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FRW Closed Bulk Viscous Cosmological Model in General Scalar Tensor Theory of Gravitation

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Abstract

Spatially homogeneous and isotropic FRW metric is considered in the framework of Nordtvedt [1] general scalar tensor theory of gravitation when the source for the energy momentum tensor is bulk viscous wet dark fluid with the help of a special case proposed by Schwinger [2]. Here, it is possible to obtained only FRW closed cosmological model. The physical aspects of the obtained model are also discussed.

Key words: FRW metric, wet dark fluid, bulk viscosity, scalar-tensor theory.

1 Introduction

Recent cosmological observations contradict the matter dominated Universe with decelerating expansion indicating that our Universe experiences accelerated expansion. Before the accelerating expansion of the Universe was revealed by high red-shift supernovae Ia (SNe Ia) observations (Riess et al. [3]; Perlmutter et al. [4]) it could hardly be presumed that the main ingredients of the Universe are dark sectors. The concept of dark energy was proposed for understanding this currently accelerating expansion of the Universe, and then its existence was confirmed by several high precision observational experiments (Bennet et al. [5]; Hinshaw et al. [6]) especially the Wilkinson Microwave Anisotropy Probe(WMAP) satellite experiment. Cosmologists have proposed many candidates for dark energy to fit the current observations such as cosmological constant, tachyon, quintessence, phantom and so on.

There is a new candidate for dark energy: Wet Dark Fluid (WDF). This model is in the spirit of generalized chaplygin gas (GCG), so, we are motivated to use the wet dark fluid as a model for dark energy which stems from an empirical equation of state proposed by Hayward [7] and Tait [8] to treat water and aqueous solution.

The equation of state for wet dark fluid is very simple

$$P_{WDF} = \omega(\rho_{WDF} - \rho^*) \tag{1.1}$$

and is motivated by the fact that it is good approximation for many fluids, including water, in which the internal attraction of the molecules makes negative pressures possible. One of the virtues of this model is that the square of the sound speed, c_s^2, which depends on $\frac{\partial P}{\partial \rho}$, can be positive, while still giving rise to cosmic acceleration in the current epoch.

We treat (1.1) as a phenomenological equation [9]. Holman et al. [10] have shown that this model can be made consistent with the most recent SNIa data [11], the WMAP results [5, 12] as well as constraints coming from measurements of the matter power spectrum [13]. The parameters ω and ρ^* are taken to be positive and we restrict ourselves to $0 \leq \omega \leq 1$. Note that if c_s denotes the adiabatic sound speed in WDF, then $\omega = c_s^2$ (ref [14]).

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To find the WDF energy density, we use the energy conservation equation

$$\dot{\rho}_{WDF} + 3H(P_{WDF} + \rho_{WDF}) = 0 \tag{1.2}$$

From equation of state (1.1) and using $3H = \frac{3\dot{v}}{v}$ in the above equation, we get

$$\rho_{WDF} = \frac{\omega}{1+\omega}\rho_* + \frac{k}{v^{1+\omega}} \tag{1.3}$$

where k is the constant of integration and v is the volume expansion. WDF naturally includes two components: a piece that behaves as a cosmological constant as well as a standard fluid with an equation of state $P = \omega\rho$. We can show that if we take $c > 0$, this fluid will not violate the strong energy condition $P + \rho \geq 0$:

$$P_{WDF} + \rho_{WDF} = (1+\omega)\rho_{WDF} - \omega\rho^* = (1+\omega)\frac{k}{v^{1+\omega}} \geq 0 \tag{1.4}$$

Singh & Chaubey [15] have studied Bianchi type I Universe with wet dark fluid in general relativity. Adhav et al. [16] have studied Einstein-Rosen Universe with wet dark fluid in general relativity. Prateek et al. [17] have investigated axially symmetric cosmological model with wet dark fluid in bimetric theory of gravitation. Samantha [18] has discussed Bianchi type V Universe filled with dark energy (DE) from a wet dark fluid (WDF) in the framework of $f(R,T)$ gravity. Recently, Mishra and Sahoo [19] have investigated Bianchi type VI_1 cosmological model with wet dark fluid in scale invariant theory of gravitation.

