Article

Gauge Field of the Self-interacting Quantum Electron

Peter Leifer [1]

Abstract

It is assumed that vacuum state of the Universe contains of omnipresent "unlocated" field motions of pure quantum degrees of freedom like spin, charge, etc., instead of quantum "elementary" particles. Localization of the quantum states of motions reveals in the observable quantum particles, i.e. in the lump-like self-interacting excitations of such a vacuum. These excitations arise as a back reaction of spin and charge "fast" degrees of freedom on "slow" driving environment during "virtual measurement" due to the self-interaction that leads to the appearance of the electromagnetic-like gauge fields in the dynamical 4D spacetime.

PASC: 03.65 Pm, 03.65 Ca

1 Introduction

The localization problem is one of the main obstacles on the way of the intrinsic unification of relativity and quantum theory. In order to get realistic nonlinear field equations with lump-like solutions one needs use not only new primordial elements of the quantum dynamics but reformulate the most fundamental physical principle - inertia principle. This principle seems to be broken not only in QCD but for a self-interacting electron too. I propose a derivation of new field equations for the self-interaction electron where affine gauge potential in $CP(3)$ replaces the Higgs potential as an analog of the Poincaré forces preventing the electron from the flying apart [1, 2].

There are a lot of attempts to build a model of electron as an extended compact object with finite self-energy and discrete spectrum of mass in existing global space-time, see for example [3]. It is commonly understood that Dirac's equation may be applied to any fundamental fermions with the spin half. Then the natural question arises: where rooted the difference between stable electron and unstable muons and tau-leptons? New achievement in the "relativistic optics" with high intensity of laser field of $10^{22}-10^{28} W/cm^2$ evokes new practical interest to the nonlinear electrodynamics of the electron/muon/tauon structure (see [5] and references therein). In this field model of the quantum electron, the spin/charge quantum degrees of freedom (QDF) have been dissolved in the non-Abelian vector fields of the $SU(4)$ generators corresponding to the matrices of Dirac. Technically, the extended self-interacting quantum electron represented by the periodic motion of QDF along closed geodesics γ obeying the equation

$$\nabla_{\dot{\gamma}}\dot{\gamma} = 0 \tag{1.1}$$

in the projective Hilbert state space $CP(3)$. Namely, the spin/charge degrees of freedom move in the affine gauge potential in the state space, whereas its "field-shell" in dynamical spacetime (DST) arises as a consequence of the local conservation law of the proper energy-momentum vector field *expressing the quantum formulation of the inertia law.* This conservation law leads to PDE's whose solution give the distribution of energy-momentum in DST that keeps the motion of spin/charge degrees of freedom along geodesic in $CP(3)$ [1]. The periodic motion of quantum spin/charge degrees of freedom along closed geodesics generated by the coset transformations from $G/H = SU(4)/S[U(1) \times U(3)] = CP(3)$ will be associated with inertial "mechanical mass" and the gauge transformations from $H = S[U(1) \times U(3)]$ rotates closed geodesics in $CP(3)$ as whole. These transformations will be associated with Jacobi fields corresponding mostly to the electromagnetic energy [2].

[1] Correspondence: Peter Leifer, Crimea Federal University by the name of V.I. Vernadskii, 4 Vernadskii Av., 295007 Simferopol, Crimea, Russia Email: peter.leifer@gmail.com.

I use in this work the mathematical tool applied to the classical gauge forces arising in deformable bodies [6]. The physical content is, however, quite different, if not opposite, since my aim is to find the equations for the "shape" of "elementary" particle embedded in DST , i.e. the "field shell" of the "elementary" particle wrapping the internal QDF. The key technical approach used here is the representation of the $SU(N)$ generators in terms of the local ray coordinates $(\pi^1, ..., \pi^{N-1})$ in $CP(N-1)$ of unlocated quantum state itself [10, 11].

2 Quantum self-interacting electron

In order to formulate the quantum (internal) energy-momentum conservation law in the state space, let us discuss the local eigen-dynamics of quantum system with finite quantum degrees of freedom N. It will be realized below in the model of self-interacting quantum electron where spin/charge degrees of freedom in C^4 have been taken into account [1]. The local dynamical variables (LDV's) like the energy-momentum and should be expressed in terms of the projective local coordinates $\pi^k, \quad 1 \le i, k, j \le N-1$ of quantum state $|\Psi>= \psi^a|a>, \quad 1 \le a \le N$, where ψ^a is a homogeneous coordinate on $CP(N-1)$

