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Kaluza-Klein Minimally Interacting Holographic Dark Energy Model in a Scalar-sensor Theory of Gravitation

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Abstract

A five dimensional Kaluza – Klein space – time filled with two minimally interacting fields, matter and holographic dark energy components is investigated in the scalar – tensor theory of gravitation proposed by Saez and Ballester (Phys, Lett, A **113**; 467, 1986). To obtain a determinate solution of the field equations we have used the fact that the scalar expansion is proportional to the shear scalar. Some physical and kinematical properties of the model are also discussed.

Keywords: Scalar – tensor theory, Kaluza – Klein space time, Holographic dark energy

1. Introduction

Recent observational data of modern cosmology based on various measurements reveals that our universe is experiencing transition from early inflation to the late time acceleration Reiss et al.1998; Perlmutter et al.!999). The main source responsible for this acceleration is supposed to be 'dark energy'. The concept of dark energy refers to a kind of exotic energy with negative pressure whose origin still remains a mystery. However, it is now believed that the universe consists of 76% dark energy, 20% dark matter and 4% ordinary matter. Two distinct approaches have been suggested to explain cosmic acceleration. The first approach deals with modifying Einstein gravity where in additional energy component is introduced to explain the concept of dark energy. In this approach a number of alternative models have been proposed but so far no suitable candidate is found. The other approach is to modify the Einstein Lagrangian by replacing the scalar curvature by a function of R known as f(R) gravity(Nojiri and Odintsov2007). Other modified theories are f(R,T) gravity (Harko et al.2011), Brans – Dicke (1961) and Saez – Ballester (1986) scalar – tensor theories of gravity. Brans – Dicke theory introduces an additional scalar field ϕ, besides the metric tensor g_{ij}, which has the dimension of the inverse of gravitational constant and which interacts equally with all forms of matter.

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Saez and Ballester have developed a new scalar – tensor theory of gravitation in which the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of the weak fields. In spite of the dimensionless character of the scalar fields an antigravity regime appears in the theory. Also this theory suggests a possible way to "missing matter problem" in non – flat FRW cosmologies.

Spatially homogeneous and anisotropic cosmological models which provide a richer structure, both geometrically and physically, than FRW models play a significant role in the description of the early universe. Bianchi type dark energy models with usual perfect fluid have been studied by Akarsu and Kilinc(2010, 2012),Yadav etal.(2010), Adhav et al.(2012),Saha and Yadav(2012) and Naidu et al.(2012a, 2012b).

Using the holographic principle of quantum gravity theory (Susskind1995) a viable holographic dark energy model was constructed by Li (2004). The holographic dark energy model is successful in explaining the observational data and has been, widely studied by several authors. In particular, Sarkar and Mahanta [2013] have discussed the evolution of holographic dark energy in Bianchi type-I space – time with constant deceleration parameter which Sarkar (2014) has investigated holographic dark energy model in Bianchi type – I universe with linearly varying deceleration parameter and established a correspondence with generalized Chapligin gas models of the universe. Also holographic scalar field dark energy models are studied by many authors. For instance, Setare (2007) studied holographic dark energy model in Brans – Dicke theory. Setare and Vanegas (2009) have discussed the cosmological dynamics of interacting holographic dark energy model. Very recently Kiran et al. (2014) have investigated Bianchi type –V minimally interacting holographic dark energy model in the scalar tensor theory of gravitation proposed by Saez and Ballester.

Higher dimensional cosmology is important because it has physical relevance to the early stages of evolution of the universe before it has undergone compactification transitions. Hence several authors (Witten, 1984; Chodos and Detweller,1980;Appelquist, et al.1987; Marchiano,1986) were attracted to the study of higher dimensional cosmology. Also, in the context of Kaluza – Klein and super string theories higher dimensions have recently acquired much significance. Several investigations have been made in higher dimensional cosmology in the frame work of different scalar – tensor theories. In particular, Reddy et al (2012) have discussed a five dimensional Kaluza – Klein cosmological model in the presence of perfect fluid in f(R,T) gravity.

