Exploration

London-Proca-Hirsch Equations for Electrodynamics of Superconductors on Cantor Sets

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Abstract

In a recent paper published at *Advances in High Energy Physics* (AHEP) journal, Yang Zhao *et al*. derived Maxwell equations on Cantor sets from the local fractional vector calculus. It can be shown that Maxwell equations on Cantor sets in a fractal bounded domain give efficiency and accuracy for describing the fractal electric and magnetic fields. However, so far there is no derivation of equations for electrodynamics of superconductor on Cantor sets. Therefore, in this paper I present for the first time a derivation of London-Proca-Hirsch equations on Cantor sets. The name of London-Proca-Hirsch is proposed because the equations were based on modifying Proca and London-Hirsch's theory of electrodynamics of superconductor. Considering that Proca equations may be used to explain electromagnetic effects in superconductor, I suggest that the proposed London-Proca-Hirsch equations on Cantor sets can describe electromagnetic of fractal superconductors. It is hoped that this paper may stimulate further investigations and experiments in particular for fractal superconductor. It may be expected to have some impact to fractal cosmology modeling too.

Key Words: Cantor sets, Hirsch theory, London equations, local fractional vector calculus, Maxwell equations, Proca equations, electrodynamics, superconductor.

1. Introduction

According to J.E. Hirsch, from the outset of superconductivity research it was assumed that no electrostatic fields could exist inside superconductors and this assumption was incorporated into conventional London electrodynamics.[2] Hirsch suggests that there are difficulties with the two London equations. To summarize, London's equations together with Maxwell's equations lead to unphysical predictions.[1] Hirsch also propose a new model for electrodynamics for superconductors. [1][2]

The present paper is intended to be a follow-up paper of our four recent papers: one paper reviews Shpenkov's interpretation of classical wave equation and its role to explain periodic table of elements and other phenomena [11], and the second one presents a derivation of GravitoElectroMagnetic Proca equations in fractional space [12], the third one presents an outline of cosmology based on the concept of fractal vibrating string [13], and the fourth one presents a derivation of Proca equations on Cantor sets [14].

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In this regard, in a recent paper Yang Zhao et al. derived Maxwell equations on Cantor sets from the local fractional vector calculus.[3] It can be shown that Maxwell equations on Cantor sets in a fractal bounded domain give efficiency and accuracy for describing the fractal electric and magnetic fields. However, so far there is no derivation of equations for electrodynamics of superconductor on Cantor sets. Therefore, in this paper I present for the first time a derivation of London-Proca-Hirsch equations on Cantor sets. The name of London-Proca-Hirsch is proposed because the equations were based on modifying London equations, Proca equations and Hirsch's theory of electrodynamics of superconductor.

Therefore, the aim of the present paper is to propose a combined version of London-Proca-Hirsch model for electrodynamics of superconductor. Then I extend further this proposed model for electrodynamics of superconductor on Cantor sets. Considering that Proca equations may be used to explain electromagnetic effects in superconductor [4]-[8], I suggest that the proposed London-Proca-Hirsch equations on Cantor sets can describe electromagnetic of fractal superconductors. It is hoped that this paper may stimulate further investigations and experiments in particular for fractal superconductor. It may be expected to have some impact to fractal cosmology modeling too.

2. Hirsch's model to revise London's equations

According to J.E. Hirsch, from the outset of superconductivity research it was assumed that no electrostatic fields could exist inside superconductors and this assumption was incorporated into conventional London electrodynamics.[2] Hirsch suggests that there are difficulties with the two London equations. Therefore he concludes that London's equations together with Maxwell's equations lead to *unphysical* predictions.[1] However he still uses four- vectors J and A according to Maxwell's equations:

$$\Box^2 A = -\frac{4\pi}{c} J, \tag{1}$$

and

$$J - J_0 = -\frac{c}{4\pi\lambda_L^2}(A - A_0). \tag{2}$$

Therefore, Hirsch proposes a new fundamental equation for electrodynamics for superconductors as follows: [1]

$$\Box^2 (A - A_0) = \frac{1}{\lambda_L^2}(A - A_0), \tag{3a}$$

where

- London penetration depth λ_L is defined as follows:[2]

$$\frac{1}{\lambda_L^2} = \frac{4\pi n_s e^2}{m_e c^2},$$

(3b)

- And d'Alembertian operator is defined as: [1]
-

$$\Box^2 = \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial_t^2}.$$

(3c)

Then he proposes the following equations: [1]

$$\Box^2 (F - F_0) = \frac{1}{\lambda_L^2}(F - F_0),$$

(4)

and

$$\Box^2 (J - J_0) = \frac{1}{\lambda_L^2}(J - J_0),$$

(5)

where F is the usual electromagnetic field tensor and F_0 is the field tensor with entries $\vec{E}_0$ and 0 from $\vec{E}$ and $\vec{B}$ respectively when expressed in the reference frame at rest with respect to the ions.

