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FRW Holographic Dark Energy Cosmological Model in Brans-Dicke Theory of Gravitation

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Abstract

In this paper, we have investigated five dimensional FRW metric with holographic dark energy in the frame work of Brans and Dicke scalar–tensor theory of gravitation. To obtain a determinate solution of the field equations we have used a particular relation between scalar field ϕ and the scale factor $a(t)$ of the universe. Also, some important properties of the model including look-back time, distance modulus and luminosity distance versus red shift with their significances are discussed.

Keywords: Five-dimensional FRW metric, holographic dark energy, perfect fluid, Brans - Dicke theory.

1. Introduction

The discovery of the accelerated mode of expansion of the Universe stands as a major breakthrough of the observational cosmology. Survey of cosmological distant type Ia supernovae (SNe Ia; Riess et al 1998; Perlmutter etal 1999) indicated the presence of a new unaccounted-for Dark energy (DE) that opposes the self-attractions of matter and causes the expansion of Universe to accelerate. This acceleration is realized with negative pressure and positive energy density that violate the strong energy condition. This violation gives a reverse gravitational effect. Due to this effect, the Universe gets a jerk and the transition from the earlier deceleration phase to the recent acceleration phase takes place (Caldwell et al. 2006). The cause of this sudden transition and the source of accelerated expansion are still unknown. The state of the art in cosmology has led to the following present distribution of the energy densities of the Universe: 4% for baryonic matter, 23% for non baryonic dark matter and 73% so-called DE (Spergel et al. 2007).

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The holographic principle emerged in the context of black-holes, where it was noted that a local quantum field theory cannot fully describe the black holes (Enqvist et al. 2005). Some long standing debates regarding the time evolution of a system, where a black hole forms and then evaporates, played the key role in the development of the holographic principle (Thorlocius 2004; Hooft 1993; Susskind 1995). Cosmological versions of holographic principle have been discussed in various literatures (e.g., Fischler and Susskind (1998); Tavakol and Ellis (1999); Easther and Lowe 1999). Easther and Lowe (1999) proposed that the holographic principle be replaced by the generalized second law of thermodynamics when applied to time dependent back grounds and found that the proposition agreed with the cosmological holographic principle proposed by Fischler and Susskind (1998) for an isotropic open and flat universe with a fixed equation of state. Verlinde (2000) studied the holographic principle in the context of an (n+1) dimensional radiation dominated closed FRW universe.

Numerous cosmological observations have established the accelerated expansion of the universe (Wang et al. 2005; Gong 2004). Since it has been proven that the expansion of the universe is accelerated, the physicists and astronomers started considering the dark energy cosmological observations indicated that at about 2/3 of the total energy of the universe is attributed by dark energy and 1/3 is due to dark matter Zhang (2005). In recent times, considerable interest has been stimulated in explaining the observed dark energy by the holographic dark energy model (Enqvist et al. 2005; Zhang 2005; Pavon and Zimdahl 2005). An approach to the problem of dark energy arises from the holographic principle stated in the first paragraph. For an effective field theory in a box size L with UV cut off Λ_c, the entropy $L^3\Lambda_c^3$. The non-extensive scaling postulated by Bekenste in suggested that quantum theory breaks down in large volume (Zhang 2005).

To reconcile this breakdown, Cohen et al. (1999) pointed out that in quantum field theory a short distance (UV) cut-off is related to a long distance (IR) cut-off due to the limit set by forming a black hole. Taking the whole universe into account the largest IR cut-off L is chosen by saturating the inequality so that we get the holographic dark energy density as Zhang (2005) $\rho_\Lambda = 3c^2 M_p^2 L^{-2}$, where c is a numerical constant and $M_p = 1/\sqrt{8\pi G}$ is the reduced Plank mass. On the basis of the holographic principle proposed by Fischler and Susskind (1998) several others have studied holographic model for dark energy Gong (2004). Employment of Friedman equation Setare (2007) $\rho = 3M_p^2 H^2$, where ρ is the total energy density and taking $L = H^{-1}$ one can find $\rho_m = 3(1-c^2)M_p^2 H^2$. Thus either ρ_m or ρ_Λ behaves like H^2. Thus, dark energy results as pressure less. But, neither dark energy, nor dark matter has laboratory evidence for its existence directly. Thus, Cardone et al. (2004) and Bento et al (2004) proposed unified dark matter/energy scenario in which two dark components are different manifestations of a single cosmic fluid. Some interesting examples of such an unification are the generalized Chaplygin gas, the tachyonic field, and the condensate cosmology (Cardone et al.2004).

