Article

Cosmological Models Filled with Perfect Fluid & Dark Energy in f(R,T) Theory of Gravity

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Abstract

In this paper we have investigated the physical behaviour of homogeneous and anisotropic Kaluza-Klein cosmological model in the frame of $f(R,T)$ theory of gravity in the presence of perfect fluid and dark energy. Where R is the Ricci scalar and T is the stress energy tensor of the matter. Here we consider the case when the perfect fluid obeys the following equations of state $P_{PF} = \gamma\rho_{PF}$ with $\gamma \in [0,1]$. Also we studied the cases when dark energy is given by a quintessence or Chaplygin gas. Some physical properties of the models are also discussed.

Keywords: Dark energy, perfect fluid, theory of gravity, Kaluza-Klein field equation.

1. Introduction

Now a day's different observation gives the suggestion that our universe is almost homogeneous and isotropic space time it is first describe by Friedman-Lemaitre-Robertson-Walker (FLRW) cosmology. Since, in early universe the FLRW statement about matter description is not correct. The cosmological view is that the universe might be anisotropic and also inhomogeneous in the vary early era and that in the course of its evaluation these characteristics has been take out by some process or mechanism and finally we get the homogeneous and isotropic universe.

In anisotropic cosmological model of the universe the large scale observation intersects with cosmic microwave background (CMB) therefore it is important to study the cosmological model within the framework of different anisotropic space time. According to the history of the expansion of the universe the cosmological observation shows that the current universe not only expanding but also accelerating. So in a late time accelerated expansion of the universe has been confirmed by high red shift Ia supernovae experiment (SNe.Ia) [1] also some observations like cosmic background radiation (CMBR) data [2] and large scale structure [3] combination shows that the late time acceleration expansion of the universe. All such results strongly imply that existence of an extra component in a universe with negative pressure i.e. the dark energy, which

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is about 70% in our universe. The dark energy is described by an equation of state EoS $p_d = \gamma\rho_d$ where p_d and ρ_d are pressure and energy density of dark energy and γ is a parameter of EoS.

This is effectively describing cosmic acceleration. The universe expand with acceleration gives the conformation of existence of dark energy and the simplest candidate for dark energy is the vacuum energy $(w=-1)$ is mathematically equal to cosmological constant Λ term in the system [4] which is first investigated by Einstein as a sure and sustainable cosmological solution to the gravitational field equation. Now in addition with cosmological constant for dark energy scenario and the proposal given by dynamical dark energy is also reliable by some scalar field mechanism which suggests that energy form with negative pressure is provided by scalar field evolved down a proper potential. Now some more examples of dark energy models are quintessence [5].Quintessence is a hypothetical form of dark energy and is thought to be the fifth fundamental force. Since cosmological constant stays constant throughout time and quintessence changes over time due to its dynamic character which is obtained by the equation of state EoS. This is the most common type of dark energy [6]. Chaplygin gas models [7] are used to combine the two different concepts as a dark energy and dark matter and thus reduce the two physical parameters in one by exotic equation of state EoS $p_C = \dfrac{-\alpha}{\rho_C}$. Since the original Chaplygin gas was introduce into Aerodynamics [8]. On chaplygin model there are some good number of works [9-16], Quintom field [18], Phantom field [19] ,K-essence[20],Tachyon field [21],

Holographic[22-23] and agegraphic [24] from these candidates to play the role of dark energy the quintessence and the Chaplygin gas have more concerned a lot in recent years . Since quintessence behave like a cosmological constant by combining positive energy density and negative pressure. Which obeys equation of state EoS $p_d = \gamma\rho_d$. Many researchers are working on dark energy in different theories of gravity like [25-31].

In a theory of gravity the development is going on whose results is that the new theories of gravity $f(T)$ and $f(R)$ are recently developed. $f(T)$ Theory has interesting features .That it may explain the current acceleration without involving dark energy and is also the generalized version of teleparallel gravity in which Weitzenbock connection is used instead of Levi-Civita connection. Many considerable works has been done in this $f(T)$ theory so far like [32-33]. In $f(R)$ theory of gravity the Ricci scalar is used in standard Einstein –Hilbert Lagrangian. In different ways this $f(R)$ is investigated by many researchers like [34-35]. For $f(R)$ the mostly studied solution is that of spherically symmetric solution because of its closeness to the nature. Felice A.D., Tsujikawa and Nojiri et al. present details reviews of a number of popular models of modified $f(R)$ gravity.

