Article

On the Role of Up & Down Quarks in Understanding Nuclear Binding Energy (Part I)

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Abstract

In a paper published in 2010, the authors proposed that, up quark mass is $m_u \cong 4.4$ MeV/c^2 and down quark mass is $m_d \cong 9.473$ MeV/c^2 and their mass ratio is 0.4645. By considering the harmonic mean of up and down quarks, a new mass unit of 6.0 MeV/c^2 can be defined. It is suggested that, when $A > A_{stable}$, nuclear binding energy is close to $k(A + Z)$ 6.0 MeV where $k \cong 0.74$ to 1 and seems to be connected with Z. When $A < A_{stable}$ nuclear binding energy is close to $k(A + Z - \delta)$ 6.0 MeV where δ is a number which seems to be connected with (Z, A, A_{stable}).

Part I of this two-part article includes: 1. Introduction, 2. About the semi empirical mass formula, 3. Estimation of stable mass number with Proton number, 4. Nuclear binding energy with up and down quark masses, and 5. Discussion.

Keywords: SEMF, up quark, down quark, strong interaction.

1. Introduction

In nuclear physics, the semi-empirical mass formula [1-7] is used to approximate the mass and various other properties of an atomic nucleus. As the name suggests, it is based partly on theory and partly on empirical measurements. The theory is based on the liquid drop model proposed by George Gamow, which can account for most of the terms in the formula and gives rough estimates for the values of the coefficients. It was first formulated in 1935 by German physicist Carl Friedrich von Weizsacker and although refinements have been made to the coefficients over the years, the structure of the formula remains the same today. The semi empirical mass formula constitutes five different energy coefficients and 5 different energy terms.

In this paper, by considering the up and down quark masses and their mass ratios, the authors proposed very simple and compact semi empirical relations for fitting the stable mass number and nuclear binding energy of atomic nuclides starting from $Z = 2$ to 100.

2. About the semi empirical mass formula

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In the following formulae, let A be the total number of nucleons, Z the number of protons, and N the number of neutrons. The mass of an atomic nucleus is given by

$$m = Zm_p + Nm_n - \left(B/c^2\right) \tag{1}$$

where m_p and m_n represent the rest masses of proton and neutron respectively and B is the binding energy of the nucleus. According to the semi-empirical mass formula nuclear binding energy takes the following form:

$$B = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} \pm \frac{a_p}{\sqrt{A}} \tag{2}$$

Here a_v = volume energy coefficient, a_s is the surface energy coefficient, a_c is the coulomb energy coefficient, a_a is the asymmetry energy coefficient and a_p is the pairing energy coefficient. If we consider the sum of the volume energy, surface energy, coulomb energy, asymmetry energy and pairing energy, then the picture of a nucleus as a drop of incompressible liquid roughly accounts for the observed variation of binding energy of the nucleus. See table-1 for the currently accepted semi empirical mass formula energy coefficients.

Table 1. Current SEMF binding energy coefficients

$a_v \cong 15.78$ MeV	$a_s \cong 18.34$ MeV	$a_c \cong 0.71$ MeV	$a_a \cong 23.21$ MeV	$a_p \cong 12.0$ MeV

3. Estimation of stable mass number with Proton number

Despite the protons' mutual electromagnetic repulsion, a stronger attractive force was postulated to explain how the atomic nucleus was bound together. This hypothesized force was called the 'strong force', which was believed to be a fundamental force that acted on nucleons: the protons and neutrons that make up the nucleus. It was later discovered that protons and neutrons were not fundamental particles, but were made up of constituent particles called 'quarks' [8]. In particle physics, the 'strong interaction' is the mechanism responsible for the strong nuclear force. It is approximately 100 times stronger than electromagnetism, a million times stronger than the weak force interaction. It ensures the stability of ordinary nuclear matter that constitutes observable neutrons and protons. It is also believed that, 'residual strong force' plays a key role in the context of binding protons and neutrons together to form atoms. Clearly speaking, it is the residuum of the strong interaction between the up and down quarks that make up the protons and neutrons. In this paper, by considering the up and down quark masses, the authors proposed a very simple relation for understanding the nuclear binding energy. If B_U is the characteristic unified nuclear binding energy constant, it is possible to show that,

1) When $A >= A_{stable}$, nuclear binding energy is close to $k\left(A+Z\right)B_U$ where $k \cong 0.74$ to 1 and seems to be connected with Z.

2) When $A < A_{stable}$, nuclear binding energy is close to $k\left(A+Z-\delta\right)B_U$ where δ is a number which seems to be connected with $\left(Z, A, A_{stable}\right)$.