At the early stages of the Universe when neutrinos decoupling occurred, the matter behaved like a viscous fluid. The coefficient of viscosity decreases as the Universe expands. Misner [20, 21] studied the effect of viscosity on the evolution of the Universe and suggested that the strong dissipation, due to the neutrino viscosity, may considerably reduce the anisotropy of the black body radiation. Murphy [22] developed a uniform cosmological model filled with fluid which possesses pressure and bulk viscosity exhibiting the interesting feature that the big-bang type singularity appears in the infinite past. The possibility of bulk viscosity leading to inflationary phase. The possibility of bulk viscosity leading to inflationary-like solution in general relativistic FRW models has been discussed by several authors like Barrow [23], Padmanabhan and Chitre [24], Pavon et al. [25], Martens [26], Lima et al. [27]. It is well known that the bulk viscosity contributes a negative pressure term giving rise to an effective total negative pressure leading to a repulsive gravity. This overcomes the attractive gravity of the matter and gives an impetus for rapid expansion of Universe. Bali and Pradhan [28], Mahesh et al. [29], Tripathy et al. [30], Singh and Kale [31], Shri Ram et al. [32], Rao and Sireesha [33] and Naidu et al. [34] are some of the authors who have investigated cosmological models with bulk viscosity in different theories of gravitation. Recently, Saadat and Pourhassan [35] have studied effect of varying bulk viscosity on generalized chaplygin gas.

Nordtvedt [1] proposed a general scalar-tensor gravitational theories in which the parameter ω of the Brans-Dicke theory is allowed to be an arbitrary function of the scalar field ($\omega \to \omega(\phi)$). This general class of scalar-tensor gravitation theories includes the Jordan [36] and Brans-Dicke [37] theories as special cases.

Ruban and Finkelstein [38], Barker [39], Banerjee and Santos [40, 41], and Santhi & Rao [42, 43] are some of the authors who have investigated several aspects of Nordtvedt general scalar-tensor theory. Rao and Kumari [44] have discussed a cosmological model with negative constant deceleration parameter in this theory. Rao et al. [45] have obtained Kaluza-Klein radiating model in a general scalar-tensor theory of gravitation. Rao et al. [46] have studied Kantowski-Sachs string cosmological model with bulk viscosity in general scalar-tensor theory of gravitation. Rao and Neelima [47] have studied Bianchi type-VI_0 space time with strange quark matter attached to string cloud in general scalar-tensor theory. Recently, Rao et al. [48] have investigated Kantowski-Sachs dark energy cosmological model in general scalar-tensor theory of gravitation.

The field equations of general scalar-tensor theory are

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi\phi^{-1}T_{ij} - \omega\phi^{-2}\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) - \phi^{-1}\left(\phi_{i;j} - g_{ij}\phi^{,k}_{;k}\right), \tag{1.5}$$

$$\phi^{;k}_{;k} = \frac{8\pi T}{3+2\omega} - \frac{\phi_{,k}\phi^{,k}}{3+2\omega}\frac{d\omega}{d\phi}, \tag{1.6}$$

where R_{ij} is the Ricci tensor, R is the scalar curvature, T_{ij} is the stress energy tensor of the matter, comma and semicolon denote partial and covariant differentiation, respectively.
Also, we have energy conservation equation as

$$T^{ij}_{;j} = 0. \tag{1.7}$$

In this paper, we investigate FRW bulk viscous cosmological model with wet dark fluid in general scalar tensor theory proposed by Nordtvedt. This paper is organized as follows: In section 2, we discuss metric, energy momentum tensor of matter fluid and field equations. In section 3, the solutions of the field equations are obtained. In section 4, we discuss some other important properties of the model. Finally, the conclusions of the obtained model are presented in section 5.

2 Metric and the field equations

We consider the spatially homogeneous and isotropic FRW space-time given by

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + d\phi^2 sin^2\theta)\right], \tag{2.1}$$

where $a(t)$ is the scale factor and the curvature constant k takes the values -1, 0, +1 for open, flat and closed models of the Universe respectively.