Motivated by the above investigations and discussions we investigate, in this paper, Kaluza-Klein minimally interacting holographic dark energy model in a Saez-Ballester scalar- tensor theory of gravitation. The plan of this paper is as follows: In section2, Saez-Ballester field equations are obtained in Kaluza-Klein space- time in the presence of matter and holographic dark energy. Section 3, deals with the exact solution of the field equations. Section 4, is devoted to the discussions of the physical and kinematical properties of the model. Some conclusions are presented in the last section.

2. Metric and Field Equations

We consider five dimensional Kaluza – klein metric in the form

$$ds^2 = -dt^2 + A^2\left(dx^2 + dy^2 + dz^2\right) + B^2 d\omega^2 \tag{1}$$

where A and B are functions of cosmic time t and the fifth coordinate is taken to be space – like unlike Wesson (1983). Here the spatial curvature has been taken as zero (Gron(1988).

The energy momentum tensors for matter and the holographic energy are defined as

$$T_{ij} = \rho_m u_i u_j \tag{2}$$

$$\bar{T}_{ij} = \left(\rho_\lambda + p_\lambda\right) u_i u_j + p g_{ij} \tag{3}$$

Where $\rho_{m,}\rho_\lambda$ are the energy densities of matter and the holographic dark energy and p_λ is the pressure of the holographic dark energy.

Saez – ballester field equations for the combined scalar and tensor fields are given by

$$R_{ij} - \frac{1}{2} g_{ij} R - \omega\phi^n\left(\phi_{,i}\phi_{,j} - \frac{1}{2} g_{ij}\phi_{,k}\varphi^{,k}\right) = \left(T_{ij} + \bar{T}_{ij}\right) \tag{4}$$

The scalar field ϕ satisfies the following equation

$$2\phi^n\phi^{;i}_{;i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \tag{5}$$

Also, the energy conservation equation is $T^{ij}_{;j} + \bar{T}^{ij}_{;j} = 0$ (6)

In a commoving coordinate system Saez – Ballester field equations (4) and (5) for the metric (1) with the help of Eqs. (2) and (3) can be, explicitly written as

$$\frac{3\dot{A}^2}{A^2} + \frac{3\dot{A}\dot{B}}{AB} - \frac{\omega}{2}\phi^n\dot{\phi}^2 = \rho_m + \rho_\lambda \tag{7}$$

$$\frac{2\ddot{A}}{A}+\frac{\dot{A}^2}{A^2}+\frac{2\dot{A}\dot{B}}{AB}+\frac{\ddot{B}}{B}+\frac{\omega}{2}\phi^n\dot{\phi}^2=-p_\lambda \tag{8}$$

$$\frac{3\ddot{A}}{A}+\frac{3\dot{A}^2}{A^2}+\frac{\omega}{2}\phi^n\dot{\phi}^2=-p_\lambda \tag{9}$$

$$\frac{\ddot{\phi}}{\phi}+\frac{\dot{\phi}}{\phi}\left(\frac{3\dot{A}}{A}+\frac{\dot{B}}{B}\right)+\frac{n}{2}\frac{\dot{\phi}^2}{\phi^2}=0 \tag{10}$$

where an over head dot denotes differentiation with respected to t. Using barotropic equation of state (EOS)

$$p_\lambda=\omega\rho_\lambda \tag{11}$$

We can write the continuity equation (6) of the matter and dark energy as

$$\dot{\rho}_m+\rho_m\left(\frac{3\dot{A}}{A}+\frac{\dot{B}}{B}\right)+\dot{\rho}_\lambda+\left(\frac{3\dot{A}}{A}+\frac{\dot{B}}{B}\right)(1+\omega)\rho_\lambda=0 \tag{12}$$

Here we are considering the minimally interacting matter and holographic dark energy components. Hence both components conserve separately, so that we have (Sarkar, 2014)

$$\dot{\rho}_m+\rho_m\left(\frac{3\dot{A}}{A}+\frac{\dot{B}}{B}\right)=0 \tag{13}$$

$$\dot{\rho}_\lambda+\left(\frac{3\dot{A}}{A}+\frac{\dot{B}}{B}\right)(1+\omega)\rho_\lambda=0 \tag{14}$$

The following are the physical and geometrical parameters to be used in solving the Saez–Ballester field equations for the space – time given by Eq. (1).