Clearly speaking, above and close to the stable mass number of Z,

$$\left.\begin{array}{r} B \propto \left(kA\right)B_U \\ \text{and } B \propto \left(kZ\right)B_U \end{array}\right\} \tag{3}$$

This proposal seems to be simple in understating and compact in presentation. Now the basic questions to be answered are: How to fit/estimate the stable mass number of Z? How to fit/estimate the characteristic nuclear binding energy constant?

In the earlier published paper the authors proposed a 'super symmetry' based simple method for estimating the six quark masses [9,10]. Proposed up quark mass is $m_u \cong 4.4 \text{ MeV}/c^2$ and its current estimate is 2.15 MeV. Proposed down quark mass is $m_d \cong 9.47 \text{ MeV}/c^2$ and its current estimate is 4.70. These proposed magnitudes are roughly two times higher than the current quark estimates [11,12] and the very interesting thing is that, current estimated up and down quark mass ratio is 0.46(5) and is almost matching with the authors' basic assumption in estimating the quark masses [9]. With these two mass units, nuclear stability and nuclear binding can be understood in a very simplified and compact form. It is quite interesting and seems to be unique at fundamental level.

In the semi empirical mass formula, by maximizing $B\left(A,Z\right)$ with respect to Z, we find the number of protons Z of the stable nucleus of atomic weight A as,

$$Z \approx \frac{A}{1+\left(a_c/2a_a\right)A^{2/3}}. \tag{4}$$

This is roughly $A/2$ for light nuclei, but for heavy nuclei there is an even better agreement with nature. By substituting the above value of Z back into B one obtains the binding energy as a function of the atomic weight, $B\left(A\right)$. Maximizing $B\left(A\right)/A$ with respect to A gives the nucleus which is most strongly bound and most stable. Considering the up and down quark masses, and without considering the semi empirical mass formula it is also possible to show that [4],

$$\left.\begin{aligned} A_{up} &\cong 2Z+\left[Z\left(\frac{2m_u m_d}{m_e\left(m_u+m_d\right)}\right)^{-1}\right]^2 \cong 2Z+\left[\frac{Z}{11.758}\right]^2 \\ A_{mean} &\cong 2Z+\left[Z\left(\frac{\sqrt{m_u m_d}}{m_e}\right)^{-1}\right]^2 \cong 2Z+\left[\frac{Z}{12.632}\right]^2 \\ A_{low} &\cong 2Z+\left[Z\left(\frac{m_u+m_d}{2m_e}\right)^{-1}\right]^2 \cong 2Z+\left[\frac{Z}{13.57}\right]^2 \end{aligned}\right\} \quad (5)$$

where, m_e is rest mass of electron and $\left(A_{up}, \mathrm{A}_{\mathrm{mean}}, A_{low}\right)$ represent the lower, mean and upper stable mass numbers of Z respectively. It can be applied to super heavy elements also. Very interesting observation is that,

$$\begin{aligned} &\frac{\left(m_n-m_p\right)c^2}{m_e c^2} \cong \ln\left(\frac{\sqrt{m_u m_d}}{m_e}\right) \\ &\rightarrow \left(m_n-m_p\right)c^2 \cong \ln\left(\frac{\sqrt{m_u m_d}}{m_e}\right) m_e c^2 \cong 1.296 \text{ MeV} \end{aligned} \quad (6)$$

See table-2 for fitting the stable nucleon number with its corresponding proton number.

Table 2. To fit the stable mass numbers of Z

Z	A_{low}	A_{mean}	A_{up}		Z	A_{low}	A_{mean}	A_{up}
2	4.0	4.0	4.0		53	121.3	123.6	126.3
3	6.0	6.1	6.1		54	123.8	126.3	129.1
4	8.1	8.1	8.1		55	126.4	129.0	131.9
5	10.1	10.2	10.2		56	129.0	131.7	134.7
6	12.2	12.2	12.3		57	131.6	134.4	137.5
7	14.3	14.3	14.4		58	134.3	137.1	140.3
8	16.3	16.4	16.5		59	136.9	139.8	143.2
9	18.4	18.5	18.6		60	139.5	142.6	146.0
10	20.5	20.6	20.7		61	142.2	145.3	148.9
11	22.7	22.8	22.9		62	144.9	148.1	151.8
12	24.8	24.9	25.0		63	147.6	150.9	154.7
13	26.9	27.1	27.2		64	150.2	153.7	157.6
14	29.1	29.2	29.4		65	152.9	156.5	160.6
15	31.2	31.4	31.6		66	155.7	159.3	163.5
16	33.4	33.6	33.9		67	158.4	162.1	166.5
17	35.6	35.8	36.1		68	161.1	165.0	169.4

