

## Article

# Does the Physics of SmB<sub>6</sub> Make the Fundamental Dynamics of TGD Directly Visible?

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## Abstract

The group of Suchitra Sebastian has discovered very unconventional condensed matter system, which seems to be simultaneously both insulator and conductor of electricity but only in presence of magnetic field. The obvious TGD inspired explanation is that the flux tubes of the magnetic field serve as current wires. If so, this finding would be one further direct evidence for several new physics notions predicted by TGD: many-sheeted space-time and topological field quantization, strong form of holography and the generalization of AdS/CFT correspondence predicted by it, TGD as almost topological QFT at the level of single space-time sheet, and hierarchy of Planck constants predicted by quantum criticality of TGD and identified as dark matter hierarchy.

## 1 Observations

The group of Suchitra Sebastian has discovered very unconventional condensed matter system, which seems to be simultaneously both insulator and conductor of electricity but only in presence of magnetic field. Science article is entitled “Unconventional Fermi surface in an insulating state” [1] (<http://www.sciencemag.org/content/early/2015/07/01/science.aaa7974>). There is also a popular article with title “Paradoxical Crystal Baffles Physicists” in Quanta Magazine summarizing the findings (<https://www.quantamagazine.org/20150702-paradoxical-crystal-baffles-physicists/>). I learned about the finding first from the blog posting of Lubos Motl.

The crystal studied at superlow temperatures was Samarium hexaboride - briefly SmB<sub>6</sub>. The high resistance implies that electron cannot move more than one atom’s width in any direction. Sebastian et al however observed electrons traversing over a distance of millions of atoms - a distance of order 10<sup>-4</sup> m, the size of a large neuron. So high mobility is expected only in conductors. SmB<sub>6</sub> is neither metal or insulator or is both of them! The finding is described by Sebastian as a “big shock” and as a “magnificent paradox” by condensed matter theorists Jan Zaanen. Theoreticians have started to make guesses about what might be involved but according to Zaanen there is no even remotely credible hypothesis has appeared yet.

On basis of its electronic structure SmB<sub>6</sub> should be a conductor of electricity and it indeed is at room temperature: the average number conduction electrons per SmB<sub>6</sub> is one half. At low temperatures situation however changes. At low temperatures electrons behave collectively. In superconductors resistance drops to zero as a consequence. In SmB<sub>6</sub> just the opposite happens. Each Sm nucleus has the average 5.5 electrons bound to it at tight orbits. Below 223 degrees of Celsius the conduction electrons of SmB<sub>6</sub> are thought to “hybridize” around samarium nuclei so that the system becomes an insulator. Various signatures demonstrate that SmB<sub>6</sub> indeed behaves like an insulator.

During last five years it has been learned that SmB<sub>6</sub> is not only an insulator but also so called topological insulator. The interior of SmB<sub>6</sub> is insulator but the surface acts as a conductor. In their experiments Sebastian et al hoped to find additional evidence for the topological insulator property and attempted to measure quantum oscillations in the electrical resistance of their crystal sample. The variation of quantum oscillations as sample is rotated can be used to map out the Fermi surface of the crystal. No quantum oscillations were seen. The next step was to add magnetic field and just see whether something interesting happens and could save the project. Suddenly the expected signal was there! It

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was possible to detect quantum oscillations deep in the interior of the sample and map the Fermi surface! The electrons in the interior travelled 1 million times faster than the electrical resistance would suggest. Fermi surface was like that in copper, silver or gold. A further surprise was that the growth of the amplitude of quantum oscillations as temperature was decreased, was very different from the predictions of the universal Lifshitz-Kosevich formula for the conventional metals.

## 2 Could TGD help to understand the strange behavior of SmB<sub>6</sub>?

There are several indications that the paradoxical effect might reveal the underlying dynamics of quantum TGD. The mechanism of conduction must represent new physics and magnetic field must play a key role by making conductivity possible by somehow providing the “current wires”. How? The TGD based answer is completely obvious: magnetic flux tubes - one of the basic distinctions between electrodynamics of Maxwell and its TGD variant! Also the failure of Lifshitz-Kosevich formulas should be understood.

### 2.1 Single sheet of many-sheeted space-time resembles topological insulator

One should also understand topological insulator property at deeper level, that is the conduction along the boundaries of topological insulator. One should understand why the current runs along 2-D surfaces. In fact, many exotic condensed matter systems are 2-dimensional in good approximation. In the models of integer and fractional quantum Hall effect electrons form a 2-D system with braid statistics possible only in 2-D system. High temperature super-conductivity is also an effectively 2-D phenomenon. By strong form of holography these aspects are also key aspects of quantum TGD at the fundamental level of single space-time sheet when the approximation replacing many-sheeted space-time with that of GRT and standard model does not mask the simplicity of the fundamental dynamics.