Recently holographic principle is incorporated in cosmology (Hsu 2004; Li 2004) to track the dark energy content of the universe. This principle was first put forward by G.'t Hooft(2009) in the context of black hole physics. According to the holographic principle, the entropy of a system scales not with its volume, but with its surface area.In the cosmological context, Fischler and Susskind (1998) have proposed a new version of the holographic principle, viz. at any time during cosmological evolution, the gravitational entropy within a closed surface should not be always larger than the particle entropy that passes through the past light-cone of that surface. In the context of the dark energy problem, though the holographic principle proposes a relation between the holographic dark energy density ρ_Λ and the Hubble parameter H as $\rho_\Lambda = H^2$, it does not contribute to the present accelerated expansion of the universe.

Li (2004) has obtained an accelerating universe considering event horizon as the cosmological scale. The model is consistent with the cosmological observations. Granda and Oliveros (2008) proposed a holographic density of the form $\rho_\Lambda \approx \alpha H^2 + \beta \dot{H}$ where H is the Hubble parameter and α, β are constants which must satisfy the restrictions imposed by the current observational data. They showed that this new model of dark energy represents the accelerated expansion of the universe and is consistent with the current observational data. Granda and Oliveros (2009) have also studied the correspondence between the quintessence, tachyon, *k*-essence and dilation dark energy models with this holographic dark energy model in the flat FRW universe. Surajit (2009), Farajollahi et al. (2012), Debnath (2012), Malekjani (2013) are some of the authors who have investigated several aspects of holographic dark energy. Recently, Kiran et al. (2014, 2015) have studied minimally interacting dark energy models in some scalar tensor theories. Adhav et al. (2015) have discussed interacting dark matter and holographic dark energy in Bianchi type-V universe.

The dimensionality of the world has long been a subject of discussion due to fact that our sense perceived only four dimensions, but there is nothing in the equation of relativity which restricts them to four dimensions. Witten (1984), Applequist et al. (1987), Chudos and Detweiler (1980) and Marciano (1984) are some of the authors who have initiated the discussion of higher dimensional cosmological models. A number of authors (Sahdev 1984; Chatterjee 1993; Emelyanov et al.1986) have studied physics of the universe in higher-dimensional space-time.

These models are believed to be physical relevance possibly at the early times before the universe has undergone compactification transitions. Recently, Rao et al. (2014) have obtained five dimensional bulk viscous cosmological models with wet dark fluid in Saez-Ballester theory of gravitation.

Brans-Dicke theory of gravitation is a natural extension of general relativity which introduces an additional scalar field ϕ besides the metric tensor g_{ij} and dimensionless coupling constant ω. The Brans – Dicke field equations for combined scalar and tensor field are given by

$$G_{ij} = -8\pi\phi^{-1}T_{ij} - \omega\phi^{-2}\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,r}\phi^{,r}\right) - \phi^{-1}(\phi_{i;j} - g_{ij}\phi_{;r}{}^{,r}) \tag{1.1}$$

and

$$\phi_{;r}{}^{,r} = 8\pi(3+2\omega)^{-1}T \tag{1.2}$$

where $G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij}$ is an Einstein tensor, R is the scalar curvature, ω and n are constants, T_{ij} is the stress energy tensor of the matter and comma and semicolon denote partial and covariant differentiation respectively.

Also, we have energy – conservation equation

$$T^{ij}_{;j} = 0 \tag{1.3}$$

This is a consequence of the field equations (1.1) and (1.2).