18	37.8	38.0	38.3		69	163.9	167.8	172.4
19	40.0	40.3	40.6		70	166.6	170.7	175.4
20	42.2	42.5	42.9		71	169.4	173.6	178.5
21	44.4	44.8	45.2		72	172.2	176.5	181.5
22	46.6	47.0	47.5		73	174.9	179.4	184.5
23	48.9	49.3	49.8		74	177.7	182.3	187.6
24	51.1	51.6	52.2		75	180.5	185.3	190.7
25	53.4	53.9	54.5		76	183.4	188.2	193.8
26	55.7	56.2	56.9		77	186.2	191.2	196.9
27	58.0	58.6	59.3		78	189.0	194.1	200.0
28	60.3	60.9	61.7		79	191.9	197.1	203.1
29	62.6	63.3	64.1		80	194.8	200.1	206.3
30	64.9	65.6	66.5		81	197.6	203.1	209.5
31	67.2	68.0	69.0		82	200.5	206.1	212.6
32	69.6	70.4	71.4		83	203.4	209.2	215.8
33	71.9	72.8	73.9		84	206.3	212.2	219.0
34	74.3	75.2	76.4		85	209.2	215.3	222.3
35	76.7	77.7	78.9		86	212.2	218.4	225.5
36	79.0	80.1	81.4		87	215.1	221.4	228.7
37	81.4	82.6	83.9		88	218.1	224.5	232.0
38	83.8	85.0	86.4		89	221.0	227.6	235.3
39	86.3	87.5	89.0		90	224.0	230.8	238.6
40	88.7	90.0	91.6		91	227.0	233.9	241.9
41	91.1	92.5	94.2		92	230.0	237.0	245.2
42	93.6	95.1	96.8		93	233.0	240.2	248.6
43	96.0	97.6	99.4		94	236.0	243.4	251.9
44	98.5	100.1	102.0		95	239.0	246.6	255.3
45	101.0	102.7	104.6		96	242.0	249.8	258.7
46	103.5	105.3	107.3		97	245.1	253.0	262.1
47	106.0	107.8	110.0		98	248.2	256.2	265.5
48	108.5	110.4	112.7		99	251.2	259.4	268.9
49	111.0	113.0	115.4		100	254.3	262.7	272.3
50	113.6	115.7	118.1		101	257.4	265.9	275.8
51	116.1	118.3	120.8		102	260.5	269.2	279.3
52	118.7	120.9	123.6		103	263.6	272.5	282.7

4. Nuclear binding energy with up and down quark masses

In this section the authors proposed a very simple method for understanding the nuclear binding energy with one energy constant. Let the unified nuclear binding energy constant be:

$$B_U \cong \frac{2m_u m_d}{(m_u + m_d)} \cong 6.0 \text{ MeV} \tag{7}$$

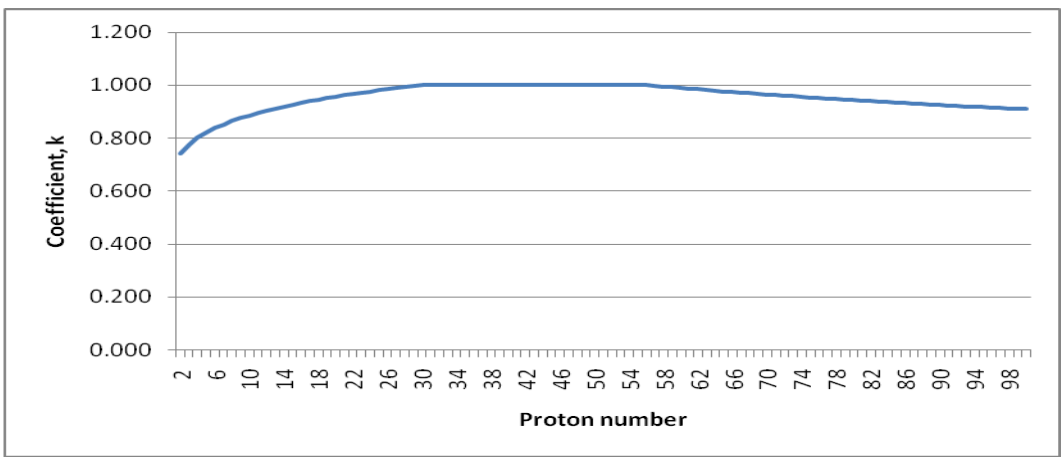
With 6.0 MeV as a characteristic single binding energy coefficient, a one term nuclear binding energy formula can be developed. Here the authors would like to stress the fact that, with further research and analysis, qualitatively a further simplified and unified method/formula can be developed [13].

