Article

Cosmological Solutions in the Framework of Lorentz-Invariant Gravitation Theory

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Abstract

All solutions of the equations of General Relativity concerning the movement of single massive bodies relative to each other and which are tested experimentally, were obtained by us within the framework of LIGT in the previous article. It is obvious that if we want to fully confirm the equivalence of General Relativity to LIGT, it seems necessary to obtain the corresponding cosmological solution in framework of LIGT. The present article is dedicated to this subject.

Keywords: Lorentz-invariant gravitation theory, cosmological solutions

1.0 Introduction

All solutions of the equations of General Relativity concerning the movement of single massive bodies relative to each other (planets, stars, etc.) and which are tested experimentally, were obtained by us within the framework of LIGT in the previous article.

Moreover, there are solutions that are interpreted as cosmological, that is, related to the entire universe. At the moment, as a tested solution is also considered the solution, obtained by means of the postulates of the homogeneity and anisotropy of Universe, jointly with the results of general relativity and thermodynamics.

The question of the legality of such description of the Universe that contains, along with an almost infinite number of stars, planets and smaller bodies also an almost infinite number of other objects (microwave cosmic background, gases, dust, supernovae, neutron and many other types of stars, different types of galaxies and so forth.), will be left outside the limits of this article. Also, we will not consider the contribution of electromagnetic field (in particular, its lower state - physical vacuum) and elementary particles, although their presence in the universe is primary. Thus, according to the Hans Alfven theory (Alfven, 1942; Alfven and Arrhenius, 1976) (for which he received the Nobel Prize), electric and magnetic fields play a crucial role in the formation of the solar and other star systems.

Let us only note that direct experimental proofs of correctness of cosmological postulates and solutions do not exist (Baryshev, 1995). However, under the current cosmological paradigm are accepted interpretations of observational data, which was recognized as confirmation of abovementioned solutions. At the same time, there are numerous alternative explanations for these

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observational data, which, as was repeatedly noted by Alfven (Alfven, 1984).) and other scientists, are not taken into account (which means that they can not be published in official publications):
«Perhaps never in the history of science has so much quality evidence accumulated against a model so widely accepted within a field. Even the most basic elements of the theory, the expansion of the universe and the fireball remnant radiation, remain interpretations with credible alternative explanations. One must wonder why, in this circumstance, that four good alternative models are not even being comparatively discussed by most astronomers» (Flandern, 2002).

2.0. Formulation of the problem

It is obvious that if we want to fully confirm the equivalence of general relativity and LIGT, it seems necessary to obtain the corresponding cosmological solution in framework of LIGT. The present article will be dedicated to this subject. At the same time, our paper bears a feature which the previous article also bore. We have practically no need to present this solution since it has long been known, and is even taken into consideration at the pedagogical level.

Basic cosmological solutions of general relativity (for three types of curvature of space-time Universe) were obtained by Friedman (1922). Their derivation is reported in numerous textbooks, lectures and monographs; See, for example. (Bogorodsky, 1971; Dullemond et al. 2011, Ch. 4.).

The basis upon which the solution of Friedman is built (Dullemond et al . 2011, Ch. 4) are the two postulates mentioned above about the state of the universe. Besides that, it was proven by Robertson and Walker that the only one choice of metric exists, that satisfies these postulates.

Let us consider this metric (Dullemond et al . 2011, Ch. 4):

2.1 Robertson-Walker geometry of space

“The Universe is homogeneous and isotropic. Isotropy means that the metric must be diagonal. Because, as we shall see, space is allowed to be curved, it will turn out to be useful to use spherical coordinates (r,θ,φ) for describing the metric. The center of the spherical coordinate system is us (the observers) as we look out into the cosmos. Let us focus on the spatial part of the metric. For flat space the metric is given by the following line element:

$$ds^2 = dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{2.1}$$

where θ is now measured from the north pole and is π at the south pole. It is useful to abbreviate the term between brackets as

