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Five-Dimensional Cosmological Model with Quadratic Equation of State in a Scalar-Tensor Theory of Gravitation

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Abstract

Five dimensional Kaluza – Klein space – time is considered in the presence of perfect fluid in the scalar – tensor theory of gravitation formulated by Saez and Ballester (Phys. Lett. A **113** : 467, 1986). A determinate solution of the field equations has been obtained under the assumption of quadratic equations of state (EoS) $p=\alpha\rho^2-\rho$ where α is a constant and strictly $\alpha\neq 0$. The physical and geometrical aspects of the model are also discussed.

Keywords. Kaluza-Klein spacetime. scalar-tensor theory, quadratic equations of state.

1. Introduction

Brans and Dicke (1961) formulated a scalar – tensor theory of gravitation in which the gravitational interaction is mediated by a scalar field ϕ as well as the tensor field g_{ij} of Einstein's theory. In this theory, the scalar field ϕ has the dimension of the inverse of the gravitational constant. Saez and Ballester (1986) have proposed a scalar – tensor theory of gravitation in which the metric is coupled with a dimensionless scalar field ϕ in a simple manner. This coupling gives a satisfactory description of the weak fields. In spite of the dimensionless character of the scalar field an antigravity regime appears. This theory also suggests a possible way to solve 'missing mass' problem in non-flat FRW cosmologies. The field equations given by Saez and Ballester (1986) for the combined scalar and tensor fields are

$$G_{ij}-\omega\phi^n\left(\phi_{,i}\phi_{,j}-\frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right)=-T_{ij} \qquad (1)$$

and the scalar field ϕ satisfies the equation

$$2\phi^n\phi_{;k}^{,k}+n\phi^{n-1}\phi_{,k}\phi^{,k}=0 \qquad (2)$$

where $G_{ij}=R_{ij}-\frac{1}{2}g_{ij}R$ is the Einstein tensor, ω and n are constants, T_{ij} is the energy momentum tensor of matter and comma and semicolon denote partial and covariant differentiation respectively.

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Also

$$T^{ij}_{;j} = 0 \qquad (3)$$

is a consequence of the field equations (1) and (2) which physically describe the energy conservation of the field and equation of motion.

Saez and Ballester (1986), Singh and Agarwal (1991), Shri Ram and Tiwari (1998), Reddy and Venkateswara Rao (2001), Adhav et al. (2007), Reddy and Naidu (2007), Reddy et al. (2008), Naidu et al. (2012a), Naidu et al. (2012b), Reddy et al. (2012), Reddy et al (2013a), Reddy and Santhikumar (2013), Reddy et al. (2013b) and Kiran et al. (2014) are some of the authors who have investigated several cosmological models in the presences of different physical sources.

The recent observations of large scale structure (Tegmark et al. 2004) and cosmic microwave background radiation (Bennett et al. 2003; Spergel et al. 2003 a, 2003 b) indicate that the universe is highly homogenous and isotropic on large scales. Also the basic problem in modern cosmology is that of dark energy which is supposed to drive the accelerated expansion of the universe (Riess et al. 1998; Perlmutter et al. 1999). Hence dark energy models of the universe with different equations of state have been discussed in literature. Nojiri and Odintsov (2005) have discussed dark energy universe with different equations of state with homogenous Hubble parameter term. Recently Reddy et al. (2015) discussed Bianchi type – I cosmological model with quadratic equation of state. Also Capozziello et al. (2006) presented the observational constraints on dark energy with quadratic equation of state. According to them, quadratic equation of state may describe dark energy of unified dark matter. Rahaman et al. (2009), Feroze and Siddique(2011), Maharaj and Takisa (2013), Chavanis (2013a, 2013b), Sharma and Ratnapal (2013) and Malaver (2014) are some of the authors who have discussed cosmological models with quadratic equation of state under some physical situations.

Motivated by the above investigations we discuss here, a five dimensional Kaluza – Klein thcosmological model in Saez – Ballester scalar – tensor theory of gravitation with quadratic equation of state.