Several aspects of Brans-Dicke cosmology have been extensively investigated by many authors. The work of Singh and Rai (1983) gives a detailed survey of Brans-Dicke cosmological models discussed by several authors. Nariai(1972), Belinskii and Khalatnikov (1973), Reddy and Rao(1981), Banerjee and Santos (1982), Singh et al. (1983), Shri Ram (1983), Shri Ram and Singh (1984), Berman etal. (1989), Reddy (2003), Reddy et al. (2007) and Adhav et al. (2007) are some of the authors who have investigated several aspects of this theory. Rao and Vijaya santhi (2011) have obtained Bianchi type-II, VIII and IX string cosmological models in Brans - Dicke theory of gravitation. Recently, Rao et al. (2012) have discussed LRS Bianchi type-I dark energy cosmological model in Brans - Dicke theory of gravitation.

Motivated by the above discussions and investigations, in this paper, we propose a cosmological model for minimally interacting holographic dark energy in Brans - Dicke scalar tensor theory of gravitation. The plan of the paper is as follows. In section 2, we have obtained the Brans - Dicke field equations for five dimensional FRW metric in the presence of matter and holographic dark energy. In section 3, we obtained the solution of the field equations. In section 4, we discuss some important properties of the obtained model. Some conclusions are presented in the last section.

2. Metric and Energy Momentum Tensor

We consider spatially homogeneous five dimensional FRW metric in the form

$$ds^2 = -dt^2 + R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right] + A^2(t)d\mu^2 \tag{2.1}$$

where $R(t)$ is the scale factor and $k = 0, -1 \, and \, +1$ is the curvature parameter for flat, open and closed Universe, respectively. The fifth coordinate μ is also assumed to be space like coordinate.

The energy momentum tensors for matter and the holographic energy are defined as

$$T_{ij} = \rho_m u_i u_j \tag{2.2}$$

and

$$\bar{T}_{ij} = (\rho_\Lambda + p_\Lambda) u_i u_j + g_{ij} p_\Lambda \tag{2.3}$$

where ρ_m , ρ_Λ are energy densities of matter and holographic dark energy and p_Λ is the pressure of holographic dark energy.

In a co moving coordinate system, we get

$$T_1^{\ 1} = T_2^{\ 2} = T_3^{\ 3} = T_5^{\ 5} = 0, \ T_4^{\ 4} = -\rho_m$$

And

$$\bar{T}_1^{\ 1} = \bar{T}_2^{\ 2} = \bar{T}_3^{\ 3} = \bar{T}_5^{\ 5} = p_\Lambda \, , \bar{T}_4^{\ 4} = -\rho_\Lambda \tag{2.4}$$

where the quantities ρ_m, ρ_Λ and p_Λ are functions of 't' only.

3. Solutions of the Field Equations

The field equations for the metric (2.1) with the help of equations (2.2) - (2.4) can be written as

$$\frac{3\dot{R}\dot{A}}{RA}+\frac{3\dot{R}^2}{R^2}-\frac{\omega\dot{\phi}^2}{2\phi^2}+\frac{3\dot{R}\dot{\phi}}{R\phi}+\frac{\dot{A}\dot{\phi}}{A\phi}+\frac{3k}{R^2}=8\pi\phi^{-1}\left(\rho_m+\rho_\Lambda\right) \tag{3.1}$$

$$\frac{3\ddot{R}}{R}+\frac{3\dot{R}^2}{R^2}+\frac{\omega\dot{\phi}^2}{2\phi^2}+\frac{\ddot{\phi}}{\phi}+\frac{3\dot{R}\dot{\phi}}{R\phi}+\frac{3k}{R^2}=-8\pi\phi^{-1}p_\Lambda \tag{3.2}$$

$$\frac{\ddot{A}}{A}+\frac{2\ddot{R}}{R}+\frac{2\dot{R}\dot{A}}{RA}+\frac{\dot{R}^2}{R^2}+\frac{\omega\dot{\phi}^2}{2\phi^2}+\frac{\ddot{\phi}}{\phi}+\frac{2\dot{R}\dot{\phi}}{R\phi}+\frac{\dot{A}\dot{\phi}}{A\phi}+\frac{k}{R^2}=-8\pi\phi^{-1}p_\Lambda \tag{3.3}$$