$$d\omega^2 = d\theta^2 + \sin^2\theta d\varphi^2 , \tag{2.2}$$

because it is a measure of angle on the sky of the observer. Because the universe is isotropic the angle between two galaxies as we see it is in fact the true angle fromour vantage point: The expansion of the universe does not change this angle. Therefore we can use $d\omega$ for the remainder of this lecture. So, for flat space we have

$$ds^2 = dr^2 + r^2 d\omega^2 , \tag{2.3}$$

It was proven by Robertson and Walker that the only alternative metric that obeys both isotropy and homogeneity is:

$$ds^2 = dr^2 + f_K(r)^2 d\omega^2, \tag{2.4}$$

where the function $f_K(r)$ is the curvature function given by

$$f_K(r) = \begin{cases} K^{-1/2}\sin\left(K^{1/2}r\right) & for\ K > 0 \\ r & for\ K = 0 \\ K^{-1/2}\sinh\left(K^{1/2}r\right) & for\ K < 0 \end{cases}, \tag{2.5}$$

The constant K is the curvature constant. We can also define a “radius of curvature”

$$R_{curv} = K^{-1/2}, \tag{2.6}$$

which, for our 2-D example of the Earth’s surface, is the radius of the Earth. In our 3- D Universe it is the radius of a hypothetical (!) 3-D “surface” sphere of a 4-D“sphere” in 4-D space.

Note that the metric given in Eq. (2.4) can be written in another way if we define an alternative radius r as $r \equiv f_K(r)$. The metric is then:

$$ds^2 = \frac{dr^2}{1 - Kr^2} + r^2 d\omega^2 , \tag{2.7}$$

Note that this metric is different only in the way we choose our coordinate r; it is not in any physical way different from Eq. (2.4)”.

The Robertson-Walker metric allows to build a solution Friedman. The Friedmann Equations are two simple first order ordinary differential equations. Solutions to these equations yield the cosmological model we are interested in (Dullemond et al. 2011, Ch. 4).

2.2. The Robertson-Walker Universe metric in framework of LIGT

As we have shown (Kyriakos, 2015), the square of the interval, which in SRT and GRT is considered as a geometry object, in the physics of elementary particles and within LIGT is a mathematical notation of the Lorentz-invariant energy-momentum conservation law.

That is why the Lorentz transformation can be found formally as a group of transformations preserving invariant the squared interval.

In the presence of a gravitational field this quadratic form contains a metric tensor, in which the amendment of changing the scale of coordinates derived due to the effects of the Lorentz transformation, is taken into account. As we have shown, this tensor is identical to the one obtained from the solution of equations of general relativity. Thus there is no need to interpret this interval as belonging to a Riemann space. It may be written in any (including rectangular) coordinate system.

Thus, all above arguments in section 2.1 may be repeated in LIGT as well as the further calculations of Friedman. Since Newton's equation is a first approximation of the equations of gravitation LIGT, you can expect that the results of Friedman's (at least to a first approximation) can be derived from Newton's theory of gravitation.

Such solutions were indeed found in 1934 (Milne, 1934; McCrea and Milne, 1934) (see the original solution in brief in Annex 1) and refined later (Leizer, 1954; ets). Moreover, it appears that these solutions are the same as the solutions of Fridman.

A modern formulation of this solution in Russian can be found, for example, in the presentation of the expert in the field of general relativity, academician Ya.B.Zeldovich; see Appendix I to the book (Weinberg, 2000), p. 190, titled "The classical non-relativistic cosmology" , who note here:
"All the calculations could have been made not only in the nineteenth century, but also in the eighteenth century".

In English, lecture 2 from the modern cosmology course ((Dullemond et al. 2011, Ch. 2) is dedicated to this subject.

Closing notes

This concludes our presentation of LIGT itself. It would be interesting to analyze the question of whether the Hilbert-Einstein's general relativity has some advantages over non-geometrical approach except for the gaudy mathematical interpretation that goes beyond the usual physics. Some thoughts on this matter will be set out in other articles.

