Article

Five-dimensional Bulk Viscous Cosmological Model with Wet Dark Fluid in Saez-Ballester Theory of Gravitation

V. U. M. Rao*, M. Vijaya Santhi & N. Sandhya Rani

Department of Applied Mathematics, Andhra University, Visakhapatnam - 530003, India

Abstract

In this paper, we obtain and present a five-dimensional LRS Bianchi type-I cosmological model with wet dark fluid (WDF) in Saez-Ballester theory of gravitation with the matter field described as bulk viscosity. Some physical and geometrical properties of the model are also discussed.

Keywords: Five-dimensional, LRS Bianchi type-I metric, bulk viscosity, wet dark fluid, Saez-Ballester, gravitation.

1. Introduction

It is well known that a gravitational scalar field, beside the metric of the space - time, must exist in the frame work of the present unified theories. Hence, there has been much interest in scalar tensor theories of gravitation. Several theories are proposed as alternatives to Einstein's theory to reveal the nature of the universe in the early stage of evolution. The most important among them are scalar-tensor theories proposed by Lyra (1951), Brans-Dicke (1961), Nordtvedt (1970), Wagoner (1970) and Saez and Ballester (1985).

Saez and Ballester (1986) formulated a scalar-tensor theory of gravitation in which the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of the weak fields. In spite of the dimensionless character of the scalar field an antigravity regime appears. This theory also suggests a possible way to solve missing matter problem in non flat FRW cosmologies.

The field equations given by Saez- Ballester (1986) for the combined scalar and tensor fields (using geometrized units with $c = 1$, $8\pi G = 1$) are

$$G_{ij} - \omega\phi^{n}\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) = -T_{ij} \quad (1.1)$$

*Correspondence: V. U. M. Rao, Department of Applied Mathematics, Andhra University, Visakhapatnam, A.P., India.
E-mail: umrao57@hotmail.com

and the scalar field ϕ satisfies the equation

$$2\phi^n \phi^{i}_{,i} + r\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \qquad (1.2)$$

where $G_{ij} = R_{ij} - \frac{1}{2}R g_{ij}$ is an Einstein tensor, R the scalar curvature, ω and n are constants, T_{ij} is the stress energy tensor of the matter. The energy conservation equation is

$$T^{ij}{}_{,j} = 0 \qquad (1.3)$$

The study of cosmological models in the framework of scalar–tensor theories has been the active area of research for the last few decades. In particular, Rao et al. (2008a, 2008b) have investigated several aspects of the cosmological models in Saez-Ballester (1986) scalar-tensor theory. Rao et al. (2013a) have discussed Bianchi type - II, VIII & IX perfect fluid dark energy cosmological models in Saez - Ballester and general theory of gravitation. Also Rao et al. (2013b) have obtained perfect fluid dark energy cosmological models in this theory and Einstein's theory of gravitation.

The possibility that space - time has more than four dimensions has attracted many researchers to the field of higher dimension. Study of higher-dimensional space - time is also important because of the underlying idea that the cosmos at its early stage of evolution might have had a higher - dimensional era Witten (1984). The extra space reduced to a volume with the passage of time which is beyond the ability of experimental observation at the moment. Attempts have been made to explain why the universe presently appears to have only four space-time dimensions, if it is dynamically evolving (4 + k) dimensional manifolds (k being the number of extra dimensions). It has been claimed that the solutions to Einstein's equation for (4+k) dimension indicate that there is an expansion of four-dimensional space - time while fifth dimension contracts or remain constant [Freund (1982); Appelquist and Chodos (1983); Daemi et al. (1984)]. Further, it has been reported that during contraction process, extra dimensions produce large amount of entropy which provides an alternative resolution to the flatness and horizon problem, as compared to usual inflationary scenario[Guth (1981); Alvax and Gavela (1983)]. Marciano (1984) has suggested that the experimental observation of fundamental constant with varying time could produce the evidence of extra dimensions.