In the meantime, it is known that Proca equations can also be used to described electrodynamics of superconductors, see [4]-[8]. The difference between Proca and Maxwell equations is that Maxwell equations and Lagrangian are based on the hypothesis that the photon has zero mass, but the Proca's Lagrangian is obtained by adding mass term to Maxwell's Lagrangian.[17] Therefore, the Proca equation can be written as follows:[17]

$$\partial_\mu F^{\mu\nu} + m_\gamma^2 A_\nu = \frac{4\pi}{c}J^\nu,$$

(6a)

where $m_\gamma = \dfrac{\omega}{c}$ is the inverse of the Compton wavelength associated with photon mass. [17] In terms of the vector potentials, equation (6a) can be written as [17]:

$$(\Box + m_\gamma)A_\mu = \frac{4\pi}{c}J_\mu.$$

(6b)

Similarly, according to Kruglov [15] the Proca equation for a free particle processing the mass m can be written as follows:

$$\partial_\nu \varphi_{\mu\nu}(x) + m^2 \varphi_\mu(x) = 0,$$

(6)

Now, the similarity between equations (1) and (6b) are remarkable with exception that equation (1) is in quadratic form. Therefore I propose to consider a modified form of Hirsch's model as follows:

$$(\Box^2 - m_\gamma^2)(F - F_0) = \frac{1}{\lambda_L^2}(F - F_0),$$ (8a)

and

$$(\Box^2 - m_\gamma^2)(J - J_0) = \frac{1}{\lambda_L^2}(J - J_0).$$ (8b)

The relevance of the proposed new equations in lieu of (4)-(5) should be verified by experiments with superconductors [16]. For convenience, the equations (8a)-(8b) can be given a name: London-Proca-Hirsch equations.

3. A review of previous result - Maxwell equations on Cantor sets

I will not re-derive Maxwell equations here. For a good reference on Maxwell equations, see for example Julian Schwinger et al.'s book: *Classical Electrodynamics* [9].

Zhao et al. were able to write the local fractional differential forms of Maxwell equations on Cantor sets as follows [3, p.4-5]:

- Gauss's law for the fractal electric field: $\nabla^\alpha \cdot D = \rho$, (9)

- Ampere's law in the fractal magnetic field: $\nabla^\alpha \times H = J_a + \dfrac{\partial^\alpha D}{\partial t^\alpha}$, (10)

- Faraday's law in the fractal electric field: $\nabla^\alpha \times E = -\dfrac{\partial^\alpha B}{\partial t^\alpha}$, (11)

- magnetic Gauss's law in the fractal magnetic field: $\nabla^\alpha \cdot B = 0$, (12)

and the continuity equation can be defined as:

$$\nabla^\alpha \cdot J = -\frac{\partial^\alpha \rho}{\partial t^\alpha},$$ (13)

where $\nabla^\alpha \cdot r$ and $\nabla^\alpha \times r$ are defined as follows:

2.1. In Cantor coordinates [10, p. 2]:

$$\nabla^\alpha \cdot u = div^\alpha u = \frac{\partial^\alpha u_1}{\partial x_1^\alpha} + \frac{\partial^\alpha u_2}{\partial x_2^\alpha} + \frac{\partial^\alpha u_3}{\partial x_3^\alpha},$$ (14)

$$\nabla^\alpha \times u = curl^\alpha u = \left(\frac{\partial^\alpha u_3}{\partial x_2^\alpha} - \frac{\partial^\alpha u_2}{\partial x_3^\alpha}\right)e_1^\alpha + \left(\frac{\partial^\alpha u_1}{\partial x_3^\alpha} - \frac{\partial^\alpha u_3}{\partial x_1^\alpha}\right)e_2^\alpha + \left(\frac{\partial^\alpha u_2}{\partial x_1^\alpha} - \frac{\partial^\alpha u_1}{\partial x_2^\alpha}\right)e_3^\alpha.$$ (15)

2.2. In Cantor-type cylindrical coordinates [3, p.4]:

$$\nabla^{\alpha} \cdot r = \frac{\partial^{\alpha} r_R}{\partial R^{\alpha}} + \frac{1}{R^{\alpha}} \frac{\partial^{\alpha} r_{\theta}}{\partial \theta^{\alpha}} + \frac{r_R}{R^{\alpha}} + \frac{\partial^{\alpha} r_z}{\partial z^{\alpha}}, \tag{16}$$

$$\nabla^{\alpha} \times r = \left(\frac{1}{R^{\alpha}} \frac{\partial^{\alpha} r_{\theta}}{\partial \theta^{\alpha}} - \frac{\partial^{\alpha} r_{\theta}}{\partial z^{\alpha}} \right) e_R^{\alpha} + \left(\frac{\partial^{\alpha} r_R}{\partial z^{\alpha}} - \frac{\partial^{\alpha} r_z}{\partial R^{\alpha}} \right) e_{\theta}^{\alpha} + \left(\frac{\partial^{\alpha} r_{\theta}}{\partial R^{\alpha}} + \frac{r_R}{R^{\alpha}} - \frac{1}{R^{\alpha}} \frac{\partial^{\alpha} r_R}{\partial \theta^{\alpha}} \right) e_z^{\alpha}. \tag{17}$$

3. London-Proca-Hirsch Equations on Cantor Sets

It can be shown that Proca equations can be derived from first principles [6], and also that Proca equations may have link with Klein-Gordon equation [7]. However, in this paper I will not attempt to re-derive Proca equations. Instead, I will use Proca equations as described in [6]. Then I will derive the London-Proca-Hirsch equations on Cantor Sets, in accordance with Zhao *et al.*'s approach as outlined above [3].