1. Many-sheeted space-time is second fundamental prediction TGD. The dynamics of single sheet of many-sheeted space-time should be very simple by the strong form of holography implying effective 2-dimensionality. The standard model description of this dynamics masks this simplicity since the sheets of many-sheeted space-time are replaced with single region of slightly curved Minkowski space with gauge potentials sums of induced gauge potentials for sheets and deviation of metric from Minkowski metric by the sum of corresponding deviations for space-time sheets. Could the dynamics of exotic condensed matter systems give a glimpse about the dynamics of single sheet? Could topological insulator and anyonic systems [3] provide examples of this kind of systems?
2. Second basic prediction of TGD is strong form of holography: string world sheets and partonic 2-surfaces serve as kind of “space-time genes” and the dynamics of fermions is 2-D at fundamental level. It must be however made clear that at QFT limit the spinor fields of imbedding space replace these fundamental spinor fields localized at 2-surface. One might argue that the fundamental spinor fields do not make them directly visible in condensed matter physics. Nothing however prevents from asking whether in some circumstances the fundamental level could make itself visible.

In particular, for large  $h_{eff}$  dark matter systems (, whose existence can be deduced from the quantum criticality of quantum TGD) the partonic 2-surfaces with  $CP_2$  size could be scaled up to nano-scopic and even longer size scales. I have proposed this kind of surfaces as carriers of electrons with non-standard value of  $h_{eff}$  in QHE and FQHE [3].

The long range quantum fluctuations associated with large,  $h_{eff} = n \times h$  phase would be quantum fluctuations rather than thermal ones. In the case of ordinary conductivity thermal energy makes it possible for electrons to jump between atoms and conductivity becomes very small at low temperatures. In the case of large scale quantum coherence just the opposite happens as observed. One therefore expects that Lifshitz-Kosevich formula for the temperature dependence of the amplitude does not hold true.

The generalization of Lifshitz-Kosevich formula to quantum critical case deduced from quantum holographic correspondence by Hartnoll and Hofman [2] (<http://journals.aps.org/prb/abstract/10.1103/PhysRevB.81.155125>) is expected to hold true qualitatively also for quantum criticality in TGD sense. The first guess is that by underlying super-conformal invariance scaling laws typical for critical systems hold true. In the proposed formula the dependence on temperature is via a power of dimensionless parameter  $x = T/\mu$ , where  $\mu$  is chemical potential for electron system. As a matter fact, exponent of power of  $x$  appears and reduces to first for Lifshitz-Kosevich formula. Since magnetic field is important, one also expects that the ratio of cyclotron energy scale  $E_c \propto \hbar_{eff} e B / m_e$  to Fermi energy appears in the formula. One can even make an order of magnitude guess for the value of  $\hbar_{eff}/\hbar \sim 10^6$  from the facts that the scale of conduction and conduction velocity were millions times higher than expected.

Strings are 1-D systems and strong form of holography implies that fermionic strings connecting partonic 2-surfaces and accompanied by magnetic flux tubes are fundamental. At light-like 3-surfaces fermion lines can give rise to braids. In TGD framework AdS/CFT correspondence generalizes since the conformal symmetries are extended. This is possible only in 4-D space-time and for the imbedding space  $H = M^4 \times CP_2$  making possible to generalize twistor approach [4].

3. Topological insulator property means from the perspective of modelling that the action reduces to a non-abelian Chern-Simons term. The quantum dynamics of TGD at space-time level is dictated by Kähler action. Space-time surfaces are preferred extremals of Kähler action and for them Kähler action reduces to Chern-Simons terms associated with the ends of space-time surface opposite boundaries of causal diamond and possibly to the 3-D light-like orbits of partonic 2-surfaces. Now the Chern-Simons term is Abelian but the induced gauge fields are non-Abelian. One might say that single sheeted physics resembles that of topological insulator.
4. The effect appears only in magnetic field. I have been talking a lot about magnetic flux tubes carrying dark matter identified as large  $\hbar_{eff}$  phases: topological quantization distinguishes TGD from Maxwell's theory: any system can be said to possess "magnetic body", whose flux tubes can serve as current wires. I have predicted the possibility of high temperature super-conductivity based on pairs of parallel magnetic flux tubes with the members of Cooper pairs at the neighboring flux tubes forming spin singlet or triplet depending on whether the fluxes are have same or opposite direction.

Also spin and electric currents assignable to the analogs of spontaneously magnetized states at single flux tube are possible. The obvious guess is that the conductivity in question is along the flux tubes of the external magnetic field. Could this kind of conductivity explains the strange behavior of SmB<sub>6</sub>. The critical temperature would be that in which the parallel flux tubes are stable. The interaction energy of spin with the magnetic field serves as a possible criterion for the stability if the presence of dark electrons stabilizes the flux tubes.