2. Metric and field equations

We consider the five dimensional Kaluza – Klein space – time given by

$$ds^2 = dt^2 - A^2\left(dx^2 + dy^2 + dz^2\right) - B^2 d\phi^2 \qquad (4)$$

where A and B are functions of cosmic time t. Unlike Wesson (1983), the fifth coordinate is taken to be space – like. Here the spatial curvature has been taken as zero (Gron 1988).
The energy momentum tensor T_{ij} for the perfect fluid is given by

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij} \qquad (5)$$

where ρ is the energy density, p is the pressure and u^i is the four velocity vector satisfying $g_{ij}u^i u_j = 1$

We assume an equation of state EoS in the general form $p = p(\rho)$ for this will affect the quadratic nature of equation of state matter distribution. We consider it in this case in the quadratic form as

$$p = \alpha p^2 - \rho \qquad (6)$$

where α is a constant and strictly $\alpha \neq 0$ (Reddy et al. 2015).

The motivation to consider a quadratic equation of state is because of its importance in the world models and in the study of dark energy and general relativistic dynamics for different models. So it is quite natural to choose quadratic form of equation of state to study anisotropy problems. Hence the anisotropic Kaluza – Klein homogenous cosmological model containing perfect fluid with quadratic equation of state is being studied here.

Using co moving coordinate system, the Saez – Ballester field equations (1) – (3), for the metric (4) with the help of Eq. (5), can be written as

$$3\left(\frac{\dot{A}}{A}\right)^2 + \frac{3\dot{A}\dot{B}}{AB} - \frac{\omega}{2}\phi^n\dot{\phi}^2 = \rho \qquad (7)$$

$$\frac{2\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} + \frac{2\dot{A}\dot{B}}{AB} + \frac{\ddot{B}}{B} + \frac{\omega}{2}\phi^n\dot{\phi}^2 = -p \qquad (8)$$

$$\frac{3\ddot{A}}{A} + \frac{3\dot{A}^2}{A^2} + \frac{\omega}{2}\phi^n\dot{\phi}^2 = -p \qquad (9)$$

$$\ddot{\phi} + \dot{\phi}\left(\frac{3\dot{A}}{A} + \frac{\dot{B}}{B}\right) + \frac{n}{2}\frac{\dot{\phi}^2}{\phi^2} = 0 \qquad (10)$$

$$\dot{\rho} + (\rho + p)\left(\frac{3\dot{A}}{A} + \frac{\dot{B}}{B}\right) = 0 \qquad (11)$$

where an overhead dot indicates differentiation with respect to time t.

We define the spatial volume V and average scale factor $a(t)$ for Kaluza – Klein space – time as

$$V = a^4(t) = A^3 B \qquad (12)$$

The mean Hubble parameter H is given by

$$H = \frac{\dot{a}}{a} = \frac{1}{4}\left(\frac{3\dot{A}}{A} + \frac{\dot{B}}{B}\right) \qquad (13)$$

The scalar expansion θ and shear scalar σ^2 are given by

$$\theta = 4H = \left(\frac{3\dot{A}}{A} + \frac{\dot{B}}{B}\right) \qquad (14)$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{4} H_i^{\,2} - 4H^2\right) \qquad (15)$$

The average anisotropy parameter Δ is defined as

$$\Delta = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i - H}{H}\right)^2 \qquad (16)$$

where $H_i = 1,2,3,4$ represent the directional Hubble parameters in x, y, z, ψ directions respectively and $\Delta = 0$ corresponds to isotropic expansion.

3. Solutions of the field equations

The set of field equations (7) – (11)) reduces to the following independent system of equations:

$$3\left(\frac{\dot{A}}{A}\right)^2 + \frac{3\dot{A}\dot{B}}{AB} - \frac{\omega}{2}\phi^n\dot{\phi}^2 = \rho \qquad (17)$$

$$\frac{\ddot{A}}{A} + \frac{2\dot{A}^2}{A^2} - \frac{2\dot{A}\dot{B}}{AB} - \frac{\ddot{B}}{B} = 0 \qquad (18)$$

$$\ddot{\phi} + \dot{\phi}\left(\frac{3\dot{A}}{A} + \frac{\dot{B}}{B}\right) + \frac{n}{2}\frac{\dot{\phi}^2}{\phi^2} = 0 \qquad (19)$$

Eqs. (17) – (19) are as system of three independent equations in five unknowns A, B, p, ρ and ϕ. Hence to obtain a determinate solution we use two physical conditions. They are:

(i) The scalar expansion θ is proportional to the shear scalar σ^2 so that we can take (Collins et al. 1983)

$$A = B^m \qquad (20)$$

where $m \neq 0$ is constant which takes care of anisotropy of the space – time.