$$\left.\begin{array}{l} \text{If } (\text{A}>=\text{A}_{\text{mean}}),\ B \cong k(A+Z)6.0 \text{ MeV and} \\ \text{If } (\text{A}<\text{A}_{\text{mean}}),\ B \cong k(A+Z-\delta)6.0 \text{ MeV} \\ \text{where } \delta \cong (2Z/A_{mean})(2Z/A)^3(A_{mean}-A) \end{array}\right\} \tag{8}$$

where k is a coefficient that seems to be related with proton number. For the observed data it can be suggested that,

$$\left.\begin{array}{l} \underline{\text{Case-1}}: Z \cong 2 \text{ to } 30,\ \text{k} \cong \left(\frac{Z}{30}\right)^{\frac{1}{9}} \\ \underline{\text{Case-2}}: Z \cong 30 \text{ to } 56,\ \text{k} \cong 1.0 \text{ and} \\ \underline{\text{Case-3}}: Z >= 57,\ \text{k} \cong \left(\frac{56}{Z}\right)^{\frac{1}{6}} \end{array}\right\} \tag{9}$$

See the following figure for the proposed magnitude of k.



Binding energy per nucleon can be expressed as follows.

$$\left.\begin{aligned} \text{If } \left(\text{A}>=\text{A}_{\text{mean}}\right),\ \left(\frac{B}{A}\right) &\cong k\left[\frac{(A+Z)}{A}\right]6.0\text{ MeV} \\ &\cong k\left[1+\frac{Z}{A}\right]6.0\text{ MeV and} \\ \text{If } \left(\text{A}<\text{A}_{\text{mean}}\right),\ \left(\frac{B}{A}\right) &\cong k\left[\frac{(A+Z-\delta)}{A}\right]6.0\text{ MeV} \\ &\cong k\left[1+\frac{Z}{A}-\frac{\delta}{A}\right]6.0\text{ MeV} \end{aligned}\right\} \tag{10}$$

5. Discussion

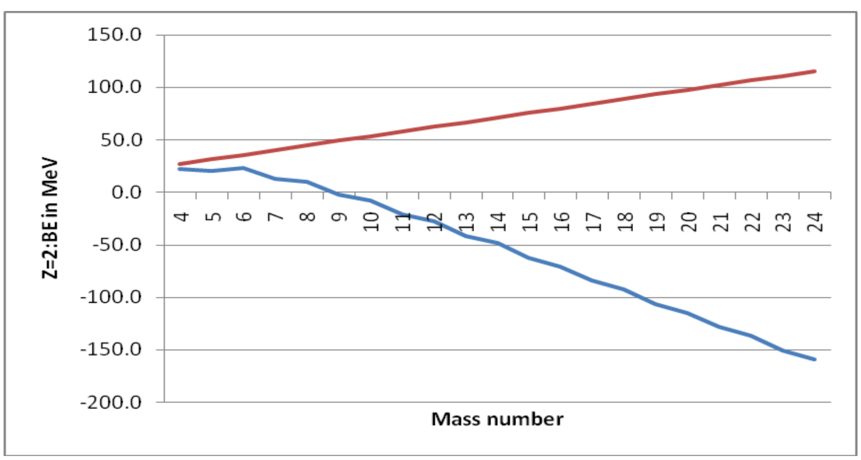
With the data presented in the lengthy table-3 (pasted at the end of References), it is clear that,