$$\pi^i_{(j)} = \begin{cases} \frac{\psi^i}{\psi^j}, & if \quad 1 \le i < j \\ \frac{\psi^{i+1}}{\psi^j} & if \quad j \le i < N \end{cases} \tag{2.1}$$

since $SU(N)$ acts effectively only on the space of rays, i.e. on equivalent classes relative the relation of equivalence of quantum states distanced by a non-zero complex multiplier. LDV's will be represented by linear combinations of $SU(N)$ generators in local coordinates of $CP(N-1)$ equipped with the Fubini-Study metric [9]

$$G_{ik^*} = [(1 + \sum |\pi^s|^2)\delta_{ik} - \pi^{i^*}\pi^k](1 + \sum |\pi^s|^2)^{-2}. \tag{2.2}$$

I will use the proximity between rays of quantum states $dS^2 = G_{ik^*} d\pi^i d\pi^{k^*}$ in the projective Hilbert space $CP(N-1)$ as a fundamental concept instead of the spacetime distance. Dynamics of the superposition state $(\pi^1, ..., \pi^{N-1})$ will be given by the Lagrangian

$$\mathcal{L} = G_{ik*} \frac{d\pi^i}{d\tau} \frac{d\pi^{k*}}{d\tau} \tag{2.3}$$

for dynamics of the LDV's corresponding spin and charge of the self-interacting electron (where $N = 4$). Then the canonical momentum is as follows $p^i = \frac{d\pi^i}{d\tau}$. Such Lagrangian leads to zeroth canonical Hamiltonian function that corresponds to the vacuum character of dynamics of the omnipresent quantum degrees of freedom. $N^2 - 1$ generators of $G = SU(N)$ may be divided in accordance with the Cartan decomposition: $[B, B] \in H, [B, H] \in B, [H, H] \in H$. Namely, $(N-1)^2$ generators

$$\Phi^i_h \frac{\partial}{\partial \pi^i} + c.c. \in H, \quad 1 \le h \le (N-1)^2 \tag{2.4}$$

of the isotropy group $H = U(1) \times U(N-1)$ of some ray and $2(N-1)$ generators

$$\Phi^i_b \frac{\partial}{\partial \pi^i} + c.c. \in B, \quad 1 \le b \le 2(N-1) \tag{2.5}$$

are the coset $G/H = SU(N)/S[U(1) \times U(N-1)]$ generators realizing the breakdown of the $G = SU(N)$ symmetry. Here $\Phi^i_\sigma, \quad 1 \le \sigma \le N^2 - 1$ are the coefficient functions of the generators of the non-linear $SU(N)$ realization [10] as follows

$$\Phi^i_\sigma = \lim_{\epsilon \to 0} \epsilon^{-1} \left\{ \frac{[\exp(i\epsilon\hat{\lambda}_\sigma)]^i_m \psi^m}{[\exp(i\epsilon\hat{\lambda}_\sigma)]^j_m \psi^m} - \frac{\psi^i}{\psi^j} \right\} = \lim_{\epsilon \to 0} \epsilon^{-1} \{\pi^i(\epsilon\hat{\lambda}_\sigma) - \pi^i\}. \tag{2.6}$$

The affine connection in $CP(N-1)$ will be used below in the formulation of conservation laws of intrinsic LDV's is as follows

$$\Gamma^i_{mn} = \frac{1}{2} G^{ip^*} (\frac{\partial G_{mp^*}}{\partial \pi^n} + \frac{\partial G_{p^* n}}{\partial \pi^m}) = - \frac{\delta^i_m \pi^{n^*} + \delta^i_n \pi^{m^*}}{1 + \sum |\pi^s|^2}. \quad (2.7)$$

Its "geodesic profile" is similar to the Higgs potential but this is finite as it is depicted in Fig. 1.