Annex 1

A Newtonian Expanding Universe*
By E. A. Milne (Oxford)
[Received 7 March 1934]
Quart. J. Math. Oxford **5**, 64 (1934)

1. The phenomenon of the expansion of the universe has usually been discussed by students of relativity by means of the concept of 'expanding space'. This concept, though mathematically significant, has by itself no physical content; it is merely the choice of a particular mathematical apparatus for describing and analysing phenomena. An alternative procedure is to choose a static space, as in ordinary physics, and analyse the expansion-phenomenon as actual motions in this space. Moving particles in a static space will give the same observable phenomena as stationary particles in 'expanding' space. In each case the space is a construct built up by the mathematician out of observations that could in principle be made; it is built up around the matter in motion according to certain rules. The formulation of the relevant laws of nature depends on the rules adopted, and the laws will be quite different if different rules are adopted, as I have elsewhere† explained.

The alternative procedures have been tersely described in a recent paper by S. R. Milner.‡ He explained that we can either modify our geometry in order to retain $\delta\int ds = 0$ as the paths of free particles, or retain Euclidean geometry and Minkowski space-time and modify the variational principle by weighting the elements of path ds with appropriate invariant weighting factors.

*Paper reprinted with persmission of the Oxford University Press.

† *Zeits. f ̈ur Astrophys.* 6 (1933), 29.
‡ *Proc. Roy. Soc.* A, 139 (1933), 349 (§2).

In this paper I show how the same locally observable results can be obtained from elementary Newtonian theory (using flat, static space, Newtonian time, and the Newtonian dynamics and law of gravitation) as are given by Einstein and de Sitter's well-known relativistic model† of a universe in flat, *expanding* space and the relativistic theory of gravitation. It will be shown that the latter model corresponds to a Newtonian universe in which every particle has the parabolic velocity of escape from the matter 'inside it' as judged by any arbitrary observer situated on any particle of the system... The identity is exact — no approximation is involved, nor is any neglect made of inverse powers of the velocity of light. These results will be extended to a general class of relativistic universes and the corresponding Newtonian universes in a joint paper by the author and Dr. W. H. McCrea.

In the Newtonian cases the symbol *t* occuring in the differential equations and their integrals denotes Newtonian time. In the relativistic cases it denotes 'cosmic time', i.e. the time kept by a clock moving with the particle concerned. In the Newtonian case such a clock keeps the same time as the observer's clock, assuming the usual definition of simultaneity by means of light-signals; in the relativistic cases, it can be shown that the 'cosmic time' of an event does not coincide with the epoch assigned to it by a distant observer, using the same definition of simultaneity. Thus the two identical sets of differential equations have different interpretations in the two cases… Apart from this question of interpretation, the relativistic and Newtonian theories as regards models of the universe are indistinguishable in their predictions of local phenomena…

† *Proc. Nat. Acad. Sci.* 18 (1932), 213.

2. It seems to have escaped previous notice that whereas the theory of the expanding universe is generally held to be one of the fruits of the theory of relativity, actually all the at-present-observable phenomena could have been predicted by the founders of mathematical hydrodynamics in the eighteenth century, or even by Newton himself. The velocity of light, *c*, does not enter into the formula determining the law of expansion or the relation between the rate of expansion and the local mean-density. This point is obscured in treatments which take the velocity of light as unity; actually *c* cancels out, and a knowledge of the numerical value of *c* is not required. All that is necessary is the Newtonian theory of dynamics and gravitation, combined with the hydrodynamical equation of continuity…

3. Velocity- and density-laws. Let us adopt Euclidian space and Newtonian time for all observers, and the Newtonian formulation of dynamics and gravitation. Consider a swarm or cloud of freely moving particles in this space; in the system we shall construct collisions do not occur. The problem is to find a cloud of particles, possibly in motion (i.e. to determine its motion and density behaviour) such that it is described in the same way as viewed from any particle of the system as place of observation. Einstein showed long ago‡ that a *static* universe of this kind led to contradictions within the walls of Newtonian theory.