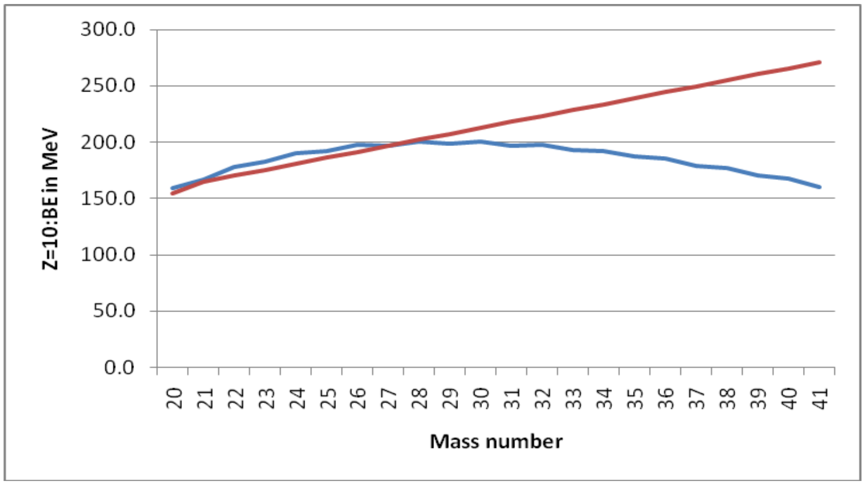
1) In case of ${}_{2}^{4}He,$ actual binding energy is 28.3 and obtained binding energy is 26.6 MeV. In case of ${}_{26}^{56}Fe,$ actual binding energy is 492.258 MeV and obtained binding energy is 484.2 MeV. In case of ${}_{50}^{116}Sn,$ actual binding energy is 988.684 MeV and obtained binding energy is 996.0 MeV. In case of ${}_{70}^{170}Yb,$ actual binding energy is 1378.13 MeV and obtained binding energy is 1384.8 MeV. In case of ${}_{82}^{206}Pb,$ actual binding energy is 1622.325 MeV and obtained binding energy is 1621.6 MeV. In case of ${}_{92}^{238}U,$ actual binding energy is 1801.69 MeV and obtained binding energy is 1822.8 MeV.
2) Starting from $Z \cong 2 \text{ to } 100,$ binding energy can be understood with the proposed relations (5) to (10). Even though some error is persisting in fitting the actual binding it is very interesting to see that, starting from $Z = 2 \text{ to } 100$, close to and above the stable mass number, binding energy is almost proportional to $k(A+Z)$.
3) As the proposed procedure is entirely new, simple and different from the currently believed semi empirical mass formula, it needs further investigation at fundamental level connected with strong interaction and quark masses[13].
4) With further research and analysis, significance of the coefficient k and δ both can be understood and their unified expressions starting from $Z = 2 \text{ to } 118$ can be developed and thereby accuracy can be improved.
5) With reference to the Semi empirical mass formula and with trial-error method, energy coefficients can be fitted in the following way.

$$\left.\begin{array}{l}\left(\frac{a_p}{B_U}\right) \cong 2 \to a_p \cong 2B_U \cong 12.0 \text{ MeV} \\ \left(\frac{a_a}{B_U}\right) \cong 4 \cong 2a_p \to a_a \cong 4B_U \cong 24.0 \text{ MeV} \\ \ln\left(\frac{a_v}{B_U}\right) \cong 1 \to a_v \cong 16.31 \text{ MeV} \\ a_v + a_s \cong a_a + a_p \cong 36.0 \text{ MeV} \\ \quad \to a_s \cong 19.69 \text{ MeV} \\ \frac{1}{2}\left(\frac{a_p^2}{a_v} - \frac{a_p^2}{a_s}\right) \cong a_c \cong 0.758 \text{ MeV.} \\ \text{Here, } \frac{a_p^2}{a_v} \cong 8.83 \text{ MeV, } \frac{a_p^2}{a_s} \cong 7.313 \text{ MeV} \\ \text{and } \quad \frac{a_p^2}{\sqrt{a_v a_s}} \cong 8.035 \text{ MeV.}\end{array}\right\} \tag{11}$$

For ${}^{170}_{70}Yb,$ actual binding energy is 1378.13 MeV and obtained binding energy is 1381.6 MeV. For ${}^{206}_{82}Pb,$ actual binding energy is 1622.325 MeV and obtained binding energy is 1616.16 MeV.

6) One can see the following 11 graphs. Note that, in these graphs, X-axis represents the number of nucleons and Y-axis represents the binding energy in MeV. Blue curve represents the binding energy calculated with the semi empirical mass formula and red curve represents the binding energy calculated with the proposed relations. It is very clear to say that curve fitting is remarkable for medium and heavy atomic nuclides.