The average scale factor a(t) of the Kaluza – Klein space – time is defined as

$$a(t)=\left(A^3B\right)^{1/4} \tag{15}$$

The spatial volume of the universe is $V=a^4(t)=A^3B$ (16)

The average Hubble's parameter is $H = \frac{1}{4}\left(\frac{3\dot{A}}{A} + \frac{\dot{B}}{B}\right)$ (17)

The average anisotropic parameter A_h is defined as $A_h = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{\Delta H_i}{H}\right)^2$ (18)

Where $\Delta H_i = H_i - H$ and H_i (i=1,2,3,4) represent the directional Hubble parameters.

The dynamical scalar expansion θ and the shear scalar σ^2 are $\theta = \frac{3\dot{A}}{A} + \frac{\dot{B}}{B}$

$$\theta = 4H = \frac{3\dot{A}}{A} + \frac{\dot{B}}{B} \tag{19}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{4}\left(\frac{3\dot{A}}{A} + \frac{\dot{B}}{B}\right)^2 \tag{20}$$

3. Solutions and the model

Field equations (7) – (10) yield the following independent equations

$$\frac{3\dot{A}^2}{A} + \frac{3\dot{A}\dot{B}}{AB} - \frac{\omega}{2}\phi^n\dot{\phi}^2 = \rho_m + \rho_\lambda \tag{21}$$

$$\frac{3\ddot{A}}{A} + \frac{3\dot{A}^2}{A^2} + \frac{\omega}{2}\phi^n\dot{\phi}^2 = -p_\lambda \tag{22}$$

$$\phi^{n/2}\dot{\phi}A^3B = \phi_0 \tag{23}$$

$$\frac{\ddot{A}}{A} + \frac{2\dot{A}^2}{A^2} - \frac{2\dot{A}\dot{B}}{AB} - \frac{\ddot{B}}{B} = 0 \tag{24}$$

where ϕ_0 is a constant of integration.

Now equations (21) – (24) are a system of four independent equations in six unknowns

$A, B, p_{\lambda,} \rho_{\lambda,} \rho_m$ and ϕ. To find a determinate solution we use the fact that scalar expansion θ is proportional to the shear scalar σ^2, so that we have (Collins et al .1980)

$$A = B^m \tag{25}$$

where $m \neq 1$ is a constant and preserves the isotropic character of the space – time.

Hence from equations (24) and (25) we obtain expressions for metric coefficients as

$$A = \left[(3m+1)(ct+d)\right]^{\frac{m}{3m+1}}$$

$$B = \left[(3m+1)(ct+d)\right]^{\frac{1}{3m+1}} \tag{26}$$

where $c \neq 0$ and d are constants of integration.

Also from equations (26) and (23) we obtain

$$\phi^{\frac{n+2}{2}} = \left(\frac{\phi_0}{2c}\right)\left(\frac{n+2}{3m+1}\right)\log(ct+d) + \psi_0 \tag{27}$$

where ψ_0 is a constant of integration.

Now through a proper choice of coordinates and constants (choosing c=1, d=0) the metric (1) with the help of Eq .(26) can be written as

$$ds^2 = -dt^2 + \left[(3m+1)t\right]^{\frac{2m}{3m+1}}\left(dx^2 + dy^2 + dz^2\right) + \left[(3m+1)t\right]^{\frac{2}{3m+1}} d\psi^2 \tag{28}$$

And the scalar field ϕ in the model is

$$\phi^{\frac{n+2}{2}} = \frac{\phi_0}{2}\left(\frac{n+2}{3m+1}\right)\log t \tag{29}$$

where the constant ψ_0 is omitted

4. Some physical and kinematical properties of the model

Eq. (28) along with Eq. (29) represents five dimensional Kaluza – Klein minimally interacting holographic dark energy universe in Saez – Ballester scalar – tensor theory of gravitation with the following physical and geometrical parameters which are significant in the discussion of cosmology.