*To be distinguished from Einstein's principle of relativity.
† *Phil. Mag.* (7) 14 (1932), 386.
‡ *Sitz. d. Preussischen Akad.* (1917), 150. [full name of the journal: *Sitzungsberichte der K ̈oniglich-Preussischen Akademie der Wissenschaften zu Berlin*—Editor].

We therefore investigate the possibility of constructing a homogeneous universe in which the density ρ at any point changes with the time. Since here there is an absolute simultaneity, there is no difficulty in defining homogeneity, and we therefore have $\rho = \rho(t)$, a function of time *t* only. Of the possible motions, a particular case will be that in which the direction of motion is strictly radial as seen by an assigned observer. (We exclude the possibility of rotation.) We now investigate the form of the function $\rho(t)$ and the law of dependence of velocity on position and epoch.

Let υ be the outward velocity of a particle at time *t*, at distance *r* from the observer, relative to the particle on which the observer* is situated. Let $M(r)$ be the mass contained in the sphere of radius *r*. Consider the particular case† in which the distant particle has the *parabolic* velocity of escape from the mass contained in the sphere of radius *r*. The observer considers the material outside this sphere as having no influence on the motions inside it, in accordance with Newtonian gravitational theory; the observer, in fact, supposes that conditions 'at infinity' are compatible with this assumption. Then he writes down

$$\frac{1}{2}\upsilon^2 = \frac{GM(r)}{r}. \qquad (1)$$

Since the mass $M(r)$ remains constant 'following the motion', the particle will always possess the velocity given by (1) if it once possesses it. In writing down (1) we are not using the notion of a gravitational potential, here inapplicable, but are employing (1) simply as an integral of the equation of motion with a particular value of the constant of integration. Equation (1) gives

$$\upsilon^2 = \frac{8\pi G}{3} r^2\rho. \qquad (2)$$

The motion must be such that the hydrodynamical equation of continuity‡ is satisfied. This, in polar coordinates, runs in Eulerian notation

$$\frac{\partial\rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\rho\upsilon) = 0, \qquad (3)$$

where υ is a function of *r* and *t* given by (2). Inserting this, we have

*A typo in the original: "observers" instead of "observer is" [Editor]. † This will be generalized later, in the joint paper which follows, but the particular case offers a better introduction and in any case requires separate treatment in the resulting integrations.
‡ *Circa* 1750.

$$\rho^{-3/2}\frac{d\rho}{dt} + 3\left(\frac{8\pi G}{3}\right)^{1/2} = 0.$$

Integrating, we have

$$-2\rho^{-1/2} + (24\pi G)^{1/2}t = \text{const.},$$

or, choosing a suitable origin of *t*,

$$-2\rho^{-1/2} + (24\pi G)^{1/2}t = 0.$$

This gives

$$\rho = 1/6\pi Gt^2. \qquad (4)$$

Equation (2) then gives*

$$\upsilon = \frac{2r}{3t}. \qquad (5)$$

We now verify that this is a solution of the problem. The acceleration of the particle is

$$\frac{D\upsilon}{Dt} = \frac{D}{Dt}\left(\frac{2r}{3t}\right) = \frac{2}{3}\left(\frac{\upsilon}{t} - \frac{r}{t^2}\right) = -\frac{2r}{9t^2}, \qquad (6)$$

and this is precisely the Newtonian acceleration, since

$$-\frac{GM(r)}{r^2} = -\frac{4}{3}\pi G r^3 \frac{1}{6\pi G t^2}\frac{1}{r^2} = -\frac{2r}{9t^2}. \qquad (6')$$

Lastly (5) satisfies the Newtonian principle of relativity and Einstein's cosmological principle. If we transform our origin to another of the moving particles, at distance **R**, where the velocity V is (2/3)**R**/ *t*, the Newtonian formulae of transformation are

$$\mathbf{r}' = \mathbf{r} - \mathbf{R}, \quad \mathbf{v}' = \mathbf{v} - \mathbf{V},$$

*The minus sign is also permissible, giving $\upsilon = -(2/3)\, r/t$.

whence

$$\mathbf{v}' = \frac{2\mathbf{r}}{3t} - \frac{2\mathbf{R}}{3t} = \frac{2\mathbf{r}'}{3t}. \qquad (7)$$

The acceleration also obeys the Newtonian transformation law.