The energy-momentum tensor of the matter source is given by

$$T^i_j = (P_{eff} + \rho_{WDF})u^i u_j - P_{eff} \tag{2.2}$$

where ρ_{WDF} is the energy density of wet dark fluid, u_i is the four-velocity vector satisfying the relation $u^i u_i = 1$ and P_{eff} represents the effective pressure defined by

$$P_{eff} = P_{WDF} + P_{vis}. \tag{2.3}$$

Here P_{WDF} denotes the pressure due to wet dark fliud and P_{vis} represents the pressure due to viscosity. The bulk viscous pressure is defined by Eckart's [49] expression in terms of fluid expansion scalar and is given by $P_{vis} = -\xi u^i_{;i}$, where ξ represents the bulk viscous coefficient. For FRW model, the viscous pressure is found to be $P_{vis} = -3\xi\frac{\dot{a}}{a}$ and hence the effective pressure becomes

$$P_{eff} = P_{WDF} - 3\xi\frac{\dot{a}}{a}. \tag{2.4}$$

In the comoving coordinate system, from equation (2.2) we have,

$$T^1_1 = T^2_2 = T^3_3 = -P_{eff} \quad and \quad T^4_4 = \rho_{WDF} \tag{2.5}$$

For metric (2.1), using equation (2.5), the field equations (1.5) and (1.6) takes the form

$$\frac{\dot{a}^2}{a^2} + 2\frac{\ddot{a}}{a} + \frac{k}{a^2} + \frac{1}{2}\omega\phi^2\dot{\phi}^2 + 2\frac{\dot{a}\dot{\phi}}{a\phi} + \frac{\ddot{\phi}}{\phi} = -8\pi P_{eff} \tag{2.6}$$

$$3\frac{\dot{a}^2}{a^2} + 3\frac{k}{a^2} - \frac{1}{2}\omega\phi^2\dot{\phi}^2 + \frac{\dot{a}\dot{\phi}}{a\phi} = 8\pi\rho_{WDF} \tag{2.7}$$

$$\ddot{\phi}+\dot{\phi}\left(\frac{3\dot{a}}{a}\right)=\frac{8\pi}{3+2\omega}(\rho_{WDF}-3P_{eff})-\frac{1}{3+2\omega}\frac{d\omega}{d\phi}\dot{\phi}^2. \tag{2.8}$$

The conservation equation (1.7), yields

$$\dot{\rho}_{WDF}+3\frac{\dot{a}}{a}(P_{eff}+\rho_{WDF})=0, \tag{2.9}$$

where overhead dot represents ordinary differentiation with respect to cosmic time t.

By using the transformation $dt=a^3dT$, the above field equations can be written as

$$2\frac{a''}{a}-5\frac{a'^2}{a^2}-\frac{a'\phi'}{a\phi}+\frac{\omega\phi'^2}{6\phi^2}+\frac{\phi''}{\phi}+ka^4 = -\frac{8\pi P_{eff}}{\phi}a^6 \tag{2.10}$$

$$3\frac{a'^2}{a^2}+3\frac{a'\phi'}{a\phi}-\frac{\omega\phi'^2}{2\phi^2}+3ka^4 = \frac{8\pi\rho_{WDF}}{\phi}a^6 \tag{2.11}$$

$$(3+2\omega)\phi''+\phi'^2\frac{d\omega}{d\phi} = 8\pi(\rho_{WDF}-3P_{eff})a^6 \tag{2.12}$$

$$\rho\prime_{WDF}+3\frac{a'}{a}(P_{eff}+\rho_{WDF}) = 0 \tag{2.13}$$

where overhead dash denotes ordinary differentiation with respect to T.

3 Solutions of the field equations

The field equations (2.10) to (2.12) are three independent equations with five unknowns a, ω, ϕ, P_{eff} and ρ_{WDF}. Here we obtain bulk viscous cosmological model with wet dark fluid in Nordtvedt's general scalar-tensor theory with the help of a special case proposed by Schwinger(1970) in the form

$$3+2\omega(\phi)=\frac{1}{\lambda\phi} \tag{3.1}$$

From equations (2.10) to (2.12), we have

$$(3+2\omega)\phi''+\frac{d\omega}{d\phi}\phi'^2=6\phi\left(\frac{a''}{a}-2\frac{a'^2}{a^2}+ka^4\right)+\omega\frac{\phi'^2}{\phi}+3\phi''. \tag{3.2}$$