## 2.2 Magnetic flux tubes as dark current carriers in quantum criticality

The following represents an extremely childish attempt of a non-specialist to understand how the conductivity might be understood. The current carrying electrons at flux tubes near the top of Fermi surface are current carriers.  $\hbar_{eff} = n \times \hbar$  and magnetic flux tubes as current wires bring in the new elements. Also in the standard situation one considers cylinder symmetric solutions of Schrödinger equation in external magnetic field and introduces maximal radius for the orbits so that formally the two situations seem to be rather near to each other. Physically the large  $\hbar_{eff}$  and associated many-sheeted covering of space-time surface providing the current wire makes the situation different since the collisions of electrons could be absent in good approximation so that the velocity of charge carriers could be much higher than expected as experiments indeed demonstrate.

Quantum criticality is the crucial aspect and corresponds to the situation in which the magnetic field attains a value for which a new orbit emerges/disappears at the surface of the flux tube: in this situation dark electron phase with non-standard value of  $h_{eff}$  can be generated. This mechanism is expected to apply also in bio-superconductivity and to provide a general control tool for magnetic body.

1. Let us assume that flux tubes cover the whole transversal area of the crystal and there is no overlap. Assume also that the total number of conduction electrons is fixed, and depending on the value of  $h_{eff}$  is shared differently between transversal and longitudinal degrees of freedom. Large value of  $h_{eff}$  squeezes the electrons from transversal to longitudinal flux tube degrees of freedom and gives rise to conductivity.
2. Consider first Schrödinger equation. In radial direction one has harmonic oscillator and the orbits are Landau orbits. The cross sectional area behaves like  $\pi R^2 = n_T h_{eff} / 2m\omega_c$  giving  $n_T \propto 1/h_{eff}$ . Increase of the Planck constant scales up the radii of the orbits so that the number of states in cylinder of given radius is reduced. Angular momentum degeneracy implies that the number of transversal states is  $N_T = n_T^2 \propto 1/h_{eff}^2$ . In longitudinal direction one has free motion in a box of length  $L$  with states labelled by integer  $n_L$ . The number of states is given by the maximum value  $N_L$  of  $n_L$ .
3. If the total number of states is fixed to  $N = N_L N_T$  is fixed and thus does not depend on  $h_{eff}$ , one has  $N_L \propto h_{eff}^2$ . Quanta from transversal degrees of freedom are squeezed to longitudinal degrees of freedom, which makes possible conductivity.
4. The conducting electrons are at the surface of the 1-D "Fermi-sphere", and the number of conduction electrons is  $N_{cond} \simeq dN/d\epsilon \times \delta\epsilon \simeq dN/d\epsilon T = NT/2\epsilon_F \propto 1/h_{eff}^4$ . The dependence on  $h_{eff}$  does not favor too large values of  $h_{eff}$ . On the other hand, if scattering of electrons at flux tubes could be absent. The assumption  $L \propto h_{eff}$  increases the range over which current can flow.
5. To get a non-vanishing net current one must assume that only the electrons at the second end of the 1-D Fermi sphere are current carriers. The situation would resemble that in semiconductor. The direction of electric field would induce symmetry breaking at the level of quantum states. The situation would be like that for a mass in Earth's gravitational field treated quantally and electrons would accelerate freely. Schrödinger equation would give rise to Airy functions as its solution.

## 2.3 Quantum critical quantum oscillations

What about quantum oscillations in TGD framework?

1. Quantum oscillation refers to de Haas-van Alphen effect ([http://phy.ntnu.edu.tw/~changmc/Teach/SS/SSG\\_note/grad\\_chap14.pdf](http://phy.ntnu.edu.tw/~changmc/Teach/SS/SSG_note/grad_chap14.pdf)) - an oscillation of the induced magnetic moment as a function of  $1/B$  with period  $\tau = 2\pi e/\hbar S$ , where  $S$  is the momentum space area of the extremal orbit of the Fermi surface, in the direction of the applied field. The effect is explained to be due to the Landau quantization of the electron energy. I failed to really understand the explanation of this source and in my humble opinion the following arguments provide a clearer view about what happens.
2. If the external magnetic field corresponds to flux tubes, Fermi surface decomposes into cylinders parallel to the magnetic field since the motion in transversal degrees of freedom is along circles. In the above thought experiment also a quantization in the longitudinal direction occurs if the flux tube has finite length so that Fermi surface in longitudinal direction has finite length. One expects on basis of Uncertainty Principle that the area  $S$  of the cross section of Fermi cylinder in momentum space is given by  $S \propto h_{eff}^2/\pi R^2$ . This follows also from the equation of motion of electron in magnetic field. As the external magnetic field  $B$  is increased, the radii of the orbits decrease inside the flux tube, and in momentum space the radii increase.