One of the outstanding problems of standard cosmology is that of large entropy per baryon ratio. It has been widely discussed in the literature that during the evolution of the universe, bulk viscosity could arise in many circumstances and could lead to an effective mechanism of galaxy formation [Misner (1968); Ellis (1979); B L Hu (1983)]. The possibility of bulk viscosity leading to inflationary-like solutions in general relativistic FRW models is discussed by Padmanabhan and

Chitre (1987). Johri and Sudharsan (1989) have pointed out that the bulk viscosity leads to inflationary solution in Brans-Dicke theory. Many more efforts [Pimentel and Theor (1994); Besham (1996); Banerjee and Beesham (1996)] have been made to obtain cosmological solutions for a fluid with bulk viscosity in Brans-Dicke theory, because of the inflationary solutions due to the presence of bulk viscosity. Singh et al (1997) have studied all the FRW (flat, open and closed) cosmological models with viscous fluid in Brans - Dicke theory. In the context of open thermodynamic systems, Bianchi-type cosmological models with bulk viscosity and particle production have been studied by Krori and Mukherjee (2000).

Riess et al. (1998), perlmutter et al. (1998), sahni (2004) studied the nature of dark energy component of the universe as one of the deepest mysteries of cosmology. Holman and Naidu (2005) studied the homogeneous, isotropic Friedman-Robertson-Walker case by using the Wet dark fluid (WDF) as dark energy (DE). This model was in the spirit of Generalized Chaplygin Gas (GCG) (Gorini et al. 2004), where a physically motivated equation of state was offered with properties relevant for the dark energy problem. We are motivated to use the Wet Dark Fluid (WDF) as a model for dark energy which stems from the empirical equation of state to treat water and aqueous solution.

In this paper we will investigate a five dimensional LRS Bianchi type-I anisotropic string cosmological model with wet dark fluid in Saez - Ballester (1986) scalar-tensor theory of gravitation with the matter field described as bulk viscosity.

2. Metric and Energy Momentum Tensor

We consider a five dimensional LRS Bianchi type-I metric of the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2(dy^2 + dz^2) + C^2 dm^2 \tag{2.1}$$

where A, B, C are functions of time ‘t’ only.

The equation of state for WDF is given by

$$P_{WDF} = \gamma(\rho_{WDF} - \rho^*) \tag{2.2}$$

It is motivated by the fact that it is a good approximation for many fluids including water, in which the internal attraction of the molecules makes negative pressure possible. The parameters γ and ρ^* are taken to be positive and we restrict ourselves to $0 \le \gamma \le 1$. Note that if C_s denotes the adiabatic sound speed in WDF, then $\gamma = C_s{}^2$ (Babichev et al. 2004).

To find the WDF energy density, we use the energy conservation equation

$$\dot{\rho}_{WDF} + 3H(P_{WDF} + \rho_{WDF}) = 0 \tag{2.3}$$

From the equation of state (2.2) and using $3H = \dot{v}/v$ in Eq. (2.3), we get

$$\rho_{WDF} = \frac{\gamma}{1+\gamma} P^* + \frac{k}{v(1+v)} \tag{2.4}$$

where k is the integration constant and v is the volume expansion.

WDF naturally includes two components, a piece that behaves as a cosmological constant as well as pieces those red shifts as a standard fluid with an equation of state $P = \gamma\rho$.

We can show that if we take k >0, this fluid will not violate the strong energy condition $\rho + P = 0$. Thus, we get

$$P_{WDF} + \rho_{WDF} = (1+\gamma)\rho_{WDF} - \gamma\rho^* = (1+\gamma)\frac{k}{v^{1+\gamma}} \geq 0 \tag{2.5}$$

Holman and Naidu (2005) observed that their model is consistent with the most recent SNIa data, the WMAP results as well as the constraints coming from measurements of the power spectrum. Hence, they considered both, the case where the dark fluid is smooth (i.e. only the CDM components cluster gravitationally) as well as the case where the dark fluid also clusters.

The energy momentum tensor T_{ij} is given by

$$T_{ij} = (P_{WDF} + \rho_{WDF})u_i u_j + P_{WDF} g_{ij} - \xi u^a_{;a}(g_{ij} + u_i u_j) \tag{2.6}$$

together with the co-moving coordinates

$$g^{ij} u_i u_j = -1 \tag{2.7}$$

In a commoving coordinate system, we get

$$T_1^1 = T_2^2 = T_3^3 = -P_{WDF}, T_4^4 = \rho_{WDF} \tag{2.8}$$

where P_{WDF} & ρ_{WDF} are functions of cosmic time only.