According to Blackledge, Proca equations can be written as follows [7]:

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} - \kappa^2 \phi, \tag{18}$$

$$\nabla \cdot \vec{B} = 0, \tag{19}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \tag{20}$$

$$\nabla \times \vec{B} = \mu_0 j + \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} + \kappa^2 \vec{A}, \tag{21}$$

where:

$$\nabla \phi = -\frac{\partial \vec{A}}{\partial t} - \vec{E}, \tag{22}$$

$$\vec{B} = \nabla \times \vec{A}, \tag{23}$$

$$\kappa = \frac{mc_0}{\hbar}. \tag{24}$$

Therefore, by using the definitions in equations (14)-(17), we can arrive at Proca equations on Cantor sets from (18) through (23), as follows:

$$\nabla^{\alpha} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} - \kappa^2 \phi \,, \qquad (25)$$

$$\nabla^{\alpha} \cdot \vec{B} = 0 \,, \qquad (26)$$

$$\nabla^{\alpha} \times \vec{E} = -\frac{\partial^{\alpha} \vec{B}}{\partial t^{\alpha}} \,, \qquad (27)$$

$$\nabla^{\alpha} \times \vec{B} = \mu_0 j_a + \varepsilon_0 \mu_0 \frac{\partial^{\alpha} \vec{E}}{\partial t^{\alpha}} + \kappa^2 \vec{A} \,, \qquad (28)$$

where:

$$\nabla^{\alpha} \phi = -\frac{\partial^{\alpha} \vec{A}}{\partial t^{\alpha}} - \vec{E} \,, \qquad (29)$$

$$\vec{B} = \nabla^{\alpha} \times \vec{A} \,, \qquad (30)$$

and Del operator $\nabla^{\alpha} \phi$ can be defined as follows [10, p.2]:

$$\nabla^{\alpha} \phi = \frac{\partial^{\alpha} \phi}{\partial x_1^{\alpha}} e_1^{\alpha} + \frac{\partial^{\alpha} \phi}{\partial x_2^{\alpha}} e_2^{\alpha} + \frac{\partial^{\alpha} \phi}{\partial x_3^{\alpha}} e_3^{\alpha} \,. \qquad (31)$$

Since according to Blackledge, the Proca equations can be viewed as a *unified wavefield* model of electromagnetic phenomena [7], therefore we can also regard the Proca equations on Cantor sets as a further generalization of Blackledge's *unified wavefield* model.

Now, having defined Proca equations on Cantor Sets, we are ready to write down London-Proca-Hirsch on Cantor sets using the same definition, as follows:

$$(\square_{\alpha}^2 - \kappa^2)(F - F_0) = \frac{1}{\lambda_L^2}(F - F_0), \qquad (32)$$

And

$$(\square_{\alpha}^2 - \kappa^2)(J - J_0) = \frac{1}{\lambda_L^2}(J - J_0), \qquad (33)$$

where

$$\square_{\alpha}^2 = \nabla^{\alpha 2} - \frac{1}{c^2} \frac{\partial^{\alpha 2}}{\partial_t^{\alpha 2}} \,. \qquad (34)$$

As far as I know, the above London-Proca-Hirsch equations on Cantor Sets have never been presented elsewhere before. Provided the above equations can be verified with experiments, they can be used to describe electrodynamics of fractal superconductors on Cantor sets.

As a last note, it seems interesting to remark here that Kruglov [15] has derived a square-root of Proca equations as a possible model for hadron mass spectrum, therefore perhaps equations (32)-(34) may be factorized too to find out a model for hadron masses (on Cantor sets). However, this problem is left for other paper.

Conclusions

In a recent paper Yang Zhao *et al.* derived Maxwell equations on Cantor sets from the local fractional vector calculus. It can be shown that Maxwell equations on Cantor sets in a fractal bounded domain give efficiency and accuracy for describing the fractal electric and magnetic fields. However, so far there is no derivation of equations for electrodynamics of superconductor on Cantor sets. Therefore, in this paper I present for the first time a derivation of London-Proca-Hirsch equations on Cantor sets. The name London-Proca-Hirsch is proposed because the equations were based on modifying London equations, Proca equations and Hirsch's theory of electrodynamics of superconductor.

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