The generalized $f(R,T)$ theory of gravity developed by Harko et al.[36] where gravitational Lagrangian is given by an arbitrary function of Ricci scalar R and of trace T of stress energy tensor and obtained several models corresponding to some explicit form of the function $f(R,T)$. Currently, Samanta and Dhala constructed higher dimensional cosmological model filled with perfect fluid in $f(R,T)$ theory of gravity. Shri Ram et al. [37] have examined anisotropic cosmological models in f(R,T) theory of gravitation. Pawar and Solanke [38] have investigated the physical behaviour of LRS Bianchi type I cosmological model in f(R,T) theory of gravity. Dark energy cosmological models in f(R,T) theory of gravity studied by Pawar and Agrawal [39].Therefore it is hoped that gravity may explain the present phase of cosmic acceleration of our universe.

Now a day there is more work on binary mixture of perfect fluid and dark energy in that Bijan Saha [40] worked on self-consistent system of Binachi Type –I. T Singh and R Chaubey [41] has considered Binachi type –V model of the Universe with a binary mixture of perfect fluid and dark energy. Katore [42] have considered cosmological model of the universe with a binary mixture of perfect fluid and dark energy Akarsu and Kilinc [43] worked on specially homogeneous but totally anisotropic and non-flat Binachi type –II cosmological models with general relativity and in the presence of two minimally interacting fluids i.e. Perfect fluid as the Matter fluid and hypothetical anisotropic fluid as the dark energy fluid. Cataldo et al [44] described that a simplest non trivial cosmological scenario for an interacting mixture of two cosmic fluids by Power law scale factors i.e. expansion (contraction) as a power law in time.

Bainchi type IX two fluids cosmological models in General Relativity have obtained by Pawar and Dagwal [45]. Singh et al.[46] have obtained two-fluid cosmological model of Bianchi type-V with negative constant deceleration parameter. Two fluid cosmological models in Kaluza-Klein space-time examined by Samanta [47]. Recently, two fluids tilted cosmological model in General Relativity and Axially Bianchi type-I mesonic cosmological models with two fluid sources in Lyra Geometry presented by Pawar and Dagwal [48-49] Since the five dimensional theory introduced by Kaluza-Klein by Maxwell's theory of electromagnetism and Einstein's gravity theory. The candidate of fundamental theory is a Kaluza-Klein due to its potential function to unite the fundamental interaction. Kaluza-Klein cosmological model have been studied with different type of matter and cosmological constant with considerable inflection.

In this paper we have investigated the physical behaviour of homogeneous and anisotropic Kaluza-Klein cosmological model in the frame of $f(R,T)$ theory of gravity in the presence of perfect fluid and dark energy. Where R is the Ricci scalar and T is the stress energy tensor of the matter. The dark energy is given by Chaplygin gas or quintessence and also the perfect fluid obeys the equation of state $p = \gamma\rho$ with $\gamma \in [0,1]$ .This work is an extension of G.C. Samanta, S.N.Dhala [50].

2. Field Equations and Solutions

Here we consider specially Kaluza-Klein space time field equation in the form of

$$ds^2 = dt^2 - a^2\,(dx^2 + dy^2 + dz^2) - b^2 d\phi^2 \tag{2.1}$$

where a and b represents the functions of t only.

Kaluza-Klein universe is generalization of the open universe in FRW cosmology. Therefore its study is important in the presence of dark energy models in a universe with non-zero curvature [41].

For the matter source the energy momentum tensor is given by

$$T_i^j = (P+\rho)u_i u^j - P\delta_i^j \tag{2.2}$$

where, u^i is the flow vector satisfy the equation,

$$g_{ij}u^i u^j = 1 \tag{2.3}$$

By using equation (2.3) we find,

$$T_0^0 = \rho \ , \ T_1^1 = T_2^2 = T_3^3 = T_4^4 = -P \tag{2.4}$$

in a co-moving system of co-ordinates.