3. Solutions of Field equations

Now with the help of (2.6) & (2.8), the field equations (1.1) for the metric (2.1) can be written as

$$\frac{2\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{2\dot{B}\dot{C}}{BC}+\frac{\dot{B}^2}{B^2}-\frac{1}{2}\omega\phi^n\dot{\phi}^2=-P_{WDF}+\xi\theta \tag{3.1}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}-\frac{1}{2}\omega\phi^n\dot{\phi}^2=-P_{WDF}+\xi\theta \tag{3.2}$$

$$\frac{\ddot{A}}{A}+\frac{2\ddot{B}}{B}+\frac{2\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}-\frac{1}{2}\omega\phi^n\dot{\phi}^2=-P_{WDF}+\xi\theta \tag{3.3}$$

$$\frac{2\dot{A}\dot{B}}{AB}+\frac{2\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{B}^2}{B^2}+\frac{1}{2}\omega\phi^n\dot{\phi}^2=\rho_{WDF} \tag{3.4}$$

$$\ddot{\phi}+\dot{\phi}\left(\frac{\dot{A}}{A}+\frac{2\dot{B}}{B}+\frac{\dot{C}}{C}\right)+\frac{n\dot{\phi}^2}{2\phi}=0 \tag{3.5}$$

$$\theta+\frac{\dot{\rho}_{WDF}}{P_{WDF}+\rho_{WDF}-\xi\theta}=0 \tag{3.6}$$

Here the over head dot denotes differentiation with respect to 't'.

By taking the transformation $A=e^{\alpha}, B=e^{\beta}$ & $C=e^{\gamma}$ and $dt=AB^2CdT$, the above field equations (3.1) to (3.6) can be written as

$$2\beta''+\gamma''-\beta'^2-2\alpha'\beta'-2\beta'\gamma'-\alpha'\gamma'-\frac{1}{2}\omega k=(-P_{WDF}+\xi\theta)e^{(2\alpha+4\beta+2\gamma)} \tag{3.7}$$

$$\alpha''+\beta''+\gamma''-\beta'^2-2\alpha'\beta'-2\beta'\gamma'-\alpha'\gamma'-\frac{1}{2}\omega k=(-P_{WDF}+\xi\theta)e^{(2\alpha+4\beta+2\gamma)} \tag{3.8}$$

$$\alpha''+2\beta''-\beta'^2-2\alpha'\beta'-2\beta'\gamma'-\alpha'\gamma'-\frac{1}{2}\omega k=(-P_{WDF}+\xi\theta)e^{(2\alpha+4\beta+2\gamma)} \tag{3.9}$$

$$\beta'^2+2\alpha'\beta'+2\beta'\gamma'+\alpha'\gamma'+\frac{1}{2}\omega k=\rho e^{(2\alpha+4\beta+2\gamma)} \tag{3.10}$$

$$\phi'' + \frac{n\phi'^2}{2\phi} = 0 \tag{3.11}$$

$$\rho'_{WDF} + (\rho + P)(\alpha' + 2\beta' + \gamma') - \xi(\alpha' + 2\beta' + \gamma')^2 e^{-\alpha - 2\beta - \gamma} = 0 \tag{3.12}$$

Here the over head dash denotes differentiation with respect to 'T'.

The field equations (3.7) to (3.11) are only four independent equations with four unknowns α, β, γ, ϕ, ρ_{WDF}, P_{WDF} and θ.

From the field equations (3.8) & (3.9), we get

$$\beta = \gamma + k_1 T + k_2 \tag{3.13}$$

where k_1 & k_2 are integrating constants.

Using equation (3.13), the field equations (3.7) - (3.10) are solvable for any arbitrary function. For the sake of simplicity here we consider

$$\gamma = (k_3 T + k_4)^s \tag{3.14}$$

where k_3 & k_4 are arbitrary constants.