4. Discussion. Equations (4) and (5) provide a solution to our problem. By (5), υ obeys a velocity-distance proportionality at any one epoch, and so this Newtonian universe obeys Hubble's law of nebular velocities. If we put this law in the form $\upsilon = \alpha r$, where α is observed, then $\alpha = 2/3t$, and by (4)

$$\rho = \frac{\alpha^2}{\frac{8}{3}\pi G}. \qquad (8)$$

The data from the nebular velocities and distances then give for *t* a value of about $1\cdot 3\times 10^9$ years, and a density r of about 5×10^{-238} gram cm^{-3} These are of the usual orders of magnitude given by the 'expanding space' theories, as well as by the kinematic theory. In the kinematic theory, as I have shown,* $\rho \sim 1/(4/3)\pi\ Gt^2$ and $\upsilon = r/t$, so that here $\rho \sim \alpha^2/(4/3)\pi G$. Thus the local value of the 'age' of the universe is on the Newtonian theory two-thirds that on the kinematic theory, and the density about one-half that on the kinematic theory. Present estimates of the actual mean local density of the universe cannot discriminate between the two.

5. Comparison with the Einstein–de Sitter universe.

We have seen that the Newtonian universe constructed above is defined by the equations

$$\frac{1}{2}\upsilon^2 = \frac{GM(r)}{r} \qquad \frac{D\upsilon}{Dt} = -\frac{GM(r)}{r^2}, \qquad (9)$$

where

$$M(r) = \tfrac{4}{3}\pi\rho r^3. \qquad (10)$$

Equations (9) contain within themselves the equation of continuity, for on differentiating the first of (9) and using the second we have at once

**M.N.R.A.S.* 94 (1933), 7.

$$\frac{D}{Dt}M(r) = 0.$$

Put

$$r = fR(t), \qquad (11)$$

where f is a constant particularizing the particle considered and R is a universal function of t only. Actually, integration of

$$\upsilon = \frac{dr}{dt} = \frac{2r}{3t}$$

gives at once $r = f\,t^{2/3}$, so that $R(t) = t^{2/3}$, but we do not need this. Then $\upsilon = fdR/dt$, $D\upsilon/Dt = f\,d^2R/dt^2$. Introducing these in (9) and using (10) we see that f divides out and we get

$$\left(\frac{1}{R}\frac{dR}{dt}\right)^2 = \frac{8\pi G}{3}\rho, \qquad (12)$$

$$\frac{2}{R}\frac{d^2R}{dt^2} = -\frac{8\pi G}{3}\rho = -\left(\frac{1}{R}\frac{dR}{dt}\right)^2. \qquad (13)$$

Introduce Einstein's constant k defined by

$$\kappa = 8\pi G/c^2,$$

and write

$$cdt = d\tau..$$

Then (12) and (13) become

$$\left(\frac{1}{R}\frac{dR}{d\tau}\right)^2 = \frac{1}{3}\kappa\rho, \qquad (14)$$

$$\frac{2}{R}\frac{d^2R}{d\tau^2} + \left(\frac{1}{R}\frac{dR}{d\tau}\right)^2 = 0. \qquad (15)$$

But these are identical with the relativistic equations for an expanding universe of zero curvature with pressure $p = 0$ and cosmical constant $\lambda = 0$, as given* by Einstein and de Sitter. Conversely, from the relativistic equations (14) and (15) we can infer equations identical in form with the Newtonian equations. The equations (14) and (15) are derived from a metric

$$ds^2 = d\tau^2 - R^2(dx^2 + dy^2 + dz^2).$$

In this space a particle is assigned fixed 'coordinates' x, y, z, and the 'distance' r of such a particle is given by

$$r = fR,$$

where f is constant for the particle, depending on the particle chosen. Then .