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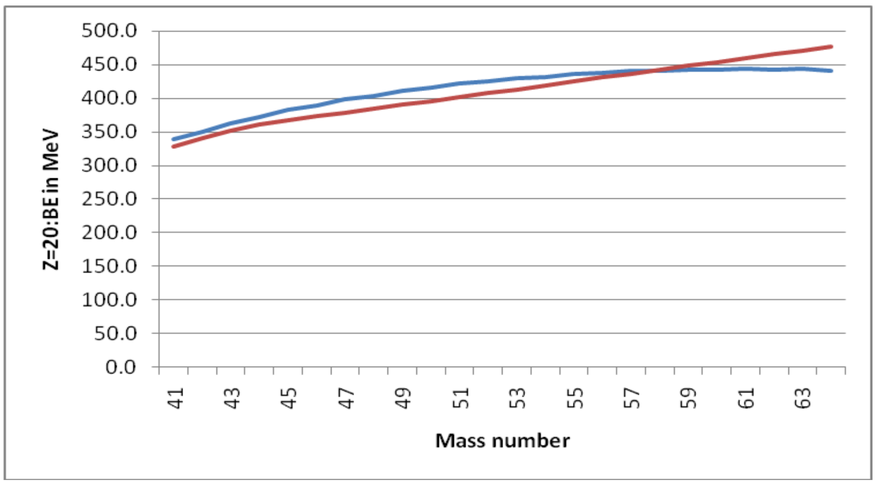
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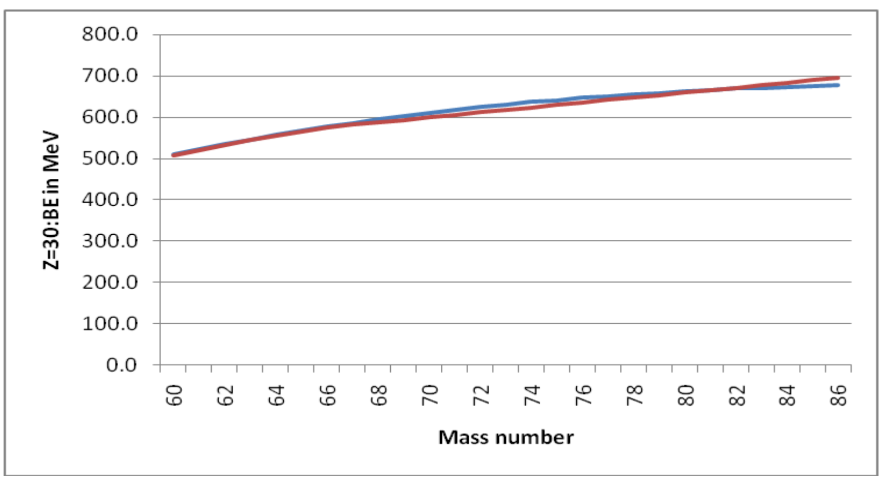
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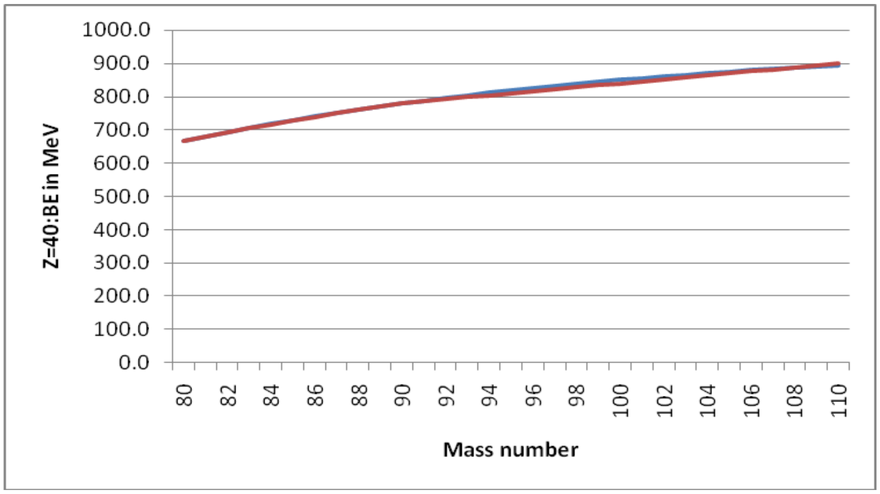