Spatial volume is $$V = (3m+1)t \tag{30}$$

The average Hubble parameter is $$H = \frac{1}{4t} \tag{31}$$

The scalar expansion is $$\theta = \frac{1}{t} \tag{32}$$

The shear scalar is $$\sigma^2 = \frac{1}{4t^2} \tag{33}$$

The average anisotropy parameter

$$A_h = \frac{1}{12} \tag{34}$$

Now using Eq. (29) and (28) in Eq. (22) we obtain

$$p_\lambda(t) = \frac{3m(m+1)}{(3m+1)^2 t^2} - \frac{2\omega\phi_0^{n+2}}{(n+2)^2 t^2} \tag{35}$$

Using Eq. (28) in Eq. (13) we get the energy density of dark matter as

$$\rho_m = \frac{\rho_0}{t} \tag{36}$$

where $\rho_0 > 0$ is a real constant of integration.

Now using Eqs. (36), (28) and (29) we obtain energy density of holographic dark energy as

$$\rho_\lambda = \frac{3m(m+1)}{(3m+1)^2 t^2} - \frac{2\omega\phi_0^{n+2}}{(n+2)^2 t^2} - \frac{\rho_0}{t} \tag{37}$$

Using Eqs. (11) , (35) and (37), the EOS parameter of holographic dark energy is

$$\omega=\frac{\frac{3m(m+1)}{(3m+1)^2t^2}-\frac{2\omega\phi_0^{n+2}}{(n+2)^2t^2}}{\frac{3m(m+1)}{(3m+1)^2t^2}-\frac{2\omega\phi_0^{n+2}}{(n+2)^2t^2}-\frac{\rho_0}{t}} \tag{38}$$

which shows that ω is a function of cosmic time.

The coincident parameter is

$$r=\frac{\rho_\lambda}{\rho_m}=\frac{\frac{3m(m+1)}{(3m+1)^2t}-\frac{2\omega\phi_0^{n+2}}{(n+2)^2t}-\rho_0}{\rho_0} \tag{39}$$

The matter density parameter Ω_m and holographic dark energy density parameter Ω_λ are given by

$$\Omega_m=\frac{\rho_m}{3H^2},\qquad \Omega_\lambda=\frac{\rho_\lambda}{3H^2} \tag{40}$$

Using Eqs. (31), (36), (37) and (40) we obtain the over all density parameter as

$$\Omega=\Omega_m+\Omega_\lambda=\frac{9m(m+1)}{(3m+1)^2}-\frac{6\omega\phi_0^{n+2}}{(n+2)^2} \tag{41}$$

We shall now discuss the behavior of the above physical and geometrical parameters of the model by observing their analytical expressions. The spatial volume V of the model increases with time which shows the spatial expansion of the universe. The Hubble parameter, scalar expansion, shear scalar diverge at t=0 while they all approach zero as $t\to\infty$. It can also be seen that the physical parameters $p_{\lambda,}\rho_{\lambda,}\rho_{m,}\omega$ and ϕ diverge at t=0 and they all vanish for infinitely large t. the coincident parameter r tends to infinity when t=0 and it becomes -1 when $t\to\infty$. It is interesting to observe that the over all density parameter Ω turns out to be constant in this model.

5. Conclusion

In this paper, we have investigated a five dimensional Kaluza – Klein universe filled with two minimally interacting fluids, matter and holographic dark energy components in the scalar –

tensor theory of gravitation proposed by Saez and Ballester (1986). We have solved the field equations of the theory by using the fact that the scalar expansion of the space – time is proportional to the shear scalar. The solution obtained represents a minimally interacting matter and holographic dark energy model in the five dimensional space – time. It is observed that the average density parameter in the universe is constant. The model exhibits early inflation and late time acceleration which is according to the present scenario of modern cosmology.

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