Article

Five-Dimensional FRW Radiating Models in Brans-Dicke Theory of Gravitation

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Abstract

A five dimensional FRW space-time is considered in the frame work of Brans-Dicke (1961) scalar-tensor theory of gravitation in the presence of perfect fluid distribution. FRW cosmological models corresponding to flat, closed and open universes are obtained which represent radiating models in five dimensions. Some physical and kinematical properties of the models are also discussed.

Keywords: Radiating models, Brans-Dicke theory, FRW models.

1. Introduction

In recent years there has been an immense interest in the study of higher dimensional space-time because of the fact that the cosmos at its early stage of evolution of the universe might have had a higher dimensional era. This fact has attracted many researchers to the field of higher dimensions (Witten, 1984; Appelquist et al.1987; Chodos and Detweller 1980). It is well known that solutions of field equations in general relativity and in scalar-tensor theories in higher dimensional space-time are of physical relevance possibly at the early times before the universe has undergone compactification transitions. Also Marciano (1984) has suggested that the experimental observation of the fundamental constants with varying time could produce the evidence of extra dimensions.

Recently, modified theories of gravitation are attracting more and more attention of researchers. In particular scalar-tensor theories of gravitation proposed by Brans and Dicke (1961) and Saez and Ballester (1986) are playing vital role in the discussion of modern cosmology. The latest inflationary models (Mathiazhagan and Johri 1984), extended inflation (La et al.1990; Steinhardt and Accetta 1990) hyper extended inflation and extended chaotic inflation (Linde 1990) are based on Brans-Dicke (BD) theory of gravitation. Brans and Dicke (1961) introduced a scalar-

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tensor theory of gravitation involving a scalar field ϕ in addition to the familiar general relativistic metric tensor field g_{ij}. In this theory, the scalar field has the dimension of inverse of the gravitational constant and its role is confined to its effects on gravitational field equations. BD field equations for combined scalar and tensor fields are

$$G_{ij} = 8\pi\varphi^{-1}T_{ij} + \omega\varphi^{-2}\left(\varphi_{,i}\ \varphi_{,j} - \frac{1}{2}g_{ij}\,\varphi_{,k}\,\varphi^{,k}\right) + \varphi^{-1}\left(\varphi_{i;j} - g_{ij}\Box\varphi\right) \tag{1}$$

and

$$\Box\varphi = \varphi^{;k}_{;k} = 8\pi\varphi^{-1}T\left(3+2\omega\right)^{-1} \tag{2}$$

where $G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R$ is the Einstein tensor, ω is the dimensionless coupling constant, T_{ij} is the stress energy tensor of the matter and comma and semicolon denote partial and covariant differentiation respectively. BD cosmological models in four dimensions have been extensively studied by several authors. The work of Singh and Rai (1983) gives a detailed discussion of BD cosmological models in four-dimensional space-time.

Several authors investigated five dimensional cosmological models in BD theory of gravitation (Reddy et al.2007; Reddy et al. 2008; Naidu et al. 2013). Very recently, Reddy and Vijayalakshmi (2014) have discussed five dimensional Kaluza- Klein radiating cosmological model in BD theory. However, as far as our knowledge goes, much work has not been done on FRW five dimensional cosmological models in BD theory.

Motivated by the above discussion and investigations, we present, in this paper, five dimensional FRW flat, closed and open radiating models in BD theory of gravitation.

2. Metric and field equations

We consider five-dimensional FRW space-time in the following form

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right) + \left(1-kr^2\right)d\psi^2\right] \tag{3}$$

Here $k = +1, -1, 0$ represent closed, open and flat models respectively. The non-vanishing components of Einstein tensor for the metric given by equation (3) are

$$G_0^0 = 6\frac{\dot{a}^2}{a^2} + \frac{6k}{a^2} \tag{4}$$

$$G_1^1 = G_2^2 = G_3^3 = G_4^4 = 3\frac{\ddot{a}}{a} + 3\frac{\dot{a}^2}{a^2} + 3\frac{k}{a^2}$$

where an overhead dot indicates differentiation with respect to t.