$$\ddot{\phi}+\dot{\phi}\left(\frac{3\dot{R}}{R}+\frac{\dot{A}}{A}\right)=8\pi(3+2\omega)^{-1}\left(\rho_m+\rho_\Lambda-4p_\Lambda\right) \tag{3.4}$$

$$\dot{\rho}_m+\dot{\rho}_\Lambda+\left(\frac{3\dot{R}}{R}+\frac{\dot{A}}{A}\right)\left(\rho_m+\rho_\Lambda+p_\Lambda\right)=0 \tag{3.5}$$

here the over head dot denotes differentiation with respect to $'t'$.

By taking the transformation $dt=R^3AdT$, the above field equations (3.1) to (3.5) can be written as

$$\frac{A''}{A}+\frac{2R''}{R}-\frac{3R'A'}{RA}-\frac{A'^2}{A^2}-\frac{5R'^2}{R^2}+\frac{\omega\phi'^2}{2\phi^2}+\frac{\phi''}{\phi}-\frac{R'\phi'}{R\phi}+kR^4A^2=-8\pi\phi^{-1}p_\Lambda R^6A^2 \tag{3.6}$$

$$\frac{3R'A'}{RA}+\frac{3R'^2}{R^2}-\frac{\omega\phi'^2}{2\phi^2}+\frac{3R'\phi'}{R\phi}+\frac{A'\phi'}{A\phi}+3kR^4A^2=8\pi\phi^{-1}\left(\rho_m+\rho_\Lambda\right)R^6A^2 \tag{3.7}$$

$$\frac{3R''}{R}-\frac{3R'A'}{RA}-\frac{6R'^2}{R^2}+\frac{\omega\phi'^2}{2\phi^2}+\frac{\phi''}{\phi}-\frac{A'\phi'}{A\phi}+3kR^4A^2=-8\pi\phi^{-1}p_\Lambda R^6A^2 \tag{3.8}$$

$$\phi''=-8\pi(3+2\omega)^{-1}\left(4p_\Lambda-\rho_m-\rho_\Lambda\right)R^6A^2 \tag{3.9}$$

here the over head dash denotes differentiation with respect to $'T'$.

We can write the conservation equation (1.3) of the matter and dark energy as

$$\rho_m'+\rho_\Lambda'+\left(\frac{3R'}{R}+\frac{A'}{A}\right)\left(\rho_m+\rho_\Lambda+p_\Lambda\right)=0 \tag{3.10}$$

Here we are considering the minimally interacting matter and holographic dark energy components. Hence both components conserve separately, so that we have (Sarkar, 2014)

$$\rho'_m + \left(\frac{3R'}{R} + \frac{A'}{A}\right)\rho_m = 0 \tag{3.11}$$

$$\rho'_\Lambda + \left(\frac{3R'}{R} + \frac{A'}{A}\right)(\rho_\Lambda + p_\Lambda) = 0 \tag{3.12}$$

From the field equations (3.6)-(3.9), we get

$$\frac{R''}{R} - \frac{2(2m+3)}{(m+3)}\frac{R'^2}{R^2} + \frac{\omega}{2(m+3)}\frac{\phi'^2}{\phi^2} + \frac{3kR^{4+2m}}{m+3} = \frac{(2\omega-1)}{3(m+3)}\frac{\phi''}{\phi} \tag{3.13}$$

The field equations (3.6)-(3.9) are four independent equations with six unknowns $A, R, p_\Lambda, \rho_\Lambda, \rho_m$ and ϕ. So, in order to get a deterministic solution we take the following possible physical conditions:

1. The shear scalar σ is proportional to scalar expansion θ, which leads to the linear relationship between the metric potentials R and A, i.e.

$$A = R^m, \tag{3.14}$$

where $m \neq 0$ is an arbitrary constant.