Now varying the action,

$$S = \frac{1}{16\pi}\int f(R,T)\sqrt{-g}\ d^5x + \int L_m\sqrt{-g}\ d^5x \tag{2.5}$$

Of the gravitational field with concerned to the metric tensor components g_{ij}, We get the field equation of $f(R,T)$ gravity model as (Harko et al [36])

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + (g_{ij}\nabla^i\nabla_j - \nabla_i\nabla_j)f_R(R,T) = 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\ \theta_{ij} \tag{2.6}$$

where $T_{ij} = \frac{-2}{\sqrt{-g}}\frac{\delta(\sqrt{-g})}{\delta g^{ij}}L_m$ and,

$$\theta_{ij} = -2T_{ij} - Pg_{ij} \tag{2.7}$$

Now we define the covariant derivative as,

$$f_R(R,T) = \frac{\partial f(R,T)}{\partial R} \quad , \quad f_T(R,T) = \frac{\partial f(R,T)}{\partial T} \ \text{ and } \nabla_i \ .$$

Here $f(R,T)$ denote arbitrary function of Ricci scalar R and the trace T of the stress energy tensor of matter T_{ij} . L_m Represent the matter Lagrangian density in the present study.

Here we have assumed that stress energy tensor of matter is given by,

$$T_{ij} = (P+\rho)u_i u_j - Pg_{ij} \tag{2.8}$$

And also assuming that the function $f(R,T)$ given by (Harko et al. [36])

$$f(R,T) = R + 2f(T) \tag{2.9}$$

Where $f(T)$ denote arbitrary function of trace of the stress energy tensor of matter.

Now use the equations (2.7) and (2.8), in an equation (2.6) we get the new form,

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} + 2f'(T)T_{ij} + (2Pf'(T) + f(T))\ g_{ij} \tag{2.10}$$

Where $f'(T)$ is differentiation $f(T)$ with respect to the argument T.
We take,

$$f(T) = \lambda T \tag{2.11}$$

Where, λ is constant (Harko et al[36])
Now in a co-moving co-ordinate systems use equations (2.2), (2.3), (2.4) and (2.11) in a field equation (2.10) ,for a metric (2.1) we get,

$$\left(\frac{a'}{a}\right)^2 + 2\left(\frac{a'b'}{ab}\right) + 2\left(\frac{a''}{a}\right) + \left(\frac{b''}{b}\right) = P(8\pi + 4\lambda) - \lambda\rho , \tag{2.12}$$

$$3\left(\frac{a'}{a}\right)^2 + 3\left(\frac{a''}{a}\right) = P(8\pi + 4\lambda) - \lambda\rho , \tag{2.13}$$

$$3\left(\frac{a'}{a}\right)^2 + 3\left(\frac{a'b'}{ab}\right) = (8\pi + 3\lambda)\rho - 2P\lambda . \tag{2.14}$$

Where overhead prime denotes differentiation with respect to cosmic time t only.
Now subtract equation (2.12) from (2.13) we get,

$$2\left(\frac{a'}{a}\right)^2 + \left(\frac{a''}{a}\right) - 2\left(\frac{a'b'}{ab}\right) - \left(\frac{b''}{b}\right) = 0 \tag{2.15}$$

$\therefore$ Rearranging equation (2.15) we get,

$$\frac{d}{dt}\left(\frac{a'}{a} - \frac{b'}{b}\right) + \left(\frac{a'}{a} - \frac{b'}{b}\right)\frac{V'}{V} = 0 \tag{2.16}$$

Since above equation has form of $\frac{dy}{dt} + P(t)y = Q(t)$ which is the linear differential equation of first order.
$\therefore$ Solution of equation (2.16) is,

$$\left(\frac{a'}{a} - \frac{b'}{b}\right)V = l_1 ,$$

where $l_1 =$ integrating constant.

$$\left(\frac{a'}{a} - \frac{b'}{b}\right) = \frac{l_1}{V}$$

$$\therefore d\left[\log\left(\frac{a}{b}\right)\right] = \frac{l_1}{V}$$

On integrating above equation both side we get,

$$\log\left(\frac{a}{b}\right) = \int \frac{l_1}{V} dt + m ,$$

here $m =$ integrating constant.