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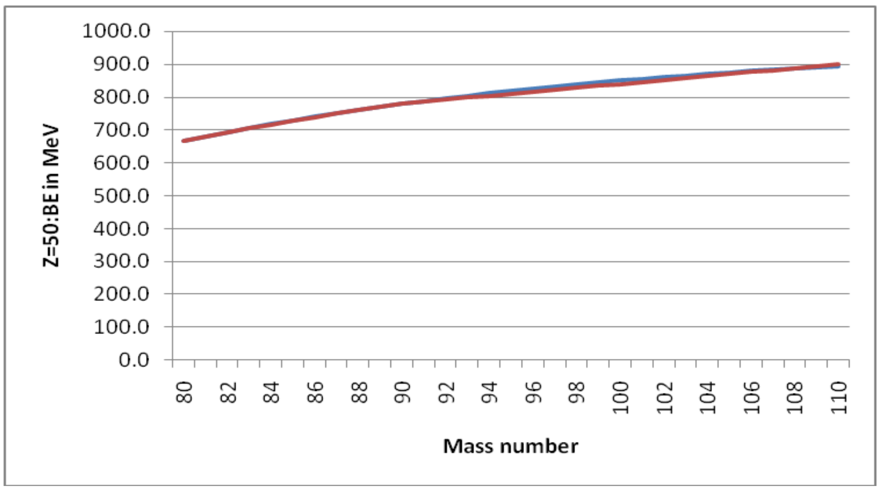
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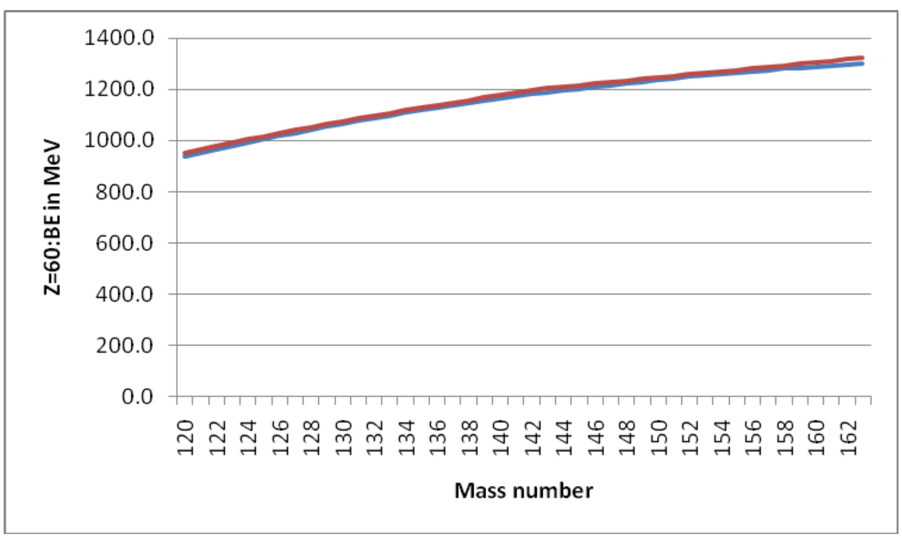
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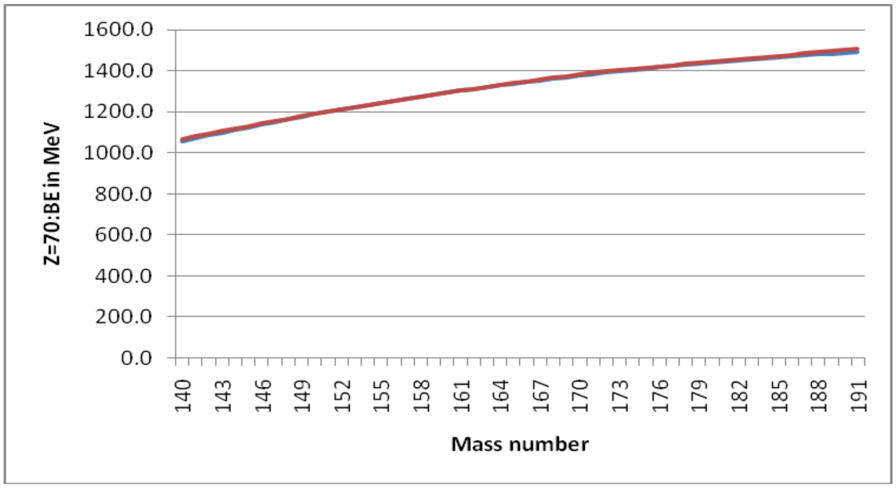