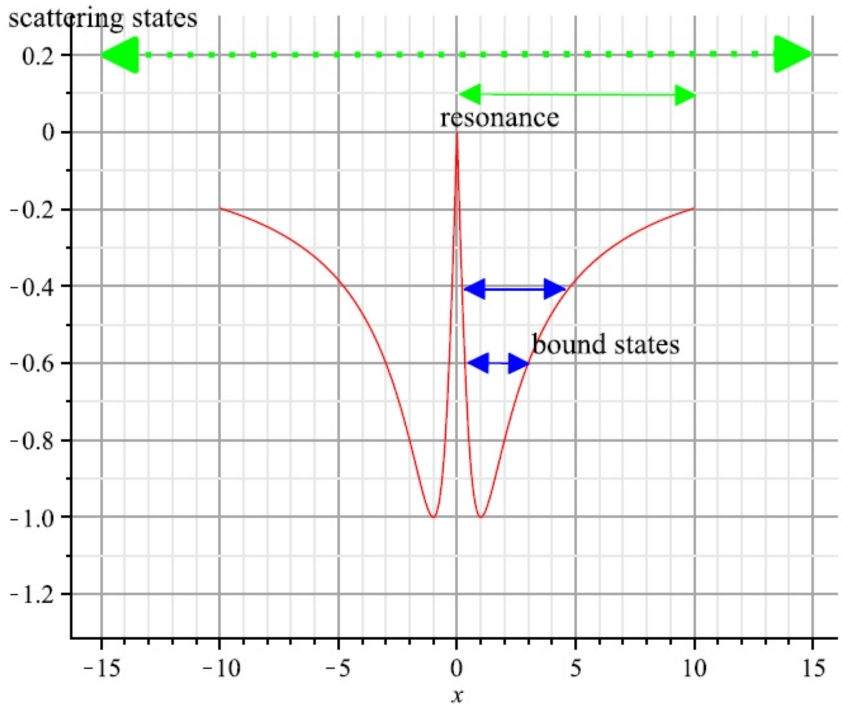

Figure 1: The geodesic profile of the affine gauge potential in CP(3). Closed geodesics in the compact totally geodesic manifold $CP(3)$ makes all states bounded with a discrete spectrum.

Canonical energy-momentum of the relativistic electron will be expressed by the contraction of unknown proper 4-potential $P^\mu = p^\mu - \frac{e}{c} A^\mu$ and the coefficient functions Φ^i_μ calculated for the matrices of Dirac according (2.6), namely

$$p^i = \frac{d\pi^i}{d\tau} = \frac{c}{\hbar} P^\mu(x) \Phi^i_\mu(\pi). \quad (2.8)$$

Such internal quantum spin/charge dynamics in $CP(3)$ cannot be directly connected with the spacetime dynamics since one needs to separate the Lorentz transformations from the motion of QDF in $CP(3)$ [6].

3 Dynamical spacetime

The process of a measurement in physics should be associated with the comparison process of some dynamical variable with corresponding scale. Newtonian physics established invariant character of a measurement in the sense of its independence on the choice of inertial reference frame (IRF). This assumption was formulated as *the inertia principle of Galileo-Newton*.

Einstein, taking into account the finite speed of light and its invariance relative choice of the IRF, found that result of the distant measurement depend on the choice of the IRF, i.e. so-called relativistic kinematics and dynamics replaced absolute character of a measurement of Newtonian mechanics. Meanwhile, general relativity already renders global spacetime coordinates in fact in physically senseless values.

Quantum theory brings a new kind of "relativity" – dependence of the result of a measurement on the apparatus used for the measurement. This kind of relativity is so radical that the indeterministic paradigm and even agnostic philosophy takes over the habitant deterministic character of predictions in exact sciences. Deep difficulties in the "standard" QFT almost insist to try to find new invariants, in fact a new quantum geometry based on the intrinsic properties of quantum states of the elementary particles [10]. Two most principle differences between fundamental approaches of the "standard" QFT and the real situation in quantum physics are as follow:

a). Attempt to divide the action variables into spacetime variables x^μ and energy-momentum P_μ as in the phase argument of the plane wave $\phi = \frac{1}{\hbar}x^\mu P_\mu$ is not accessible since internal degrees of freedom make generally the spacetime variable state-dependable.

b). Main experimental method of the investigation in high energy physics is scattering accelerated quantum particles. However all attempts to rich deep zone of the quantum particle with help of the outer "probe" particles leads to creation of new particles as a consequence of new quantum degrees of freedom de-freezing. But stable quantum particles and even unstable particles demonstrate temporal classical behavior in relatively weak external fields. Therefore their own field structure "sweeps" all zone occupied by the quantum particle without dramatic multiple particle creations. Therefore, it is reasonable to assume that (internal) quantum degrees of freedom of quantum particle forming unlocated quantum state may be "wrapped" in the "field shell" that distributed in DST spanned on state-depended energy-momentum. The general idea is that the local DST arises in a cross-section of the frame fiber bundle due to embedding of the state-dependent Lorentz frame. Thereby, we would like to save ordinary spacetime four dimension and local Lorenzian character of the metric.