$$\therefore \left(\frac{a}{b}\right) = m_1 \exp\left(l_1 \int \frac{dt}{V}\right), \text{ here } m_1 = e^m \neq 0 \tag{2.17}$$

Now equation (2.17) becomes,

$$a = K_1 V^{1/4} \exp\left(d_1 \int \frac{dt}{V}\right), \tag{2.18}$$

Where $K_1 = m_1^{1/4}$, $d_1 = \frac{l_1}{4}$

Now from equation (2.17) and $V = a^3 b$ we get,

$$b = K_2 V^{1/4} \exp\left(d_2 \int \frac{dt}{V} \right) \tag{2.19}$$

Where $K_2 = m_1^{-3/4}$, $d_2 = \frac{-3l_1}{4}$

Where $K_i(i=1,2)$ and $d_i(i=1,2)$ satisfy the relation, $K_1^3 K_2 = 1$ and $3d_1 + d_2 = 0$

Put $K_1 = K$ therefore

$$K_2 = K^{-3} \tag{2.20}$$

And,

$$\text{Put } d_1 = D \text{ therefore } d_2 = -3D \tag{2.21}$$

$\therefore$ Equation (2.18) and (2.19) becomes,

$$a = KV^{1/4} \exp\left(D \int \frac{dt}{V} \right) \tag{2.22}$$

$$b = K^{-3} V^{1/4} \exp\left(-3D \int \frac{dt}{V} \right) \tag{2.23}$$

Where $K \neq 0$ and D are constant of integration.

3. Universe filled with perfect fluid and dark energy

We consider the evaluation of Kaluza-Klein space-time universe filled with perfect fluid and dark energy in f(R,T) frame in details. For the perfect fluid and dark energy, ρ is the energy density and P is the pressure.

$$\text{i.e. } \rho = \rho_{PF} + \rho_{DE} \quad , P = P_{PF} + P_{DE} \tag{3.1}$$

The energy momentum tensor can be decomposed as,

$$T_i^j = \left[\rho_{PF} + \rho_{DE} + P_{PF} + P_{DE}\right] u_i u^j - \left(P_{PF} + P_{DE}\right)\delta_i^j \tag{3.2}$$

In above equation ρ_{DE} is the dark energy density, P_{DE} its pressure. We also use the notation of ρ_{PF} and P_{PF} to denote the energy density and the pressure of the perfect fluid respectively.

Here we consider the case when the perfect fluid obeys' the following equations of state.

$$P_{PF} = \gamma \rho_{PF} \tag{3.3}$$

Where γ is the constant and lies in the interval $\gamma \in [0,1]$ depending on its numerical value we define types of universe as following,

$\gamma = 0$ (Dust universe) , $\gamma = \frac{1}{4}$ (Radiation universe)

$\gamma \in \left(\frac{1}{4}, 1\right)$ (Hard universe) , $\gamma = 1$ (Zeldovich universe or stiff matter)

In a co-moving frame the balance of the energy density is due to conservation law of energy momentum tensor.

$$\rho'_{DE} + \rho'_{PF} = -\frac{V'}{V}\left(\rho_{DE} + \rho_{PF} + P_{DE} + P_{PF}\right) \tag{3.4}$$

The dark energy is supposed to interact with itself only and is minimally coupled to the gravitational field.
As a result the evaluation equation for the energy density decouples from that of the perfect fluid and from equation **(3.3)** we obtain two balance equations.

$$\rho'_{PF} + \frac{V'}{V}\left(\rho_{PF} + P_{PF}\right) = 0 \tag{3.5}$$

And

$$\rho'_{DE} + \frac{V'}{V}\left(\rho_{DE} + P_{DE}\right) = 0 \tag{3.6}$$

$\therefore$ From equation **(3.3)** and **(3.5)**

$$\rho_{PF} = \frac{\rho_0}{V^{1+\gamma}} \;, P_{PF} = \frac{\gamma\rho_0}{V^{1+\gamma}} \tag{3.7}$$

Where ρ_0 is integrating constant.