Using equation (3.14) in (3.13), we get

$$\beta = \left(k_3 T + k_4\right)^s + k_1 T + k_2 \tag{3.15}$$

From the field equations (3.7) & (3.8), we get

$$\alpha = (k_3 T + k_4)^s + (k_7 T + k_8) \tag{3.16}$$

where $k_7 = k_1 + k_5$, $k_8 = k_2 + k_6$ are constants of integration.

The proper energy density is given by

$$\rho_{WDF} = \frac{3k_{12}\left(k_3T+k_4\right)^{s-1}\left[2k_{12}\left(k_3T+k_4\right)^{s-1}+k_9\right]+k_{11}+\frac{1}{2}\omega l}{e^{8\left(k_3T+K_4\right)^{s}+2K_9T+2K_{10}}} \tag{3.17}$$

The isotropic pressure is given by

$$P_{WDF} = \frac{3k_{12}\left(k_3T+k_4\right)^{s-1}\left[2k_{12}\left(k_3T+k_4\right)^{s-1}-k_{13}\left(k_3T+k_4\right)^{-1}+k_9\right]+k_{11}+\frac{1}{2}\omega l}{e^{8\left(k_3T+k_4\right)^{s}+2k_9T+2k_{10}}}+ \xi\frac{4k_{12}\left(k_3T+k_4\right)^{s-1}+k_9}{e^{4\left(k_3T+k_4\right)^{s}+k_9T+k_{10}}} \tag{3.18}$$

where $k_9=2k_1+k_7$, $k_{10}=k_8+2k_2$, $k_{11}=k_1^2+2k_1k_7$, $k_{12}=sk_3$, and $k_{13}=(s-1)\,k_3$ are constants of integration .

The metric (2.1), in this case can be written as

$$ds^2 = -e^{4\left(k_3T+k_4\right)^{s}+\left(k_9T+k_{10}\right)}dT^2 + e^{2\left(k_3T+k_4\right)^{s}+2\left(k_7T+k_8\right)}dx^2 + e^{2\left(k_3T+k_4\right)^{s}+2\left(k_1T+k_2\right)}\left(dy^2+dz^2\right)+e^{2\left(k_3T+k_4\right)^{s}}dm^2 \tag{3.19}$$

Thus the metric (3.19) together with (3.17) and (3.18) constitutes a five dimensional LRS Bianchi type-I cosmological model with wet dark fluid (WDF) in Saez - Ballester theory of gravitation.

4. Some physical and kinematical properties of the model

The spatial volume for the model (3.19) is

$$V = e^{4\left(k_3T+k_4\right)^{s}+k_9T+k_{10}} \tag{4.1}$$

The expression for expansion scalar θ calculated for the flow vector u^i is given by

$$\theta = \frac{4k_{12}\left(k_3T+k_4\right)^{s-1}+k_9}{e^{4\left(k_3T+k_4\right)^s+k_9T+k_{10}}} \tag{4.2}$$

and the shear scalar σ is given by

$$\sigma^2 = \frac{7}{18}\frac{\left[4k_{12}\left(k_3T+k_4\right)^{s-1}+k_9\right]^2}{e^{8\left(k_3T+k_4\right)^s+2k_9T+2k_{10}}} \tag{4.3}$$

The deceleration parameter q is given by

$$q = -\left[\frac{12k_{15}\left(k_3T+k_4\right)^{s-2}}{\left(4k_{12}\left(k_3T+k_4\right)^{s-1}+k_9\right)^2}-2\right] \tag{4.4}$$

where $k_{15} = s(s-1)k_3^2$.

The Hubble parameter is one of the most important numbers in cosmology as it is used to estimate the size and age of the universe. It indicates the rate at which the universe is expanding.

The components of Hubble parameter in three directions H_x, H_y & H_z are given by

$$H_x = \frac{sk_3(k_3T+k_4)^{s-1}+k_7}{e^{4(k_3T+k_4)^s+k_9T+k_{10}}}, H_y = H_z = \frac{k_{12}(k_3T+k_4)^{s-1}+k_1}{e^{4(k_3T+k_4)^s+k_9T+k_{10}}} \;\&$$

$$H_m = \frac{k_{12}(k_3T+k_4)^{s-1}}{e^{4(k_3T+k_4)^s+k_9T+k_{10}}}$$

Therefore the generalized mean Hubble parameter (H) is

$$H = \frac{1}{4}\left(H_x+H_y+H_z+H_m\right) = \frac{1}{4}\frac{4k_{12}\left(k_3T+k_4\right)^{s-1}+k_9}{e^{4\left(k_3T+k_4\right)^s+k_9T+k_{10}}}. \tag{4.5}$$