Quantum geometry establishes a new principle of "super-relativity", i.e. relativity to the choice of the unlocated quantum state refers to the pure quantum degrees of freedom. This principle makes accent on the objective invariant nature of the unlocated quantum states and operates with the notion of "existence" instead of "observation". Namely, existence associated with self-interaction or "self-measurement", i.e. expression of some quantum dynamical variable in the terms of quantum state itself without any reference to external apparatus. Thereby the plaque problem of the division of "classical" and "quantum" disappears ab initial. More technically all mentioned above means that *new world quantum geometry* appears instead of so-called *quantum cosmology* developed last time. Geometry of the projective Hilbert space takes the place of the fundamental geometry where gauge dynamics of the quantum degrees of freedom defines the basic properties of the "elementary" quantum particles. The habitant global omnipresent spacetime should be replaced by the specific section of the fiber bundle over $CP(N-1)$. In the framework of the affine gauge dynamics the term "existence" esquires the exact mathematical sense in relation to solution stability of quasi-linear PDE's expressing conservation of such fundamental dynamical variables as energy-momentum [1, 2]

The separation of the spacetime coordinates from pure quantum degrees of freedom is a serious problem since these coordinates are now state-dependent values too. Such separation may be associated with state-dependent Lorentz transformations of the proper energy-momentum. This fact leads to the new construction of dynamical spacetime. Namely, the spacetime distance occurs as a result of unholonomy of the state space of the unlocated quantum degrees of freedom. In this picture the global spacetime coordinates have no physical meaning at all. Only local coordinates relative state-dependent Lorentz reference frames may be defined due to more or less complicated operational procedure. This procedure is based on the analysis of the "virtual quantum measurement" in the quantum state space.

The dynamical spacetime (DST) is pure local construction built for description of energy-momentum distribution which I called the "field-shell" of the quantum particles. This spacetime is non-distinguishable from state-dependent Lorentz frame and moves together with it in a cross-section of the principle fiber bundle over $CP(N-1)$. There are two times in this theory. One time is the ordinary Einstein's time in the DST. The second one is the "quantum proper time" that serves as a measure of the distance between two quantum states in the base state manifold $CP(N-1)$ expressed in seconds. I will use anywhere symbol "t" for the Einstein time and the symbol "τ" for the quantum proper time.

Since there is no a possibility to use classical physical reference frame comprising usual clock and solid scales on the deep quantum level, I will use the "field frame" from the four components of the vector field of the proper energy-momentum $P^\mu = (\frac{\hbar\omega}{c}, \hbar\vec{k})$ instead. This means that the period T and the wave length λ of the oscillations associating with an electron's field are identified with flexible (state-dependent) scales in the DST. Thereby, the local Lorentz "field frame" is in fact the 4-momentum tetrad whose components may be locally (in $CP(3)$) adjusted by state dependent "quantum boosts" and "quantum rotations".

It is convenient to take Lorentz transformations in the following form

$$\begin{aligned} ct' &= ct + (\vec{x}\vec{a}_Q)\delta\tau \\ \vec{x'} &= \vec{x} + ct\vec{a}_Q\delta\tau + (\vec{\omega}_Q \times \vec{x})\delta\tau \end{aligned} \tag{3.1}$$

where I put for the parameters of quantum acceleration and rotation the definitions $\vec{a}_Q = (a_1/c, a_2/c, a_3/c)$, $\vec{\omega}_Q = (\omega_1, \omega_2, \omega_3)$ [7] in order to have for the "proper quantum time" τ the physical dimension of time. The expression for the "4-velocity" V^μ is as follows

$$V^\mu_Q = \frac{\delta x^\mu}{\delta\tau} = (\vec{x}\vec{a}_Q, ct\vec{a}_Q + \vec{\omega}_Q \times \vec{x}). \tag{3.2}$$

The coordinates x^μ of an imaging point in dynamical spacetime serve here merely for the parametrization of the energy-momentum distribution in the "field shell" described by quasi-linear field equations [1] that will be derived below. The embedding Lorentz transformation into isotropy group $H = S[U(1)\times U(N-1)]$ will be discussed later since state-dependent parameters $\vec{a}_Q$ and $\vec{\omega}_Q$ may be derived during the lift of the characteristics of the quasi-linear PDE's into the frame fiber bundle over $CP(3)$.