3.1 Case with a Quintessence

Let us consider the case when dark energy is given by a quintessence which obeys' the equation of state,

$$P_q = \omega_q \rho_q \tag{3.8}$$

Where the constant ω_q varies between -1 and zero, i.e., $\omega_q \in [-1,0)$.
$\therefore$ From equation **(3.8)** and **(3.6),** we get

$$\rho_q = \frac{\rho_{0q}}{V^{1+\omega_q}} \quad , \quad P_q = \frac{\omega_q \rho_{0q}}{V^{1+\omega_q}} \tag{3.9}$$

where ρ_{0q} is integration constant.

3.2 Case with Chaplygin Gas

Let us consider the case when dark energy is represented by Chaplygin gas.

$$P_C = -\frac{\alpha}{\rho_C} \tag{3.10}$$

With α being a positive constant. From equation **(3.10)** and **(3.6),**
We get,

$$\rho_C = \left(\frac{\rho_{0C}}{V^2} + \alpha\right)^{\frac{1}{2}} \quad , \quad P_C = -\alpha\left(\frac{\rho_{0C}}{V^2} + \alpha\right)^{\frac{-1}{2}} \tag{3.11}$$

With ρ_{0C} being an integration constant.

4. Analysis and Discussion

Since we have the field equations **(2.12)** to **(2.14)** are highly non-linear. Thus extra conditions are needed to solve the system completely. For that we have used two different volumetric expansion laws.

$$V = Ce^{4lt} \tag{4.1}$$

And

$$V = Ct^{4n} \tag{4.2}$$

Where C, l and n are positive constant. In this way all possible expansion histories, the exponential expansion **(4.1)** power law expansion **(4.2)** have be covered.

The model with exponential expansion and power law totally depends on n.
As $n > 1$ exhibit accelerating volumetric expansion while for $n = 1$ Exhibits volumetric expansion with constant velocity, the model for $n < 1$ exhibits decelerating volumetric expansion.

5. Some Physical Parameters

Since the isotropy of the expansion can be parameterized after defining the directional Hubble's parameters and the average Hubble's parameters of the expansion .The directional Hubble's parameters in the direction of x, y, z and ϕ for the Kaluza-Klein metric define in **(2.1)**
Is defined as,

$$H_x = H_y = H_z = \frac{a'}{a} \text{ And } H_\phi = \frac{b'}{b} \tag{5.1}$$

$\therefore$ Average Hubble's parameter is given by,

$$H = \frac{1}{4}\frac{V'}{V} \quad \text{where } V = a^3 b \quad \text{is the volume of the universe} \tag{5.2}$$

$$\therefore H = \frac{1}{4}\left[3\left(\frac{a'}{a}\right) + \left(\frac{b'}{b}\right)\right] \tag{5.3}$$

$\therefore$ The Anisotropy parameter of the expansion is defined as,

$$\Delta = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i - H}{H}\right)^2 \tag{5.4}$$

Where $H_i(i = 1,2,3,4)$ represent the directional Hubble's parameters in the direction of x, y, z and ϕ respectively.

$\Delta = 0$ Corresponds to isotropic expansion. The space approaches isotropy,In a case of diagonal energy momentum tensor $(T^{0i} = 0, i = 1,2,3,4)$ if $\Delta \to 0, V \to \infty$ and $T^{00} > 0$. $(\rho > 0)$ as $t \to \infty$
(See [70] for details)

Let us introduce the dynamical scalars such as expansion parameters (θ) and shear (σ^2) as usual.

$$\theta = u^i_{;i} = 4H \tag{5.5}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} \tag{5.6}$$

where

$$\sigma_{ij} = \frac{1}{2}\left(u_{i;\alpha}P_j^\alpha + u_{j;\alpha}P_i^\alpha - \frac{1}{3}\theta P_{ij}\right) \tag{5.7}$$

Here the projection tensor P_{ij} has the form

$$P_{ij} = g_{ij} - u_i u_j \tag{5.8}$$

For the Kaluza-Klein metric **(2.1)** these dynamical scalars have the form,

$$\theta = 3\left(\frac{a'}{a}\right) + \left(\frac{b'}{b}\right) = 4H \tag{5.9}$$

$$\sigma^2 = \frac{2}{3}\left[\left(\frac{a'}{a}\right) - \left(\frac{b'}{b}\right)\right]^2 \tag{5.10}$$