From equations (3.1) and (3.2), we get

$$\frac{1}{\lambda}\left(\frac{\phi''}{\phi}-\frac{\phi'^2}{\phi^2}\right)+\frac{3}{2}\frac{\phi'^2}{\phi}-3\phi''=6\phi\left(\frac{a''}{a}-2\frac{a'^2}{a^2}+ka^4\right). \tag{3.3}$$

From equation (3.3), we get scalar field

$$\phi=e^{k_1T+k_2}, \tag{3.4}$$

where k_1 and k_2 are arbitrary constants and

$$a=(k_3 sechk_1T)^{1/2}; \qquad k_3=\frac{k_1}{2\sqrt{k}} \tag{3.5}$$

where $k>0$, so we can obtain only FRW closed (i.e $k=+1$) bulk viscous cosmological model with wet dark fluid in general scalar-tensor theory.

The metric (2.1), can now be written as

$$ds^2 = (k_3 sechk_1T)^3 dT^2 - (k_3 sechk_1T)^{1/2}\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + d\phi^2 sin^2\theta)\right]. \tag{3.6}$$

From equation (2.10), we get effective pressure

$$P_{eff} = \frac{e^{k_1T+k_2}}{32\pi k_3^3}(cosh^3k_1T)\{k_1(5k_1^2+6)tanh^2k_1T - 2k_1tanhk_1T - 4kk_3^2sech^2k_1T - (2\omega+4)k_1^2 - 4k_1\} \tag{3.7}$$

and pressure due to wet dark fluid

$$P_{WDF} = \frac{-3\xi k_1 tanhk_1T}{2(k_3sechk_1T)^{3/2}} + \frac{e^{k_1T+k_2}}{32\pi k_3^3}(cosh^3k_1T)\{k_1(5k_1^2+6)tanh^2k_1T - 2k_1tanhk_1T - 4kk_3^2sech^2k_1T - (2\omega+4)k_1^2 - 4k_1\}. \tag{3.8}$$

From equation (2.11), we get energy density

$$\rho_{WDF} = \frac{3e^{k_1T+k_2}}{8\pi k_3^3}(cosh^3k_1T)\left\{\frac{k_1^2}{4}tanh^2k_1T - \frac{k_1^2}{2}tanhk_1T + kk_3^2sech^2k_1T - \frac{\omega}{6}k_1^2\right\} \tag{3.9}$$

Thus, the metric (3.6) together with (3.7), (3.9) and (3.4) constitutes FRW closed bulk viscous cosmological model with wet dark fluid in general scalar-tensor theory of gravitation.

Figure 1, describes the effective pressure (P_{eff}) and energy density (ρ_{WDF}) versus T. It is observed that pressure and energy density diverge with the increase of T.

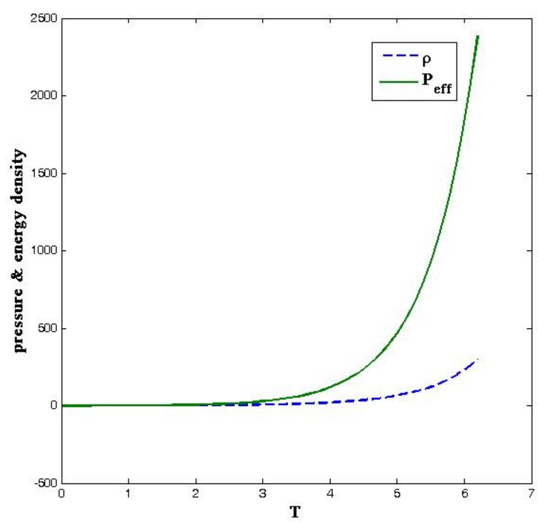

Figure 1: Plot of pressure and energy density versus T.