The conservation law of the energy-momentum vector field in $CP(3)$ during inertial evolution will be expressed by the equation of the affine parallel transport

$$\frac{\delta p^i}{\delta\tau} = \frac{c}{\hbar}\frac{\delta[P^\mu(x)\Phi^i_\mu(\pi)]}{\delta\tau} = 0, \tag{3.3}$$

which is equivalent to the following system of four coupled quasi-linear PDE's for the dynamical spacetime distribution of the energy-momentum "field-shell" of the quantum state

$$V^\mu_Q(\frac{\partial P^\nu}{\partial x^\mu} + \Gamma^\nu_{\mu\lambda}P^\lambda) = -\frac{c}{\hbar}(\frac{\partial\Phi^n_\mu(\pi)}{\partial\pi^n} + \Gamma^m_{mn}\Phi^n_\mu(\pi))P^\nu P^\mu, \tag{3.4}$$

and ordinary differential equations for relative amplitudes giving in fact the definition of the proper energy-momentum P^μ from (2.8). These equations serve as the *equations of characteristics* for the linear "super-Dirac" equation

$$\begin{aligned} i\{\frac{\hbar}{c}V^\mu_Q\frac{\partial\Psi}{\partial x^\mu} + \frac{\hbar}{c}[-V^\mu_Q\Gamma^\nu_{\mu\lambda}P^\lambda - \frac{c}{\hbar}(\frac{\partial\Phi^n_\mu(\pi)}{\partial\pi^n} + \Gamma^m_{mn}\Phi^n_\mu(\pi))P^\nu P^\mu]\frac{\partial\Psi}{\partial P^\nu} \\ + P^\mu\Phi^i_\mu(\pi)\frac{\partial\Psi}{\partial\pi^i} + c.c.\} = mc\Psi \end{aligned} \tag{3.5}$$

that equivalent to the ODE

$$i\hbar\frac{d\Psi}{d\tau} = mc^2\Psi \tag{3.6}$$

for single total state function $\Psi(x^\mu, P^\mu, \pi^i)$ of self-interacting quantum electron "cum location" moving in DST like free material point with the rest mass m.

The system of quasi-liner PDE's (3.4) following from the conservation law has been shortly discussed under strong simplification assumptions [1]. In order to provide integration for self-consistent solutions one needs to find "quantum boosts" $\vec{a}_Q$ and "quantum rotations" $\vec{\omega}_Q$ involved in the "four velocity" V^μ_Q. This leads to nonlinearity of the "quantum Lorentz transformation". We will find these parameters from the system of the characteristic equations for $\frac{dP^\nu}{d\tau}$ from (3.4). Namely, one uses

$$\frac{dP^\nu}{d\tau} = \Omega^\nu_\mu(x, P, \pi)P^\mu, \tag{3.7}$$

where

$$\Omega^\nu_\mu = \begin{pmatrix} 0 & a_1 & a_2 & a_3 \\ a_1 & 0 & -\omega_3 & \omega_2 \\ a_2 & \omega_3 & 0 & -\omega_1 \\ a_3 & -\omega_2 & \omega_1 & 0 \end{pmatrix} \tag{3.8}$$

and, therefore,

$$\Omega^\nu_\mu(x, P, \pi)P^\mu \quad = \quad -V^\mu_Q \Gamma^\nu_{\mu\lambda} P^\lambda - \frac{c}{\hbar}(\frac{\partial \Phi^n_\mu(\pi)}{\partial \pi^n} + \Gamma^m_{mn}\Phi^n_\mu(\pi))P^\nu P^\mu. \tag{3.9}$$

Changing the silent index and the cancelation of P^λ gives the system of algebraic equations for quantum boosts $\vec{a}_Q$ and quantum rotations $\vec{\omega}_Q$

$$\Omega^\nu_\lambda(x, P, \pi) \quad = \quad -V^\mu_Q \Gamma^\nu_{\mu\lambda} - \frac{c}{\hbar}(\frac{\partial \Phi^n_\lambda(\pi)}{\partial \pi^n} + \Gamma^m_{mn}\Phi^n_\lambda(\pi))P^\nu. \tag{3.10}$$

Their solutions gives quantum proper frequencies and quantum Coriolis-like accelerations of the co-moving Lorentz reference frame

$$\begin{aligned} a_\alpha &= cL_\alpha \frac{\hbar \pm \sqrt{\hbar^2 + 4P^0\hbar(L_1x + L_2y + L_3z)}}{2\hbar(L_1x + L_2y + L_3z)} \\ \omega_\alpha &= \frac{c\epsilon^\beta_{\alpha\gamma} L_\beta P^\gamma}{\hbar(1 + \frac{a_1x + a_2y + a_3z}{c})}. \end{aligned} \tag{3.11}$$