5.1 Model for Exponential Expansion

After solving the field equations **(2.12)** to **(2.14)** for the exponential volumetric expansion **(4.1)** by considering the equation **(2.22)** and **(2.23)** we obtain the scale factor as follows,

$$a = KC^{1/4}e^{lt}\exp\left(\frac{-D}{4lC}e^{-4lt}\right) \tag{5.11}$$

$$b = K^{-3}C^{1/4}e^{lt}\exp\left(\frac{3D}{4lC}e^{-4lt}\right) \tag{5.12}$$

$\therefore$ The metric **(2.1)** with the help of **(5.11)** and **(5.12)** can now be written as,

$$ds^2 = dt^2 - K^2C^{1/2}e^{2lt}\exp\left(\frac{-D}{2lC}e^{-4lt}\right)\left(dx^2+dy^2+dz^2\right) - K^{-6}C^{1/2}e^{2lt}\exp\left(\frac{3D}{2l}e^{-4lt}\right)d\varphi^2 \tag{5.13}$$

The directional Hubble's parameters H_x, H_y, H_z and H_ϕ have values given by,

$$H_x = H_y = H_z = \frac{a'}{a} = \left[\frac{D}{C}e^{-4lt} + l\right] \tag{5.14}$$

$$H_\varphi = \left[\frac{-3D}{C}e^{-4lt} + l\right] \tag{5.15}$$

$\therefore$ Mean Hubble's parameter H by using equation (5.3) we get $H = l$.

$\therefore$ Anisotropy parameter of the expansion from equation (5.4) is,

$$\Delta = \frac{3D^2}{l^2C^2}e^{-8lt} \tag{5.16}$$

Here we observe that this behaviour of Δ is equivalent to the ones obtained for the model that correspond to the exponential expansion in Bianchi type –III cosmological model with anisotropic dark energy [43].

From equation (5.9) and (5.10) the dynamical scalars are given by,

$$\theta = 4l \tag{5.17}$$

$$\sigma^2 = \frac{32D^2}{3C^2} e^{-8lt} \tag{5.18}$$

$\therefore$ To study the non-singular behaviour of the model (5.13), we consider the Riemann-curvature invariant R for the metric (2.1) given by,

$$R = 20l^2 + \frac{12D^2}{C^2} e^{-8lt} \tag{5.19}$$

$\therefore R \to 20l^2$ as $t \to \infty$. Hence the model **(**5.13) is not a singular in the finite past and in the infinite future.
Now use equation (4.1) in (3.7), (3.9) and (3.11),
$\therefore$ We get,

$$\rho_{PF} = \frac{\rho_0}{\left(Ce^{4lt}\right)^{1+\gamma}} \quad , \quad P_{PF} = \frac{\gamma\rho_0}{\left(Ce^{4lt}\right)^{1+\gamma}} \tag{5.20}$$

$$\rho_{DE} = \frac{\rho_{0q}}{\left(Ce^{4lt}\right)^{1+\omega_q}} \quad , \quad P_{DE} = \frac{\omega_q\rho_{0q}}{\left(Ce^{4lt}\right)^{1+\omega_q}} \tag{5.21}$$

$$\rho_C = \left(\frac{\rho_{0C}}{\left(Ce^{4lt}\right)^2} + \alpha\right)^{\frac{1}{2}} \quad , \quad P_C = -\alpha\left(\frac{\rho_{0C}}{\left(Ce^{4lt}\right)^2} + \alpha\right)^{\frac{-1}{2}} \tag{5.22}$$

$\therefore$ The deceleration parameter $q = \dfrac{-aa''}{(a')^2} = \dfrac{d}{dt}\left(\dfrac{1}{H}\right) - 1 = -1$ (5.23)

5.2 Model for Power Law Expansion

After solving the field equations (2.12) to (2.14) for the Power Law expansion (4.2) by considering the equation (2.22) and (2.23) we obtain the scale factor as follows

$$a = KC^{1/4} t^n \exp\left(\frac{D}{C}\frac{t^{1-4n}}{(1-4n)}\right) \tag{5.24}$$

$$b = K^{-3}C^{1/4} t^n \exp\left(\frac{-3D}{C}\frac{t^{1-4n}}{(1-4n)}\right) \tag{5.25}$$