$$\frac{1}{r}\frac{dr}{d\tau} = \frac{1}{R}\frac{dR}{d\tau}.$$

Introducing these in (14) and (15) and returning to t, we see that c cancels out, and we are left with

$$\left(\frac{1}{r}\frac{dr}{dt}\right)^2 = \frac{8\pi G}{3}\rho, \qquad (16)$$

$$\frac{2}{r}\frac{d^2r}{dt^2} = -\left(\frac{1}{r}\frac{dr}{dt}\right)^2 = -\frac{8\pi G}{3}\rho. \qquad (17)$$

Define $m(r)$ by

$$m(r) = \tfrac{4}{3}\pi r^3 \rho.$$

Then

$$\frac{D}{Dt} m(r) = \frac{4}{3}\pi \frac{D}{Dt}(r^2 \rho r) = \frac{4}{3}\pi \frac{D}{Dt}\left\{ \frac{3}{8\pi G} r \left(\frac{dt}{dt}\right)^2 \right\},$$

* Einstein and de Sitter, loc. cit., give (14); (15) is given by de Sitter, *Univ. of California Pub. Math.* 2 (1933), 161.

which by (17) is zero. Hence $m(r)$ is constant following the motion, and (16) and (17) may then be written

$$\frac{1}{2}v^2 = \frac{Gm(r)}{r}, \qquad \frac{Dv}{Dt} = -\frac{Gm(r)}{r^2},$$

which are the Newtonian equations. It follows that the two equations defining the behaviour of R and ρ in the Einstein–de Sitter universe are equivalent to an equation of motion and an equation of continuity. Since they were originally obtained from Einstein's field equations via the Riemann–Christoffel tensor, we have an interesting example of the correspondence of Einstein's field equations with Newtonian dynamics and gravitation. The density ρ in the Einstein–de Sitter universe now comes out,* as in the Newtonian case, as $\rho = 1/6\pi Gt^2$, and the 'velocity-law' as $\upsilon = (2/3)r/t$.

Since the time t or t in the relativistic case coincides locally with the Newtonian time t kept by the clock moving with the particle considered, it follows that the locally observable properties of the Einstein–de Sitter universe are identical with the properties predicted for the Newtonian universe. It can be shown that just as in the Newtonian case a particle endowed with the parabolic velocity steadily decreases in velocity, ultimately to zero, so in the Einstein–de Sitter universe the red-shift l′/l, calculated as the Doppler effect, for any given particle of the system steadily decreases as the epoch of observation of this Doppler effect advances. Thus in the Einstein–de Sitter universe, as in the Newtonian universe, each particle may be described as undergoing deceleration. This accounts for the shorter time-scale as compared with the kinematic theory, where the deceleration is zero.

In practice, 'local phenomena', or phenomena 'close to the observer' means phenomena within say 150–200 million light years' distance; they include all phenomena as yet accessible to observation. An analyst of Newton's period who had no data on nebular velocities would be unable to estimate the 'age' t or present mean density r, but he would have been led to predict a non-static universe (with either expansion or contraction), to predict a velocity-distance proportionality at any one epoch, and to obtain the formula

$$\rho = \alpha^2 / \tfrac{8}{3}\pi G$$

connecting density and rate of expansion. Thus he would have secured all the results yet capable of observational test.

6. On obtaining the above results I communicated them to Dr. McCrea. It

*De Sitter, loc. cit. p. 180, equation (58), on cancelling c.

occurred to both of us, independently, to generalize the results so as to give the elliptic and hyperbolic cases, as well as the parabolic, on Newtonian mechanics. Actually Dr. McCrea sent me his results first. The paper which follows contains features due to both of us.

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