2. Relation between scalar field ϕ and the scale factor of the universe $a(t)$ given by (Pimental 1985; Johri and Kalyani 1994)

$$\phi = \phi_0 a^n, \tag{3.15}$$

where ϕ_0 and $n > 0$ are constants.

Using (3.13) and (3.15), we get

$$R = [(m+2)(k_5 T + k_6)]^{\frac{-1}{m+2}} \tag{3.16}$$

and

$$A=\left[(m+2)(k_5T+k_6)\right]^{\frac{-m}{m+2}} \tag{3.17}$$

where $k_1=\frac{12-n(2\omega-1)}{12}$; $k_2=\frac{2(2m+3)}{3(m+3)}-\frac{n^2\omega(m+3)}{32}-\frac{n(2\omega-1)}{12}+\frac{n^2(2\omega-1)(m+3)}{48}$;

$k_3=\frac{3k}{m+3}$; $k_4=m-\frac{k_2}{k_1}$; $k_5=\left(\frac{k_3}{k_1(3+k_4)}\right)^{\frac{1}{2}}$ and k_6 is an integration constant.

The scalar field ϕ is given by

$$\phi=\phi_0\left[(m+2)(k_5T+k_6)\right]^{\frac{-(m+3)}{4(m+2)}} \tag{3.18}$$

The Hubble parameter is one of the most important numbers in cosmology as it is used to estimate the size and age of the universe. It indicates the rate at which the universe is expanding.

The directional Hubble parameters H_1, H_2, H_3 & H_4 are given by

$$H_1=H_2=H_3=\frac{-k_5}{(m+2)(k_5T+k_6)} \;\&\; H_4=\frac{-mk_5}{(m+2)(k_5T+k_6)}$$

Therefore the generalized mean Hubble parameter (H) is

$$H=\frac{1}{4}(H_1+H_2+H_3+H_4)=\frac{-(m+3)k_5}{4(m+2)(k_5T+k_6)} \tag{3.19}$$

For $M_p^{-2}=8\pi G=1$ (Granda and Oliveros 2008), the holographic dark energy density is given by

$$\rho_\Lambda=\frac{2}{\alpha-\beta}\left(\dot{H}+\frac{3\alpha}{2}H^2\right), \tag{3.20}$$

where α,β are constants and H is the average Hubble's parameter.

From the equations (3.19) and (3.20), we get

$$\rho_\Lambda=\frac{8k_5^2(m+3)(m+2)+3\alpha\, k_5^2(m+3)^2}{16(\alpha-\beta)(m+2)^2(k_5T+k_6)^2} \tag{3.21}$$

From equations (3.7) - (3.15), we get

$$\rho_m = \frac{3k + k_5^2 k_8}{8\pi\phi_0^{-1}\left[(m+2)(k_5T + k_6)\right]^{2+\frac{(m+3)(n-8)}{4(m+2)}}} - \frac{8k_5^2(m+3)(m+2) + 3\alpha k_5^2(m+3)^2}{16(\alpha-\beta)(m+2)^2(k_5T+k_6)^2} \tag{3.22}$$

Using equations (3.8) - (3.17), we calculate

$$p_\Lambda = \frac{-\left(3k + k_5^2 k_7\right)}{8\pi\phi_0^{-1}\left[(m+2)(k_5T + k_6)\right]^{2+\frac{(m+3)(n-8)}{4(m+2)}}} \tag{3.23}$$

The barotropic equation of state $p_\Lambda = w_\Lambda \rho_\Lambda$ is

$$w_\Lambda = \frac{-16(\alpha-\beta)\left(3k + k_5^2 k_7\right)}{8\pi\phi_0 k_5^2(m+3)\left[8(m+2) + 3\alpha(m+3)\right]\left[(m+2)(k_5T+k_6)\right]^{\frac{n(m+3)-8}{4(m+2)}-2}} \tag{3.24}$$

The coincident parameter is

$$r = \frac{\rho_\Lambda}{\rho_m} = -\frac{16(\alpha-\beta)\left(3k + k_5^2 k_7\right)\left[(m+2)(k_5T+k_6)\right]^{-\frac{(m+3)(n-8)}{4(m+2)}}}{16(\alpha-\beta)\left(3k + k_5^2 k_8\right) - 8\pi\phi_0^{-1}k_5^2(m+3)\left[8(m+2)+3\alpha(m+3)\right]} \tag{3.25}$$