4 Some other important properties of the model

- The volume element of the model (3.6), is given by

$$\begin{aligned} V &= \sqrt{-g} \\ &= \frac{r^2 sin\theta}{\sqrt{1-kr^2}}(k_3 sechk_1 T)^{3/2} \end{aligned} \tag{4.1}$$

and scale factor a is given by

$$\begin{aligned} a &= V^{1/3} \\ &= \left(\frac{r^2 sin\theta}{\sqrt{1-kr^2}}\right)^{1/3}(k_3 sechk_1 T)^{1/2} \end{aligned} \tag{4.2}$$

- The expansion scalar θ is given by

$$\begin{aligned} \theta &= 3\frac{\dot{a}}{a} \\ &= -\frac{3k_1 tanhk_1 T}{2(k_3 sechk_1 T)^{3/2}} \end{aligned} \tag{4.3}$$

- The shear scalar is given by

$$\begin{aligned} \sigma^2 &= \frac{1}{2}\sigma^{ij}\sigma_{ij} \\ &= \frac{7(k_1 tanhk_1 T)^2}{8(k_3 sechk_1 T)^3} \end{aligned} \tag{4.4}$$

- The Hubble's parameter is given by

$$\begin{aligned} H &= \frac{\dot{a}}{a} \\ &= -\frac{k_1 tanhk_1 T}{2(k_3 sechk_1 T)^{3/2}} \end{aligned} \tag{4.5}$$

- The vorticity tensor $w_{ij} = u_{i,j} - u_{j,i}$ is a measure of the rotation of the local rest-frame relative to the compass of inertia, is identically zero. Hence the fluid filling the Universe is non-rotational.

- Deceleration parameter q is given by

$$\begin{aligned} q &= -\frac{\ddot{a}a}{\dot{a}^2} \\ &= 2 + \frac{2cosechk_1 T}{k_1 tanhk_1 T} \end{aligned} \tag{4.6}$$

From figure 2, we observed that deceleration parameter q is always positive then the Universe decelerates. Viswakarma [50] has shown that decelerating models are also consistent with recent cosmic background observations made by WAMP as well as with the high-redshift supernova Ia data including SN 1997 ff at $Z = 1.775$. Also, we observed the expansion scalar (θ) is decreasing function of T.

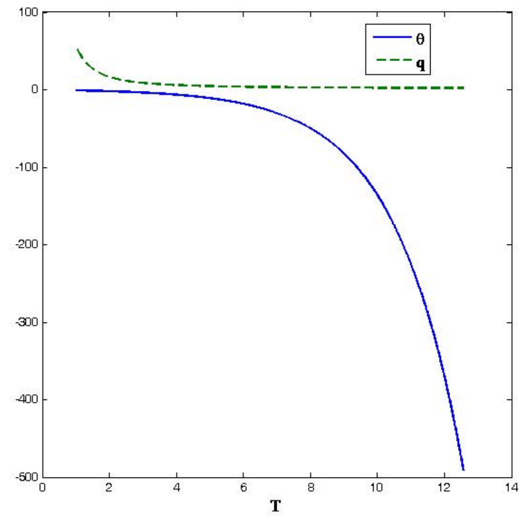

Figure 2: Plot of deceleration parameter(q) and expansion scalar(θ) versus T.

- Jerk parameter in cosmology is defined as the dimensionless third derivative of scale factor with respect to time and is given by

$$\begin{aligned} j &= \frac{1}{H^3}\frac{\dddot{a}}{a} \\ &= 10 + \frac{18cosechk_1T}{k_1tanhk_1T} + \frac{8cosech^2k_1T}{k_1^2tanh^2k_1T} - \left[\frac{2cosechk_1T}{k_1tanh^3k_1T}(1+sech^2k_1T)\right] \end{aligned} \tag{4.7}$$

5 Conclusions

In this paper, we have obtained and presented FRW closed bulk viscous cosmological model with wet dark fluid in Nordtvedt general scalar-tensor theory with the help of a special case proposed by Schwinger. The model presented here is free from singularities, and spatial volume vanishes as $T \to \infty$. The energy density and pressure are diverge with increase of T. Deceleration parameter $q > 0$ for all values of T, hence the model represents standard decelerating Universe. However, in spite of the fact that the Universe, in this case, decelerates in the standard way it will accelerate in finite time due to cosmic re collapse where the Universe in turns inflates "decelerates and then accelerates" (Nojiri and Odintsov [51]).

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