Article

Domain Wall in Ruban's Spacetime

Harshal A.Nimkar*

Government Vidarbha Institute of Science & Humanities, Amravati, India

Abstract

We have presented Ruban's spacetime in the presence of domain wall. To obtain a determinant solution, relation between metric potential and $p=\rho$ is considered. The physical and geometrical aspects are also discussed.

Keywords: Ruban's metric, domain wall, gravitation, Universe.

1. Introduction

The study of Domain walls has received considerable attention in cosmology since they play an important role in structure formation and evolution of the universe [1-2]. Domain walls are topological defects associated with spontaneous symmetry breaking whose plausible production site in cosmological phase transitions in the early universe, like others kinds of topological defects such as monopoles, cosmic string and textures.

In general, domain walls are generated at rest in the very early universe at rest, with a curvature of the order of the characteristic scale at that moment. They are accelerated later if their interaction with other types of matter is small. When they are moving, their effective equation of state becomes [3] $p=\left(\vec{v}^2-\frac{2}{3}\right)\rho$.

Hence, the effective equation of state of a network of domain walls evolves from the rest case, $p=-\frac{2}{3}\rho$ to the relativistic case $p=\frac{\rho}{3}$ .When the domain walls reach the relativistic regime, their characteristic length becomes comparable to the Hubble radius. In many weak interacting domain walls model, the relativistic regime is reached when domain walls begin to dominate the matter content of the Universe [4]. However, domain walls must feel a friction force, proportional to their velocity, due to their interaction with the other components of the matter content of the universe.

The gravitational effects of domain wall have been extensively discuss by [5-7] in General Relativity. Relativistic some topological defects in the context of Bianchi space time have been obtained by [8-12]. Recently some authors [13-15] have investigated several aspects of domain walls in general theory of relativity and scalar tensor theory of gravitation. Also, exact solution

*Correspondence: Harshal A.Nimkar, Govt.Vidarbha Institute of Science & Humanities, Amravati, India.
E-mail: Harshalnimkar53@gmail.com

of the Einstein Maxwell equations in Ruban's back ground is determined by [16]. Very recently wet dark fluid cosmological model in Ruban's background have investigated by [17].
The purpose of the present work is to obtained Ruban's cosmological models in presence of domain wall. Our paper is organized as follows. In section II, we derive the metric and field equations. Section III is mainly concerned with the physical and kinematical properties of the model. The last section contains some conclusion.

2. The Metric and field equations

We consider the space- time of Ruban's [16] in the form

$$ds^2 = dt^2 - Q^2(x,t)dx^2 - R^2(t)(dy^2 + h^2 dz^2) \tag{1}$$

where $h(y) = \frac{\sin\sqrt{k}y}{\sqrt{k}} = \begin{matrix} \sin y & if & k=1 \\ y & if & k=0 \\ \sinh y & if & k=-1 \end{matrix}$

and k is the curvature parameter of the homogeneous 2-spaces t and x constants. The functions Q and R are free and will be determined.

There are two ways of studying thick domain walls. The first way is to solve gravitational field equations with an energy momentum tensor describing a self-interacting scalar field ϕ contained in a potential $V(\psi)$ given by

$$\psi,_i \psi,_j - g_{ij}\left[\frac{1}{2}\psi,_k \psi'^k - V(\psi)\right].$$

The second approach is to assume the stress energy tensor for domain walls of the form

$$T_{ij} = \rho\left(g_{ij} + w_i w_j\right) + p w_i w_j\,, \tag{2}$$

together with $w_i w^j = -1$.

Here ρ is the energy density of the domain walls, p is the pressure of the direction normal to the plane of the domain walls and w_i is a unit space like vector in the same direction.

Here, we use the second approach to study the thick domain walls.

Using comoving coordinate system, the non-vanishing components of T_i^j can be obtained as

$$T_1^1 = -P, \quad T_2^2 = T_3^3 = T_4^4 = \rho\,, T_i^j = 0 \qquad for\ i \neq j \tag{3}$$

For the energy momentum tensor (2) for the metric (1) with the help of (3), Einstein's field equations

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi T_{ij} \tag{4}$$

yield the following three independent equations

$$2\frac{\ddot{R}}{R}+\left(\frac{\dot{R}}{R}\right)^2+\frac{k}{R^2}=-p \tag{5}$$

$$\frac{\dot{R}\dot{Q}}{RQ}+\frac{\ddot{R}}{R}+\frac{\ddot{Q}}{Q}=\rho \tag{6}$$

$$2\frac{\dot{R}\dot{Q}}{RQ}+\left(\frac{\dot{R}}{R}\right)^2+\frac{k}{R^2}=\rho \tag{7}$$