Now the metric (2.1) can be written as

$$ds^2 = -dt^2 + \left[(m+2)(k_5T+k_6)\right]^{\frac{-2}{m+2}}\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2\right] + \left[(m+2)(k_5T+k_6)\right]^{\frac{-2m}{m+2}} d\mu^2 \tag{3.26}$$

Thus the metric (3.26) together with (3.21) - (3.24) constitutes a five dimensional FRW holographic dark energy cosmological model in Brans-Dicke theory of gravitation.

4. Some physical and kinematical properties of the model

The spatial volume and average scale factor for the model (3.26) are given by

$$V = \left[(m+2)(k_5T+k_6)\right]^{\frac{-(m+3)}{m+2}} \tag{4.1}$$

$$a(t) = V^{\frac{1}{4}} = \left[(m+2)(k_5T+k_6)\right]^{\frac{-1}{4}\left(\frac{m+3}{m+2}\right)} \tag{4.2}$$

The expression for expansion scalar θ calculated for the flow vector u^i is given by

$$\theta = \frac{-(m+3)k_5}{(m+2)(k_5 T + k_6)} \tag{4.3}$$

and the shear scalar σ is given by

$$\sigma^2 = \frac{7}{18}\frac{(m+3)^2 k_5^2}{\left[(m+2)(k_5 T + k_6)\right]^2} \tag{4.4}$$

The deceleration parameter q is given by

$$q = -\left(\frac{4m+9}{m+3}\right) \tag{4.5}$$

From equation (4.5), it is observed that for all positive values of 'm' the deceleration parameter $'q'$ is negative, hence it represents an accelerated expansion of the Universe.

The average anisotropy parameter is given by

$$A_m = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{\Delta H_i}{H}\right)^2 = \frac{3(m-1)^2}{(m+3)^2} \tag{4.6}$$

The Jerk parameter is given by

$$J = \frac{1}{H^3}\frac{\dddot{a}}{a} = \frac{32(m+2)(2m+5)}{(m+3)^2} + 1 \tag{4.7}$$

The Hubble parameter H is used to estimate the size and age of the universe. It indicates the rate at which the universe is expanding. From equation (3.19), the Hubble parameter is given by

$$H = \frac{-(m+3)k_5}{4(m+2)(k_5 T + k_6)} \tag{4.8}$$

Hence

$$\frac{H}{H_0} = \frac{(k_5 T_0 + k_6)}{(k_5 T + k_6)} \tag{4.9}$$

where H_0 is the present value of Hubble parameter.

The red shift we measure for a distant source is directly related to the scale factor of the universe at the time of the photons emitted from the source. The scale factor a and red shift z are related through the equation.

Look-back time-red shift: The look-back time, $\Delta t = t_0 - t(z)$ is the difference between the age of the universe at present time (z = 0) and the age of the universe when a particular light ray at red shift z, the expansion scalar of the universe $a(t_z)$ is related to a_0 by $1+z=\frac{a_0}{a}$, where a_0 is the present scale factor. Therefore,

$$a = \frac{a_0}{1+z} \tag{4.10}$$

where a_0 is the present value of scale factor.

Using equation (4.2), the above expression (4.10) can be rewritten as

$$\frac{a_0}{a} = 1+z = \left(\frac{k_5 T_0 + k_6}{k_5 T + k_6}\right)^{-\frac{1}{4}\left(\frac{m+3}{m+2}\right)} \tag{4.11}$$

Luminosity distance:

The distance modulus (D) is given by

$$D(z) = 5\log d_L + 25 \tag{4.12}$$

where d_L stands for the luminosity distance.