We consider the energy momentum for a perfect fluid source in five dimensions as

$$T_{ij} = (\rho + p)u_i\, u_j - pg_{ij} \qquad i, j = 0,1,2,3,4 \tag{5}$$

where ρ is the energy density and p is is the isotropic pressure.

Also $u^i = \delta_0^i$ is a four velocity vector which satisfies

$$g_{ij}\, u^i\, u_j = 1 \tag{6}$$

so that we have

$$T_0^0 = \rho,\ T_1^1 = T_2^2 = T_3^3 = T_4^4 = -p,\ \ T_j^i = 0, \quad i \neq j \text{ and}$$

$$T = T_0^0 + T_1^1 + T_2^2 + T_3^3 + T_4^4 = \rho - 4p \tag{7}$$

Here we consider $\rho,\ p$ and ϕ as functions of cosmic time only.

Now, with the help of equations (4) - (7) the field equations (1) and (2) for the metric (3) can be written as

$$6\frac{\dot{a}^2}{a^2} + \frac{6k}{a^2} = 8\pi\varphi^{-1}\rho - \frac{\omega}{2}\left(\frac{\dot{\varphi}}{\varphi}\right)^2 - 4\frac{\dot{a}}{a}\frac{\dot{\varphi}}{\varphi} \tag{8}$$

$$3\frac{\ddot{a}}{a} + 3\frac{\dot{a}^2}{a^2} + \frac{3k}{a^2} = -8\pi\varphi^{-1}p + \frac{\omega}{2}\left(\frac{\dot{\varphi}}{\varphi}\right)^2 - 3\frac{\dot{a}}{a}\frac{\dot{\varphi}}{\varphi} - \frac{\ddot{\varphi}}{\varphi} \tag{9}$$

$$\ddot{\phi} + 4\frac{\dot{a}}{a}\dot{\phi} = 8\pi\phi^{-1}(3 + 2\omega)^{-1}(\rho - 4p) \tag{10}$$

Also, the equations of motion $T^{ij}_{;j} = 0$ are consequences of the field equations (1) and (2) which take the simplified form for the metric (3) as

$$\dot{\rho}+4\frac{\dot{a}}{a}(\rho+p)=0 \tag{11}$$

3. Solutions and the models

Here we have three independent field equations (8) - (10) connecting four unknown quantities a, ρ, p and ϕ. Hence to get a determinate solution one has to assume physical or mathematical conditions. Here we assume the physical condition

$$\rho=4p \tag{12}$$

and obtain physically viable models of the universe in five dimensional space-time. The condition (12) corresponds to disordered radiation in general relativity and yields a radiating FRW five dimensional model. Using equation (12) we solve the field equations (8) - (10) in the following physically important cases:

Case (i) k= +1, closed model

In this particular case, the field equations (8) - (10) with the help of equation (12) reduce to

$$6\frac{\dot{a}^2}{a^2}+\frac{6}{a^2}=8\pi\phi^{-1}\rho-\frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2-4\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} \tag{13}$$

$$3\frac{\ddot{a}}{a}+3\frac{\dot{a}^2}{a^2}+\frac{3}{a^2}=-8\pi\phi^{-1}p+\frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2+3\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi}+\frac{\ddot{\phi}}{\phi} \tag{14}$$

$$\ddot{\phi}+4\ \dot{a}\ \dot{\phi}=0 \tag{15}$$

Now using the well accepted power law relation between the scalar field and scale factor of the universe (Pimental 1985; Johri and Kalyani 1994)

$$\phi=\phi_0\ a^n \tag{16}$$

where n>0 is a constant.

The set of field equations yield the solution

$$a(t)=\left[\left(\frac{n+4}{n}\right)\left\{\left(\frac{a_0}{\phi_0}\right)t+t_0\right\}\right]^{\frac{1}{n+4}} \quad \text{and}$$

$$\phi=\phi_0\left[\left(\frac{n+4}{n}\right)\left\{\left(\frac{a_0}{\phi_0}\right)t+t_0\right\}\right]^{\frac{n}{n+4}} \tag{17}$$