Hence, one has a physically reasonable behavior of "quantum Lorentz parameters" $\vec{a}_Q$ and $\vec{\omega}_Q$ since they have finite limits at the origin of the Lorentz frame $r = 0$

$$\begin{aligned} a_\alpha(0) = \lim_{r\to 0} a_\alpha = \frac{-cL_\alpha P^0}{\hbar} \\ \omega_\alpha(0) = \lim_{r\to 0} \omega_\alpha = \frac{c\epsilon^\beta_{\alpha\gamma} L_\beta P^\gamma}{\hbar} \end{aligned} \tag{3.12}$$

under the choice of the sign " $-$ " in the expression for a_α, and in the remote zone at the limit $r \to \infty$ they are as follows

$$\begin{aligned} a_\alpha(\infty) = \lim_{r\to\infty} a_\alpha = 0 \\ \omega_\alpha(\infty) = \lim_{r\to\infty} \omega_\alpha = 0. \end{aligned} \tag{3.13}$$

Here $L_\lambda = \frac{\partial \Phi^n_\lambda(\pi)}{\partial \pi^n} + \Gamma^m_{mn}\Phi^n_\lambda(\pi)$ is the divergency in $CP(3)$ of the vector field of the energy-momentum generator and it was assumed that $\Gamma^\nu_{\mu\lambda}$ is the DST connection whose components coincide with boost and rotation instant parameters of the accelerated Lorentz tetrad [7].

The theory of the quasi-linear PDE's (3.4) is very well known [8]. It is equivalent to the system of the ordinary differential equations in symmetric form

$$\frac{dx^\mu}{\Omega^\mu_\nu(x, P, \pi)x^\nu} = \frac{dP^\mu}{\Omega^\mu_\nu(x, P, \pi)P^\nu} = \frac{\hbar d\pi^i}{cP^\mu \Phi^i_\mu(\pi)} = i\frac{\hbar d\Psi}{mc^2\Psi} = d\tau \tag{3.14}$$

Two independent first integrals may be found easily from the integrable combinations

$$\frac{d(x_\mu x^\mu)}{d\tau} = 2\Omega^\mu_\nu x_\mu x^\nu = 0; \quad x_\mu x^\mu = s^2;$$
$$\frac{d(P_\mu P^\mu)}{d\tau} = 2\Omega^\mu_\nu P_\mu P^\nu = 0; \quad P_\mu P^\mu = m^2c^2, \tag{3.15}$$

using the equality

$$\Omega^\mu_\nu x^\nu x_\mu = \Omega^\rho_\nu x^\nu g_{\rho\sigma} g^{\sigma\mu} x_\mu = \Omega_{\sigma\nu} x^\nu x^\sigma = 0. \tag{3.16}$$

Equations (3.4) for energy-momentum P^σ we can rewrite now in more convenient form

$$\Omega^\mu_\nu x^\nu \frac{\partial P^\sigma}{\partial x^\mu} - \Omega^\sigma_\nu P^\nu = 0. \tag{3.17}$$

This is complicated quasi-linear PDE's system and the general solution is unknown but it is possible to find its partial solution. Namely,

$$P^\mu(x) = C_1 x^\mu + C_2 x^\mu f(x^\mu x_\mu) \tag{3.18}$$

where C_1, C_2 are arbitrary constants and $f(x^\mu x_\mu)$ is any analytic function of the square of the interval $s^2 = x^\mu x_\mu$, say, finite non-singular function $f(x^\mu x_\mu) = \exp(-s^2)$. Notice, the first term is the pure gauge field. It is interesting that in general the four-gradient of the scalar function $G_\mu(x) = \frac{\partial \phi(x)}{\partial x^\mu}$ is not a solution.

Found solutions describe the complex-value 4-potentials (energy-momentum) that represents fundamental fermion in complex-value coordinates. It is an example of the relativistic quantum theory with the arbitrary function $f(s^2)$ mentioned by Feynman [12]. Stationary points of the system of characteristics equations (2.15) have been found too [13]. Their explicit expression in terms of divergences of the $SU(4)$ generators fields L_μ presents Coulomb-like 4-potentials.

4 Finite self-energy of the electron with scalar vacuum

The old problem of the divergency of the self-energy in classical and quantum area is one of the main obstacle on the way of the consistent quantum theory of "elementary" quantum particles. One of the interesting classical nonlinear electrodynamics was proposed by Born and Infeld that got a modern development in different directions (see [5] and references therein). I would like to discuss possible quantum model describing local (now in DST) dynamics of the self-interacting electron where relativistic scalar field $f(x^\mu x_\mu)$ included in the partial solution of (3.17) seeking the finite proper energy-momentum of the electron.