Luminosity distance is defined as the distance which will preserve the validity of the inverse law for the fall of intensity and given by

$$d_L = r_1(1+z)a_0 \tag{4.13}$$

where r_1 is the radial coordinate distance of the object at light emission and, is given by

$$r_1 = \int_t^{t_0} \frac{dt}{a} = \int_t^{t_0} \left[(m+2)(k_5 T + k_6)\right]^{\frac{m+3}{4(m+2)}} \tag{4.14}$$

The luminosity distance

$$d_L = \frac{-4k_5^2(m+3)(m+2)^2}{5m+11}(1+z)\left(H_0^{-1} - (1+z)a_0 H^{-1}\right) \tag{4.15}$$

The distance modulus

$$D(Z) = 5\log d_L + 25 \tag{4.16}$$

From equations (4.15) and (4.16), we get

$$D(z) = 5\log \frac{-4k_5^2(m+3)(m+2)^2}{5m+11}(1+z)\left(H_0^{-1} - (1+z)a_0 H^{-1}\right) + 25 \tag{4.17}$$

The tensor of rotation

$w_{ij} = u_{i,j} - u_{j,i}$ is identically zero and hence this universe is non-rotational.

5. Conclusions

In this paper, we have presented a five dimensional FRW holographic dark energy cosmological model in Brans-Dicke theory of gravitation. Observational data also suggest that dark energy is responsible for gearing up the universe some five billion years ago. But at the time the universe need not to be isotropic. So, we have assumed the universe to be anisotropic and consider the five-dimensional FRW metric filled with matter and holographic dark energy. The following are the observations and conclusions:

- The volume decreases with the increase of time i.e., as $T \to \infty$, the spatial volume vanishes. The expansion scalar θ, shear scalar σ and the Hubble parameter H are vanish with the increase of time.
- From (4.6), one can observe that $A_m \neq 0$ and this indicates that this present model is anisotropic throughout the evaluation of the universe.
- From (4.5), we observe that the deceleration parameter appears with negative sign implies accelerating expansion of the universe, which is consistent with the present day observations.
- From (3.21) & (3.22), we observe that the matter density and holographic dark energy density decreases with the increase of time.
- We have obtained expressions for look-back time ΔT, distance modulus $D(z)$ and luminosity distance d_L versus red shift and discussed their significance.
- Thus the model presented here is shearing, non-rotating and accelerating in a standard way.

References

Adhav, K.S., Nimkar, A.S., Ugale, M.R., Dawande, M.V.: Astrophys. Space Sci. **310**, 231 (2007)

Adhav,K.S., Munde,S.L.,Tayade, G.B., Bokey V.D: Astrophys Space Sci **359**,24 (2015)

Apple, T., Chodos, A., Freund, P.G.O.: Modern Kaluza-KleinTheories. Addison-Wesley, Reading (1987)

Banerjee, A., Santos, N.O.: Nuovo Cimento **67B**, 31 (1982)

Belinskii, V.A., Khalatnikov, I.M.: Sov. Phys. JETP **36**, 591 (1973)

Bento, M.C., Bertolami, O., Sen, A.A.: Phys. Rev. D **70**, 083519(2004)

Berman, M.S., Som, M.M., de Mello Gomide, F.: Gen. Relativ. Gravit. **21**, 287 (1989)

Caldwell, R.R., Komp, W., Parker, L., Vanzella, D.A.T.: Phys. Rev. D**73**, 023513 (2006)

Cardone, V.F., Troisi, A., Copoziello, S.: Phys. Rev. D **69**, 083517(2004)

Chatterjee, S., Bhui, B.: Int. J. Theor. Phys. **32**, 671 (1993)

Chodos, A., Detweller, S.: Phys. Rev. D **21**, 2167 (1980)

Cohen, A.G., et al.: Phys. Rev. Lett. **82**, 4971 (1999)

Debnath, U.: Astrophys Space Sci. **337**,503 (2012)

Easther, R., Lowe, D.: Phys. Rev. Lett. **82**, 4967 (1999)

Emelyanov, V.M., Nikitin, Yu.P., Rozental, J.L., Berkov, A.V.: Phys. Rep. **143**, 1 (1986)

Enqvist, K., Hannested, S., Sloth, M.S.: J. Cosmol. Astropart. Phys. **2**, 004 (2005)