3. Why does the induced magnetic moment (magnetization) and other observables oscillate?
  - (a) The simplest manner to understand this is to look at the situation at space-time level. Classical orbits are harmonic oscillator orbits in radial degree of freedom. Suppose that the area of flux tube is fixed and  $B$  is increased. The orbits have radius  $r_n^2 = (n + 1/2) \times \hbar/eB$  and shrink. For certain field values the flux  $eBA = n\hbar$  corresponds to an integer multiple of the elementary flux quantum - new orbit at the boundary of the flux tube emerges if the new orbit is near the boundary of Fermi sphere providing the electrons. This is clearly a critical situation.
  - (b) In de Haas- van Alphen effect the orbit  $n + 1$  for  $B$  has same radius as the orbit  $n$  for  $1/B + \Delta(1/B)$ :  $r_{n+1}(1/B) = r_n(1/B + \Delta(1/B))$ . This gives approximate differential equation with respect to  $n$  and one obtains  $(1/B)(n) = (n + 1/2) \times \Delta(1/B)$ .  $\Delta(1/B)$  is fixed from the condition the flux quantization. When largest orbit is at the surface of the flux, tube the orbits are same for  $B(n)$  and  $B(n + 1)$ , and this gives rise to the de Haas - van Alphen effect.
  - (c) It is not necessary to assume finite radius for the flux tube, and the exact value of the radius of the flux tube does not play an important role. The value of flux tube radius can be estimated from the ratio of the Fermi energy of electron to the cyclotron energy. Fermi energy about .1 eV depending only on the density of electrons in the lowest approximation and only very weakly on temperature. For a magnetic field of 1 Tesla cyclotron energy is .1 meV. The number of cylinders defined by orbits is about  $n = 10^4$ .
  
4. What happens in TGD Universe in which the areas of flux tubes identifiable as space-time quanta are finite? Could quantum criticality of the transition in which a new orbit emerges at the boundary of flux tube lead to a large  $h_{eff}$  dark electron phase at flux tubes giving rise to conduction?
  - (a) The above argument makes sense also in TGD Universe for the ordinary value of Planck constant. What about non-standard values of Planck constant? For  $h_{eff}/h = n$  the value of flux quantum is  $n$ -fold so that the period of the oscillation in de Haas - van Alphen effect becomes  $n$  times shorter. The values of the magnetic field for which the orbit is at the surface of the flux tube are however critical since new orbit emerges assuming that the cyclotron energy corresponds is near Fermi energy. This quantum criticality could give rise to a phase transition generating non-standard value of Planck constant.  
 What about the period  $\Delta(1/B)$  for  $h_{eff}/h = n$ ? Modified flux quantization for extremal orbits implies that the area of flux quantum is scaled up by  $n$ . The flux changes by  $n$  units for the same increment of  $\Delta(1/B)$  as for ordinary Planck constant so that de Haas -van Alphen effect does not detect the phase transition.
  - (b) If the size scale of the orbits is scaled up by  $\sqrt{n}$  as the semiclassical formula suggests the number of classical orbits is reduced by a factor  $1/n$  if the radius of the flux tube is not changed in the transition  $h \rightarrow h_{eff}$  to dark phase.  $n$ -sheetedness of the covering however compensates this reduction.
  - (c) What about possible values of  $h_{eff}/h$ ? The total value of flux seems to give the upper bound of  $h_{eff}/h = n_{max}$ , where  $n_{max}$  is the value of magnetic flux for ordinary value of Planck constant. For electron and magnetic field for  $B = 10$  Tesla and has  $n \leq 10^5$ . This value is of the same order as the rough estimate from the length scale for which anomalous conduction occurs.

Clearly, the mechanism leading to anomalously high conductivity might be the transformation of the flux tubes to dark ones so that they carry dark electrons currents. The observed effect would be dark, quantum critical variant of de Haas-van Alphen effect!

Also bio-superconductivity is quantum critical phenomenon and this observation would suggest sharpening of the existing TGD based model of bio-super-conductivity. Super-conductivity would

occur for critical magnetic fields for which largest cyclotron orbit is at the surface of the flux tube so that the system is quantum critical. Quantization of magnetic fluxes would quantify the quantum criticality. The variation of magnetic field strength would serve as control tool generating or eliminating supra currents. This conforms with the general vision about the role of dark magnetic fields in living matter.

To sum up, a breakthrough of TGD is taking place. I have written about thirty articles during this year - more than one article per week. There is huge garden there and trees contain fruits hanging low! It is very easy to pick them: just shatter and let them drop to the basket! New experimental anomalies having a nice explanation using TGD based concepts appear on weekly basis and the mathematical and physical understanding of TGD is taking place with great leaps. It is a pity that I must do all alone. I would like to share. I can only hope that colleagues could take the difficult step: admit what has happened and make a fresh start.

## References

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