(ii) The EoS parameter in the quadratic form given by Eq. (6), i.e.

$$p = \alpha\rho^2 - \rho \qquad (21)$$

Now from Eqs. (20), (18) and (12) we obtain the expressions for the metric coefficients as

$$A = \left[(3m+1)(a_0 t + t_0)\right]^{\frac{m}{3m+1}}$$

$$B = \left[(3m+1)(a_0 t + t_0)\right]^{\frac{1}{3m+1}} \qquad (22)$$

where $a_0 \neq 0$ and t_0 are constants of integration.

By a suitable choice of coordinates and constants (i.e. $a_0 = 1, t_0 = 0)$ the metric (4) with the help of Eqs. (22) can be written as

$$ds^2 = dt^2 - \left[(3m+1)t\right]^{\frac{2m}{3m+1}}\left(dx^2 + dy^2 + dz^2\right) - \left[(3m+1)t\right]^{\frac{2}{m+1}} d\psi^2 \qquad (23)$$

with the scalar field ϕ is given by

$$\phi^{\frac{n+2}{2}} = \left(\frac{n+2}{2}\right)\frac{\phi_0}{(3m+1)}\log t + \psi_0 \qquad (24)$$

where ϕ_0 and ψ_0 are constants of integration and ψ_0 can be set equal to zero so that we have

$$\phi = \left[\left(\frac{n+2}{2}\right)\left(\frac{\phi_0}{3m+1}\right)\log t\right]^{\frac{2}{n+2}} \qquad (25)$$

4. Physical Discussion

Eq. (25) represents the five dimensional perfect fluid cosmological model with the following physical and kinematical properties.
Using Eqs. (23) and (25) in Eq. (17) we obtain energy density in the model, as

$$\rho = \frac{6m(m+1) - \omega\phi_0^{\,2}}{2(m+1)^2 t^2} \qquad (26)$$

From Eqs .(21) and (26), we obtain isotropic pressure as

$$p = \alpha\left[\frac{6m(m+1) - \omega\phi_0^{\,2}}{2(m+1)^2 t^2}\right]^2 - \left[\frac{6m(m+1) - \omega\phi_0^{\,2}}{2(m+1)^2 t^2}\right] \qquad (27)$$

The mean Hubble parameter in the model is

$$H = \frac{1}{4t} \qquad (28)$$

The spatial volume is

$$V = (3m+1)t$$

The scalar of expansion is

$$\theta = 4H = \frac{1}{t} \qquad (29)$$

The shear scalar is

$$\sigma^2 = \frac{3(m-1)^2}{8(3m+1)^2 t^2} \qquad (30)$$

The anisotropic parameter is

$$\Delta = 0 \quad (31)$$

The spatial volume V is finite at $t=0$ and it expands as time t increases and becomes infinitely large as $t \to \infty$. The scalar expansion θ, the shear scalar σ^2 start with infinite value at $t=0$ and as t increases, they vanish as $t \to \infty$. The energy density ρ and pressure p diverge at $t=0$ and tends to zero as $t \to \infty$. When $m=1$, the universe becomes shear free since $\sigma^2 = 0$. Also, since $\Delta = 0$, the universe becomes isotropic at late times and the deceleration parameter

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = 3 \qquad (32)$$

which shows that the universe decelerates in the standard way initially and accelerates at late times which is in accordance with the recent scenario accelerated expansion of the universe. Also, the scalar field in the model diverge at $t = 0$ and vanishes as $t \to \infty$.

5. Conclusions

A five dimensional Kaluza-Klein cosmological model has been investigated in a scalar – tensor theory of gravitation proposed by Saez and Ballester (1986) with a quadratic equation of state.
The physical and kinematical parameters which play a vital role in the discussion of cosmological model have been obtained and their significance has been explained. It is observed that spatial volume of the universe increases with time. The energy density, the pressure, the average Hubble parameter, scalar expansion and shear scalar diverge at $t = 0$ and become zero as $t \to \infty$. Observations of average anisotropy parameter and the deceleration parameter shows early inflation and late time acceleration which is the scenario of modern cosmology (Riess et al. 1998 and Perlmutter et al. 1999). Also the scalar field in the model diverges at the initial epoch and vanishes for infinite time.

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