Farajollahi,H., Sadeghi,J., Pourali, M.: Astrophys Space Sci. **341**,695 (2012)

Fischler, W., Susskind, L.: hep-th/9806039 (1998)

Gong, Y.: Phys. Rev. D **70**, 064029 (2004)

Granda, L.N., Oliveros, A.: Phys. Lett. B **669**, 275 (2008)

Granda, L.N., Oliveros, A.: Phys. Lett. B **671**, 199 (2009)

Hooft, G.T.: gr-qc/9310026 (1993)

Hsu, S.D.H.: Phys. Lett. B **594**, 13 (2004)

Johri, V.B., Kalyani, D.: Gen. Relativ. Gravit. **26**, 1217 (1994)

Kiran,M., Reddy, D.R.K., Rao, V.U.M.: Astrophys Space Sci.**354**,577 (2014).

Kiran,M., Reddy, D.R.K., Rao, V.U.M.: Astrophys Space Sci **356**,407(2015).

Li, M.: Phys. Lett. B **603**, 1 (2004)

Malekjani, M.: Astrophys Space Sci.**347**,405 (2013)

Marciano, W.J.: Phys. Rev. Lett. **52**, 498 (1984)

Nariai, H.: Prog. Theor. Phys. **47**, 1824 (1972)

Pavon, D., Zimdahl, W.: hep-th/0511053 (2005)

Perlmutter, S., et al.: Astrophys. J. **517**, 565 (1999)

Pimental, L.O.: Astrophys. Space Sci. **112**, 175–183 (1985)

Rao, V.U.M. and Vijaya Santhi, M.:ISRN Math.Phys. DOI:10.5402/2012/573967 (2011)

Rao, V.U.M., Vijaya Santhi, M.,Vinutha, T.and Sreedevi kumarai, G.: Int. J. Theor. Phys., **51**, 330 (2012)

Rao, V.U.M., Sireesha, K.V.S. and Suneetha, P.: Afr. Rev. Phys., **8**, 0042(2012)

Rao, V.U.M., Vijaya Santhi, M. and Sandhya Rani, N.: Prespacetime. J. 6, 226(2014)

Reddy, D.R.K., Rao, V.U.M.: J. Phys. A: Math. Gen. **14**, 1973 (1981)

Reddy, D.R.K.: Astrophys. Space Sci. **281**, 365 (2003)
Reddy, D.R.K., Naidu, R.L.: Astrophys. Space Sci. **307**, 395 (2007)
Riess, A. G. et al.: Astron. J. 116, 1009 (1998)
Sahdev, D.: Phys. Rev. D **30**, 2495 (1984)
Sarkar, S.: Astrophys. Space Sci. **349,** 985 (2014)
Setare, M.R.: Phys. Lett. B **648**, 329 (2007a)
Shri Ram: Gen. Relativ. Gravit. **15**, 635 (1983)
Shri Ram, Singh, D.K.: Astrophys. Space Sci. **103**, 21 (1984)
Singh, T., Rai, L.N.: Gen. Relativ. Gravit. **15**, 875 (1983)
Singh, T., Rai, L.N., Tarakeswara Singh: Astrophys. Space Sci. **96**, 95(1983)
Spregel, D.N., et al.: Astrophys. J. Suppl. **170**, 377 (2007)
Surajit Chattopadhyay · Ujjal Debnath Astrophys Space Sci. **319**, 183 (2009)
Susskind, L.: J. Math. Phys. **36**, 6377 (1995)
Tavakol, R., Ellis, G.: Phys. Lett. B **469**, 33 (1999)
’t Hooft, G.: (2009). arXiv:gr-qc/9310026
Thorlocius, L.: hep-th/0404098 (2004)
Verlinde, E.: hep-th/0008140 (2000)
Wang, B., Gong, Y., Abdalla, E.: Phys. Lett. B **624**, 141 (2005)
Witten, E.: Phys. Lett. B **144**, 351 (1984)
Zhang, X.: Int. J. Mod. Phys. D **14**, 1597 (2005)