$\therefore$ The metric by using equation (5.24) and (5.25) we get,

$$ds^2 = dt^2 - K^2C^{1/2}t^{2n}\exp\left(\frac{2D}{C}\frac{t^{1-4n}}{(1-4n)}\right)\left(dx^2+dy^2+dz^2\right) - K^{-6}C^{1/2}t^{2n}\exp\left(\frac{-6D}{C}\frac{t^{1-4n}}{(1-4n)}\right)d\varphi^2 \tag{5.26}$$

The directional Hubble's parameters H_x, H_y , H_z and H_ϕ are,

$$H_x = H_y = H_z = \frac{a'}{a} = \frac{D}{Ct^{4n}} + \frac{n}{t} \quad , \quad H_\varphi = \frac{b'}{b} = \frac{-3D}{Ct^{4n}} + \frac{n}{t} \tag{5.27}$$

$\therefore$ Mean Hubble's parameter H by using equation (3.3) we get $H = \dfrac{n}{t}$.

$\therefore$ Anisotropy parameter of the expansion from equation (5.4) is,

$$\Delta = \frac{3D^2}{C^2 n^2 t^{8n-2}} \tag{5.28}$$

Form equation (5.9) and (5.10) the dynamical scalars are given by,

$$\theta = \frac{4n}{t} = 4H \tag{5.29}$$

$$\sigma^2 = \frac{32D^2}{3C^2 t^{8n}} \tag{5.30}$$

$\therefore$ Now to study the non-singular behaviour of the model (5.26).We consider the Riemann-curvature invariant R for the metric (2.1) given by,

$$R = \frac{20n^2}{t^2} + \frac{12D^2}{C^2 t^{8n}} - \frac{8n}{t^2} \tag{5.31}$$

Now use equation (4.2) in (3.7), (3.9) and (3.11),
We get,

$$\rho_{PF} = \frac{\rho_0}{\left(Ct^{4n}\right)^{1+\gamma}} \quad , P_{PF} = \frac{\gamma\rho_0}{\left(Ct^{4n}\right)^{1+\gamma}} \tag{5.32}$$

$$\rho_{DE} = \frac{\rho_{0q}}{\left(Ct^{4n}\right)^{1+\omega_q}} \quad , \quad P_{DE} = \frac{\omega_q \rho_{0q}}{\left(Ct^{4n}\right)^{1+\omega_q}} \tag{5.33}$$

$$\rho_C = \left(\frac{\rho_{0C}}{\left(Ct^{4n}\right)^2} + \alpha\right)^{\frac{1}{2}} \quad , \quad P_C = -\alpha\left(\frac{\rho_{0C}}{\left(Ct^{4n}\right)^2} + \alpha\right)^{\frac{-1}{2}} \tag{5.34}$$

$\therefore$ The deceleration parameter $q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = -(n-1)$ (5.35)

6. Some particular cases

Case-I For $\gamma = 0$, $\omega_q = 0$ the equations (5.20),(5.21),(5.32) and (5.33) becomes,

$$\rho_{PF} = \frac{\rho_0}{Ce^{4lt}} \quad , \quad P_{PF} = 0 \tag{6.1}$$

$$\rho_{DE} = \frac{\rho_{0q}}{Ce^{4lt}} \quad , \quad P_{DE} = 0 \tag{6.2}$$

$$\rho_{PF} = \frac{\rho_0}{Ct^{4n}} \quad , \quad P_{PF} = 0 \tag{6.3}$$

$$\rho_{DE} = \frac{\rho_{0q}}{Ct^{4n}} \quad , \quad P_{DE} = 0 \tag{6.4}$$

Case-II For $\gamma = 1$, $\omega_q = 0$ the equations (5.20),(5.21),(5.32) and (5.33) becomes,