The partial solution (3.17) contains arbitrary analytic function of the relativistic interval. The simplest assumption about the nature of this scalar field is that this is massive scalar field described by the ordinary Klein-Gordon equation with appropriate boundary conditions for the proper energy-momentum P^μ. Thereby one assumes investigate lump solution of such equation. The simple 2D analog is well known membrane modes [14] or more complicated 3D analog of the Wolker magneto-static modes [15]. Here we have more complicated dynamics of the ball of scalar field where dAlembertian corresponding to the metric tensor in the DST takes the place of the role of the permeability of a magnetic media. I postponed the analysis of this dynamics for a future work since for me it is interesting merely "Lorentz-radial" scalar field. Therefore, it is natural to use Klein-Gordon equation in the Lommel form

$$\frac{d^2 f}{ds^2} + \frac{3}{s}\frac{df}{ds} + \frac{m^2c^2}{\hbar^2} f = 0. \tag{4.1}$$

This equation was widely used in the Euclidian regime in the problem the metastable vacuum decay, see for example [16]. I will assume that the scalar field $f(x^\mu x_\mu)$ occupies all electron's volume. If the classical radius of an electron $r_0 = \frac{e^2}{mc^2}$ is taken as the scale of the spacetime distance then (4.1) takes the form

$$\frac{d^2 f}{d\rho^2} + \frac{3}{\rho}\frac{df}{d\rho} + \alpha^2 f = 0, \tag{4.2}$$

where dimensionless distance is $\rho = \frac{s}{r_0}$ and $\alpha = \frac{e^2}{\hbar c}$ is the fine structure constant [17].

The general solution of this equation will be expressed in Bessel functions

$$f(\rho) = c_1 J_1(\frac{\alpha\rho}{\sqrt{2}})\rho^{-1} + c_2 Y_1(\frac{\alpha\rho}{\sqrt{2}})\rho^{-1}. \tag{4.3}$$

I put $c_2 = 0$ in order to avoid singularity on the light cone. But it is easy to check that $f(\rho) = C_1 J_1(\frac{\alpha\rho}{\sqrt{2}})\rho^{-1}$ does not satisfy realistic boundary conditions

$$P_\mu P^\mu = m^2 c^2 \quad if \quad s^2 = x^\mu x_\mu = 0. \tag{4.4}$$

In order to find the scalar field capable to bring acceptable form of the proper energy-momentum I will use the additive form of the trial solution

$$P^\mu = x^\mu [C_1 + C_2(\rho^n + J_1(\frac{\alpha\rho}{\sqrt{2}})\rho^{-1}]. \tag{4.5}$$

I found that the boundary conditions will be satisfied if $n = -1$, hence I put $C_2 = mc$, and finally

$$P^\mu = x^\mu [C_1 + mc(\rho^{-1} + J_1(\frac{\alpha\rho}{\sqrt{2}})\rho^{-1}]. \tag{4.6}$$

Removing the pure gauge field $C_1 x^\mu$, one gets the the electron's energy plus internally generated scalar potential which I wrote as follows

$$P^0 = mc^2 \frac{1 + J_1(\frac{\alpha ct}{\sqrt{2}}\sqrt{1-\beta^2})}{e\sqrt{1-\beta^2}}, \tag{4.7}$$

assuming here $\beta^2 = \frac{r^2}{c^2 t^2}$ merely with the aim of the estimation of the proximity to the light cone. Thence the time t may serve as the distance from the origin of the electron. In order to estimate physically realistic values of the argument of Bessel function one needs return from the dimensionless ct to the physically dimensional values as follows: $\alpha ct \to \frac{e^2}{mc^2}\frac{mc}{\hbar}ct\frac{mc^2}{e^2} = ct\frac{mc}{\hbar} = \frac{L}{\lambda_C} = \frac{L}{2.43\times 10^{-12} m}$. This dependence is depicted in Fig.2.

One sees that this potential of the electron has explicit non-monotonic dependence on the distance. This looks like integrally represented the vacuum polarization due to pairs creation-annihilation. I took few points of the maxima and, using "CurveFitting" procedure of the Maple 15, found that the envelope of these maxima has the form $f(x) = \frac{Ax+B}{ax^2+bx+c}$, i.e. it is close "in average" to the Coulomb potential. The deviation from the Coulomb's Law in the form $\frac{1}{r^{2+q}}$ gave $q \leq (2.7 \pm 3.1) \times 10^{-16}$, alternatively the limit on the photon rest mass being $m_\gamma \leq 1.6 \times 10^{-47}$ g [18]. But all methods of investigation mentioned in this report are macroscopic and hence the "ripple" of the potential could not be found. But probably we have so complicated form of the potential since the scalar model (4.1) is too simple! On the the other hand this result may be treated as confirmation of the initial intuitive de Broglie hypothesis about electron's structure: "Moreover, what must we understand by the interior of a parcel of energy? An electron is for us the archetype of isolated parcel of energy, which we believe, perhaps incorrectly, to know well; but, by received wisdom, the energy of an electron is spread over all space with a strong concentration in a very small region, but otherwise whose properties are very poorly known." [19]