The average anisotropy parameter is given by

$$A_m = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{\Delta H_i}{H}\right)^2 = \frac{-24k_{12}\left(k_3T+k_4\right)^{s-1}\left[2k_{12}\left(k_3T+k_4\right)^{s-1}+k_9\right]-8k_1k_{14}}{\left[4k_{12}\left(k_3T+k_4\right)^{s-1}+k_9\right]^2} \tag{4.6}$$

where $k_{14} = k_1+2k_7$, $\Delta H_i = H_i - H \;\; (i=1,2,3)$.

The average scale factor

$$a(t) = V^{\frac{1}{4}} = e^{\frac{1}{4}\left[4\left(k_3T+k_4\right)^s + k_9T + k_{10}\right]} \tag{4.7}$$

Red shift

$$Z = \frac{1}{a(t)} - 1 = e^{\frac{-1}{4}\left[4\left(k_3T+k_4\right)^s + k_9T + k_{10}\right]} - 1 \tag{4.8}$$

Jerk parameter

$$J = \frac{1}{H^3}\frac{\dddot{a}}{a}$$
$$= \left[1 + \frac{16k_{12}k_{13}\left(k_3T+k_4\right)^{s-1}\left(2+\left(k_3T+k_4\right)^{-1}\right)}{\left(4k_{12}\left(k_3T+k_4\right)^{s-1}+k_9\right)^2} + \frac{64k_{16}\left(k_3T+k_4\right)^{s-3}}{\left(4k_{12}\left(k_3T+k_4\right)^{s-1}+k_9\right)^3}\right] e^{12\left(\left(k_3T+k_4\right)^s + 3k_9T + 3k_{10}\right)} \tag{4.9}$$

The Hubble parameter H is used to estimate the size and age of the universe. It indicates the rate at which the universe is expanding. From equation (4.5), the Hubble parameter is given by

$$H = \frac{1}{4}\frac{4k_{12}\left(k_3T+k_4\right)^{s-1}+k_9}{e^{4\left(k_3T+k_4\right)^s + k_9T + k_{10}}}. \tag{4.10}$$

Hence

$$\frac{H}{H_0} = \frac{4k_{12}\left(k_3T+k_4\right)^{s-1}+k_9}{4k_{12}\left(k_3T_0+k_4\right)^{s-1}+k_9} e^{4\left[\left(k_3T_0+k_4\right)^s - \left(k_3T+k_4\right)^s\right] + k_9\left(T_0 - T\right)} \tag{4.11}$$

where H_0 is the present value of Hubble parameter.

The red shift we measure for a distant source is directly related to the scale factor of the universe at the time of the photons emitted from the source. The scale factor a and red shift z are related through the equation

$$a = \frac{a_0}{1+z} \tag{4.12}$$

where a_0 is the present value of scale factor.

Using equation (4.8), the above equation (4.12) can be rewritten as

$$\frac{a_0}{a} = 1 + z = e^{\frac{-1}{4}\left[4\left(k_3 T + k_4\right)^s + k_9 T + k_{10}\right]} \tag{4.13}$$

The distance modulus (D) is given by

$$D(z) = 5 \log d_L + 25 \tag{4.14}$$

where d_L stands for the luminosity distance.

Luminosity distance is defined as the distance which will preserve the validity of the inverse law for the fall of intensity and given by

$$d_L = r_1 (1+z) a_0 \tag{4.15}$$

where r_1 is the radial coordinate distance of the object at light emission and, is given by

$$r_1 = \int_t^{t_0} \frac{dt}{a} = \int_t^{t_0} \frac{dt}{e^{\frac{1}{4}\left[4\left(k_3 T + k_4\right)^s + k_9 T + k_{10}\right]}} \tag{4.16}$$

The tensor of rotation

$w_{ij} = u_{i,j} - u_{j,i}$ is identically zero and hence this universe is non-rotational.