$$\rho_{PF} = \frac{\rho_0}{C^2 e^{8lt}} = P_{PF} \tag{6.5}$$

$$\rho_{DE} = \frac{\rho_{0q}}{Ce^{4lt}} \quad , \; P_{DE} = 0 \tag{6.6}$$

$$\rho_{PF} = \frac{\rho_0}{C^2 e^{8n}} = P_{PF} \tag{6.7}$$

$$\rho_{DE} = \frac{\rho_0}{Ce^{4n}} \qquad , \qquad P_{DE} = 0 \tag{6.8}$$

Case-III For $\gamma = 1$, $\omega_q = -1$,the equations (5.20),(5.21),(5.32) and (5.33) becomes,

$$\rho_{PF} = \frac{\rho_0}{C^2 e^{8lt}} = P_{PF} \tag{6.9}$$

$$\rho_{DE} = \rho_{0q} \qquad , \quad P_{DE} = -\rho_{0q} \tag{6.10}$$

$$\rho_{PF} = \frac{\rho_0}{C^2 t^{8n}} = P_{PF} \tag{6.11}$$

Case-IV For $\gamma = \frac{1}{4}$, $\omega_q = \frac{-1}{4}$ the equations (5.20),(5.21),(5.32) and (5.33) becomes,

$$\rho_{PF} = \frac{\rho_0}{C^{\frac{5}{4}} e^{5lt}} \qquad , \qquad P_{PF} = \frac{\rho_0}{4C^{\frac{5}{4}} e^{5lt}} \tag{6.12}$$

$$\rho_{DE} = \frac{\rho_{0q}}{C^{\frac{3}{4}} e^{3lt}} \qquad , \qquad P_{DE} = \frac{-\rho_{0q}}{4C^{\frac{3}{4}} e^{3lt}} \tag{6.13}$$

$$\rho_{PF} = \frac{\rho_0}{C^{\frac{5}{4}} t^{5n}} \qquad , \quad P_{PF} = \frac{\rho_0}{4C^{\frac{5}{4}} t^{5n}} \tag{6.14}$$

$$\rho_{DE} = \frac{\rho_{0q}}{C^{\frac{3}{4}} t^{3n}} \qquad , \qquad P_{DE} = \frac{-\rho_{0q}}{4C^{\frac{3}{4}} t^{3n}} \tag{6.15}$$

Case-V For $\gamma = \frac{1}{2}$ (Hard Universe) , $\omega_q = \frac{-1}{2}$ the equations **(5.20),(5.21)**,**(5.32)** and **(5.33)** becomes,

$$\rho_{PF} = \frac{\rho_0}{C^{\frac{3}{2}} e^{6lt}} \qquad , \qquad P_{PF} = \frac{\rho_0}{2C^{\frac{3}{2}} e^{6lt}} \tag{6.16}$$

$$\rho_{DE}=\frac{\rho_{0q}}{C^{\frac{1}{2}}e^{2lt}} \quad , \quad P_{DE}=\frac{-\rho_{0q}}{2C^{\frac{1}{2}}e^{2lt}} \tag{6.17}$$

$$\rho_{PF}=\frac{\rho_{0}}{C^{\frac{3}{2}}e^{6n}} \quad , \quad P_{PF}=\frac{\rho_{0}}{2C^{\frac{3}{2}}e^{6n}} \tag{6.18}$$

$$\rho_{DE}=\frac{\rho_{0q}}{C^{\frac{1}{2}}e^{2n}} \quad , \quad P_{DE}=\frac{-\rho_{0q}}{2C^{\frac{1}{2}}e^{2n}} \tag{6.19}$$

7. Conclusion

We consider the Kaluza-Klein space-time with binary mixture of dark energy and perfect fluid in the frame of f(R,T) theory. To obtain the complete solution of field equations we assume the two expansion laws namely: exponential law and power law. We have been discussed the solution for both volumetric laws of expansion for constant deceleration parameter. An accelerated expansion of the model is due to the presence of the dark energy into the system. As a consequence the initial anisotropy of the model rapidly passes away. When $n>\frac{1}{4}$, anisotropy goes to decays and it becomes null monotonically in both volumetric laws of expansion, if $n>1$ then the universe tends to isotropy. Also for $\gamma\in[0,1]$, we studied five particular cases in details. It is note that our cosmological models bear a resemblance to [50].

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