Here over head dot represent partial differentiation with respect to t

To get a determinate solution one has to assume condition.

$$p=\rho \tag{8}$$

By using (8) and relation between metric potential

$$Q=x^n R^n \tag{9}$$

The solution of field equations (5-7), we have

$$R=N(k_1t+k_2)^{\frac{1}{M+1}} \tag{10}$$

$$Q=x^n N^n (k_1t+k_2)^{\frac{n}{M+1}} \quad , \tag{11}$$

where $M=\dfrac{n^2-n}{(n+2)}$ and $N=(M+1)^{\frac{1}{M+1}}$

$$p=\rho=\frac{=\left(-Mk_1+k_1n^2-k_1n\ M\right)^2}{(M+1)^2}\ \frac{1}{(k_1t+k_2)^2} \tag{12}$$

With the help of (10) and (11) the Metric (1) becomes

$$ds^2=dt^2-x^{2n}N^2n(k_1t+k_2)^{\frac{2n}{M+1}}dx^2-N^2(k_1t+k_2)^{\frac{2}{M+1}}(dy^2+h^2dz^2)$$

Through a proper choice of coordinates and constants the model can be written as

$$ds^2=\frac{dT^2}{k_1{}^2}-x^{2n}N^2(T)^{\frac{2}{M+1}}dx^2-N^2(T)^{\frac{2}{M+1}}(dy^2+h^2dz^2) \tag{13}$$

3. The physical and kinematical property

The physical quantities that are important in cosmology are spatial volume, Scalar Expansion and Shear Scalar and have the following expression for the model given by (13)

$$\text{Spatial volume:}\quad V=x^nN^{n+2}h(T)^{\frac{n+2}{M+1}} \tag{14}$$

$$\text{Scalar Expansion:}\ \theta=\left(\frac{n+2k_1}{M+1}\right)\frac{1}{T} \tag{15}$$

Shear Scalar: $$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{6}\left(\frac{n+2k_1}{M+1}\right)\frac{1}{T^2} \tag{16}$$

It may be observed that at initial moment when $T=0$. The special volume will be zero. While pressure density and tension density diverges. When $T \to o$, then scalar expansion and shear scalar tends to ∞.

For large value of T ($T \to \infty$) we observed that scalar expansion and shear scalar becomes zero.

4. Conclusion

In this paper, we have studied domain wall cosmological model in Ruban's spacetime. For finding the exact solution the relation between the metric potential and the equation of state is used. The model is free from initial singularities and they are expanding, shearing and non rotating in the standard way.

References

1.Vilenkin, A.: Phys. Rev.**D24**,2082(1981).
2. Goetz,G.:J.Math Phys.**31**, 2683(1990).
3.Julio Csar Fabris and serjio Vitorino de Borba Goncalves:Brazilian journal of physics, Vol.**33**,no.4,(2003).
4.Vilenkin, A. and Shellard, E.P.S.:Cosmic strings and other topological defects, Cambridge University press, Cambridge (1994).
5.Gott,J.R.:Astrophys,J 288,422(1985).
6.Letelier,P.S.:Phys.Rev.**D28**, 2414(1983).
7.Satchel, j.: Phys.Rev.**D21**, 2171(1980).
8.Krori,K.D. *et.al* :Gen.Rel.Grav.**22**,123(1990).
9.Reddy, D.R.K.,Naidu, R.L.: Astrophys space sci.**312**,99(2007).
10.Bhattacharjee, R. and Baruah,K.K.:J.Pure.Appl.Math.**32**,47(2001).
11.Reddy, D. R. K. ,Subha Rao, M. V.:Astrophys space sci.**302**,157-160(2006).
12.Adhav K.S., Nimkar A.S.,Naidu R.L.: Astrophys space sci.**312**,165-169(2007).
13.Adhav, K.S.,Mete, V.G., Pund, A.M., Raut,R.B.:JVR **5**,4,26-33(2010).
14. Adhav, K.S.,Nimkar, A.S.,Ugle, M. R.,Thakare, R.S.:JVR**6**,2,1-11(2011)
15. Pund, A. M., Avachar,G. R.: Prespace time journal Vol.**5**,3,207-211(2014).
16.Lima, J. A. S., Nobre, M. A. S.: Class. Quantum Grav.**7** 399-409 (1990).
17.Nimkar, A. S., Pund, A. M.: IOSR journal of mathematics vol.**11,** issue 4,ver II,47-50(2015).