Using proper choice of the coordinates and the integration constants (choosing $a_0=1$ and $t_0=0$) we can write the metric (3) in the form

$$ds^2=dt^2-\left[\left(\frac{n+4}{n\,\varphi_0}\right)t\right]^{\frac{2}{n+4}}\left[\frac{dr^2}{1-r^2}+r^2\left(d\theta^2+\sin^2\theta\,d\varphi^2\right)+\left(1-r^2\right)d\psi^2\right] \tag{18}$$

which represents radiating flat FRW model in five dimensions with the following physical properties:

The spatial volume of the model is

$$V=a^4=\left[\left(\frac{n+4}{n\,\phi_0}\right)t\right]^{\frac{4}{n+4}} \tag{19}$$

The scalar field ϕ in the model is

$$\phi=\phi_0\left[\left(\frac{n+4}{n\,\phi_0}\right)t\right]^{\frac{n}{n+4}} \tag{20}$$

The Hubble's parameter H of the model is

$$H=\frac{\dot{a}}{a}-\frac{1}{(n+4)t} \tag{21}$$

The energy density and pressure in the model is(in view of equation (12))

$$8\pi\rho=\varphi_0\left[\left(\frac{n+4}{n\,\varphi_0}\right)t\right]^{\frac{n}{n+4}}\left[\frac{\omega n^2+4n+12}{(n+4)^2t^2}+6\left\{\frac{n+4}{n\,\varphi_0}\right\}t\right]^{\frac{-2}{n+4}}=32\pi\,p \tag{22}$$

The deceleration parameter q in the model is

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = n + 3 \tag{23}$$

Case (iii) k= -1, open model

In this particular case the model is open and is given by

$$ds^2 = dt^2 - \left[\left(\frac{n+4}{n\,\phi_0}\right)t\right]^{\frac{2}{n+4}}\left[\frac{dr^2}{1+r^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right) + \left(1+r^2\right)d\psi^2\right] \tag{24}$$

with energy density and the pressure given by

$$8\pi\rho = \phi_0\left[\left(\frac{n+4}{n\,\phi_0}\right)t\right]^{\frac{n}{n+4}}\left[\frac{\omega n^2 + 4n + 12}{(n+4)^2 t^2} - 6\left\{\frac{n+4}{n\,\phi_0}\right\}t\right]^{\frac{-2}{n+4}} = 32\pi\, p \tag{25}$$

The scalar field is given by $\phi = \phi_0\left[\left(\frac{n+4}{n\,\phi_0}\right)t\right]^{\frac{n}{n+4}}$

Case (iii): $k = 0$, **Flat model**

In this case, again, the model is given by

$$ds^2 = dt^2 - \left[\left(\frac{n+4}{n\,\phi_0}\right)t\right]^{\frac{2}{n+4}}\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right) + d\psi^2\right] \tag{26}$$

with $k = 0$, energy density and pressure given by

$$8\pi\rho = \phi_0\left[\left(\frac{n+4}{n\,\phi_0}\right)t\right]^{\frac{n}{n+4}}\left[\frac{\omega n^2 + 4n + 12}{(n+4)^2 t^2}\right] = 32\pi\, p \tag{27}$$

The scalar field is, again, given by

$$\phi = \phi_0\left[\left(\frac{n+4}{n\,\phi_0}\right)t\right]^{\frac{n}{n+4}} \tag{28}$$

4. Discussion

The models given by equation (18), (24) and (26) represent radiating closed, open or flat models in five dimensional FRW space-time respectively. The models are singularity free at the initial epoch, i.e., at $t = 0$. As t increases the spatial volume, the scalar field ϕ increases in all the above cases while the Hubble's parameter, the energy density and the pressure decrease with time. Here, in each case, the deceleration parameter is positive. Hence the radiating models in five dimensions decelerate in the standard way. However, the universes will accelerate in finite time after compactification transition and cosmic re-collapse (Nojiri & Odintsov 2003).

5. Conclusions

In this paper, cosmological models have been obtained in the context of Brans-Dicke (1961) theory by considering FRW five dimensional space -time in the presence of perfect fluid which can be considered as analogous radiating models in closed, open and flat space-times. It is observed that the model in all the case is expanding and free from initial singularity. Also the model decelerates in the standard way which will accelerate in finite time after compactification transition and 'cosmic re-collapse' which will be in accordance with the present day scenario of accelerated expansion of the universe.

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