5. Discussion and Conclusions

In this paper, we have presented a five dimensional LRS Bianchi type-I cosmological model with wet dark fluid (WDF) in Saez - Ballester theory of gravitation with the matter field described as bulk viscosity.

The following are the observations and conclusions:

- For the model (3.17), we observe that the spatial volume, the expansion scalar θ and the shear scalar σ increase continuously with time. This shows that the universe expands continuously approaching to infinite volume.
- From (4.1) & (4.2), we can see that matter pressure and density increases with the increase of time.
- From (4.6), we observe that the deceleration parameter appears with negative sign implies accelerating expansion of the universe, which is consistent with the present day observations.
- From (4.8), we can observe that $A_m \neq 0$, which indicates that this model is always anisotropic. Thus the model presented here is expanding, non-rotating and accelerating in a standard way.

References

Alvax, Z. E. & Gavela, M. B.: Phys. Rev. Lett. **51**, 931 (1983)
Appelquist, T. & Chodos, A.: Phys. Rev. Lett. **50**, 141 (1983)
Babichev, et al.: arXiv:astro-ph/0407190(2004)
Banerjee,N.& Beesham,A.: Aust. J. Phys. **49**, 899 (1996)
Besham,A.: Quaestiones Mathematicae. **19**, 219l (1996)
Brans,C.,& Dicke. R. H.: Phys. Rev. **124**,925 (1961)
Ellis, G. F. R., Sachs, R.K.: General relativity and cosmology, Enrico Fermi course, Academic, New York, p. 47 (1979)
Freund, P.G. O.: Nucl. Phys. B.**209**, 146 (1982)
Gorini, V., et al.: arXiv:gr-qc/0403062, (2004)
Guth, A. H.: Phys. Rev. D.**23**, 347 (1981)
Holman, R., Naidu, S.: arXiv:astro-ph/0408102, (2005)
Hu, L.B., Fang, L. Z. and Ruffini. R.: Advances in astrophysics, World Scientific, Singapore (1983)
Johri,V.B.& Sudharsan,R.: Aust. J. Phys. **42**, 215 (1989)
Krori,K D.,& Mukherjee,A.: Gen. Relativ. Gravit. **32**, 1429 (2000)
Lyra,G., Math. Z. **54**, 52 (1951).
Marciano, W. J.: Phys. Rev. Lett. **52**, 489 (1984)
Misner, C. W.: Astrophys. J. **151**, 431 (1968)

Nordtvedt,K., Jr.: Astrophys. J. **161**, 1059(1970).

Padmanabhan, T. & Chitre,S.M.: Phys. Lett. A.**120**, 433 (1987)

Perlmutter, S., et al.: Astrophys. J. **157**, 565 (1998)

Pimentel,L O., Int. J. Theor. Phys. **33**, 1335 (1994, 1987)

Randjbar. Daemi, S., Salaus, A. & Strathdee, J.: Phys. Lett. B.**135**, 388 (1984)

Rao V.U.M., Vijaya Santhi M., Vinutha T: Astrophys. Space Sci. **314**, 73 (2008a)

Rao V.U.M., Vijaya Santhi M., Vinutha T: Astrophys. Space Sci. **317**, 27 (2008b)

Rao, V.U.M., Sireesha, K.V.S., Neelima, D.: ISRN Astronomy and Astrophysics DOI: 10.1155/2013/924834 (2013a)

Rao, V.U.M., Rao, B.J.M., Vijaya Santhi, M., Sireesha, K.V.S.: Prespacetime Journal, **4**, 807 (2013b)

Riess, A.G., et al.: Astron. J. **116**, 1009 (1998)

Saez,D.& Ballester,A.: Phys. Lett. **A113**, 467 (1986)

Sahni, V.: arXiv:astro-ph/0403324(2004)

Samanta, G.C., Dhal, S.,Mishra,B.: Astrophys. Space Sci. **346**, 233(2013)

Singh, GP, Ghosh, SG. & Beesham, A.: Aust. J. Phys. **50**, 1 (1997)

Wagoner, R.V.: Phys. Rev. **D1**, 3209(1970)

Witten, E.:Phys. Lett. B.**144**, 351 (1984)