is possible to achieve a STR in which observers will always agree *in-principle* on the numerical value of any length and time interval.

2. In its bare form, the new STR does not have a provision for a special length or time-scale except when justified – as has been done in the reading Nyambuya (2010); on the bases of a number theoretic approach where the speed of light is assumed to be the optimum speed in the Universe.

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