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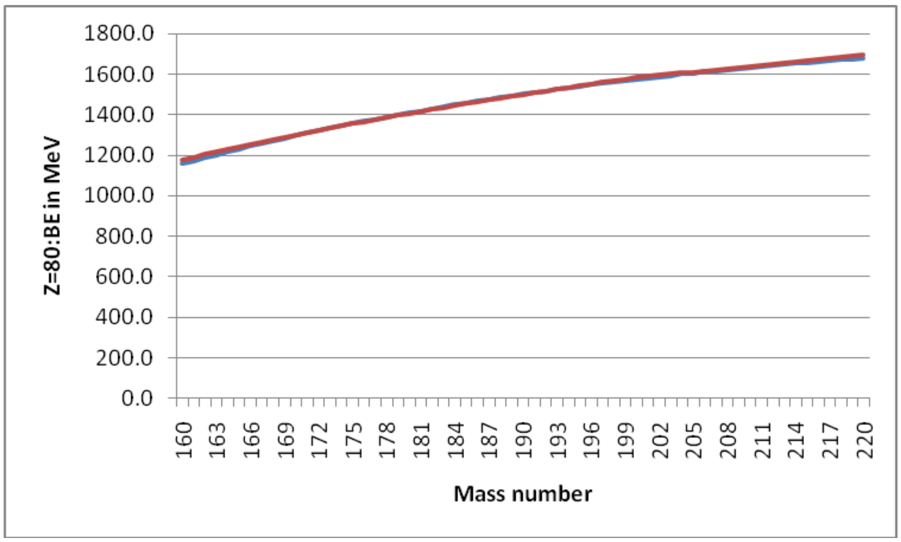
Mass number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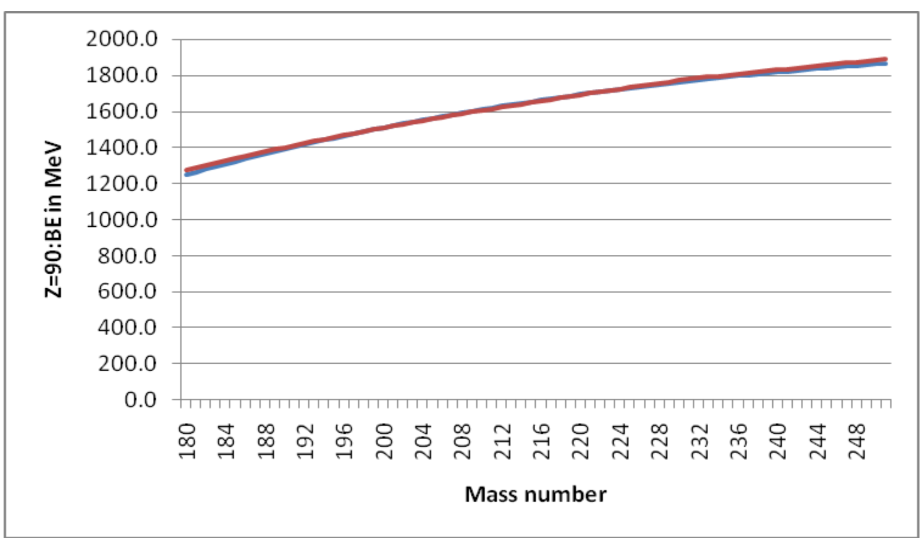
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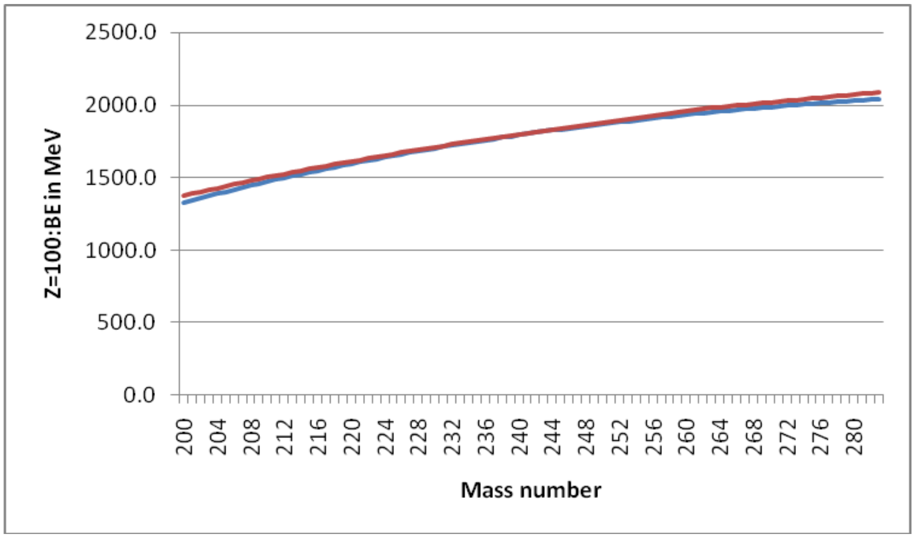

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