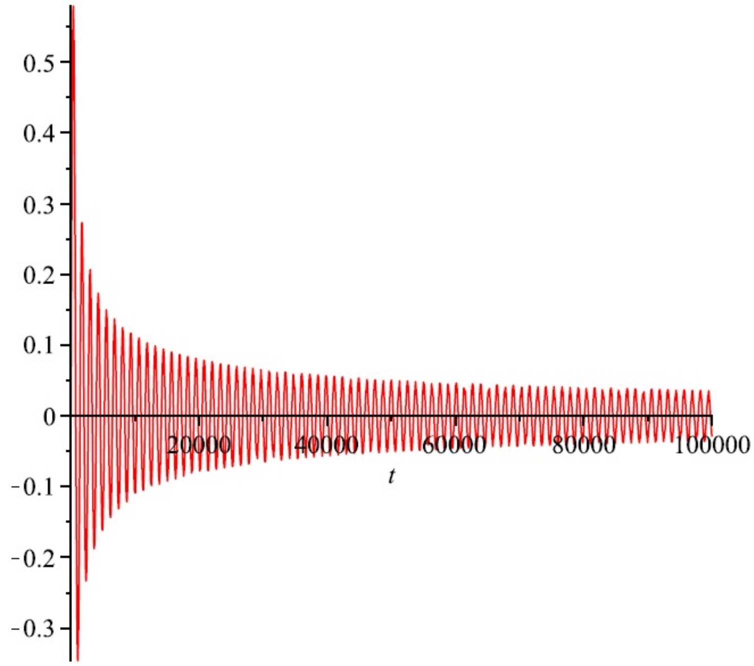

Figure 2: The scalar potential of the moving charge with the velocity v=c/100. Dimensionless argument of the Bessel function is given in the scale of the Compton wavelength. I put for simplicity m=c=e=1.

Now one may calculate electric and magnetic fields around electron and, after all, the density of the electric field. I avoid to show elementary formulas for the components of electric and magnetic field but provide instead their 3d pictures, see Figures 3, 4. These fields are singular free and similar to classical field of a pointwise charged particle.

I calculated the density of the electric field of the rest charge $U = \frac{\epsilon_0 E^2}{2}$ with the help of Maple 15. Expression for E^2 is as follows:

$$E^2 = \frac{2m^2c^6t^2r^2}{e^2}\{c^2t^2\frac{J_0(\frac{\alpha\sqrt{c^2t^2-r^2}}{\sqrt{2}})^2\alpha^2}{(c^2t^2-r^2)^3} - r^2\frac{J_0(\frac{\alpha\sqrt{c^2t^2-r^2}}{\sqrt{2}})^2\alpha^2}{(c^2t^2-r^2)^3}$$
$$-4\alpha\sqrt{2}\frac{J_0(\frac{\alpha\sqrt{c^2t^2-r^2}}{\sqrt{2}})J_1(\frac{\alpha\sqrt{c^2t^2-r^2}}{\sqrt{2}})}{(c^2t^2-r^2)^{5/2}}$$
$$-2\alpha\sqrt{2}\frac{J_0(\frac{\alpha\sqrt{c^2t^2-r^2}}{\sqrt{2}})}{(c^2t^2-r^2)^{5/2}} + 8\frac{J_1(\frac{\alpha\sqrt{c^2t^2-r^2}}{\sqrt{2}})^2}{(c^2t^2-r^2)^3}$$
$$+8\frac{J_1(\frac{\alpha\sqrt{c^2t^2-r^2}}{\sqrt{2}})}{(c^2t^2-r^2)^3} + \frac{2}{(c^2t^2-r^2)^3}\}. \tag{4.8}$$

This function has zeroth limits $E^2(t=0, r=0) = 0; E^2(t=\infty, r=\infty) = 0$. The integral $4\pi\int E^2r^2dr$ may be calculated analytically by the change of the variable $r \to Q = \sqrt{c^2t^2-r^2}$ that gives the explicit antiderivative in terms of hypergeometric and Bessel functions. I will represent here result for the first

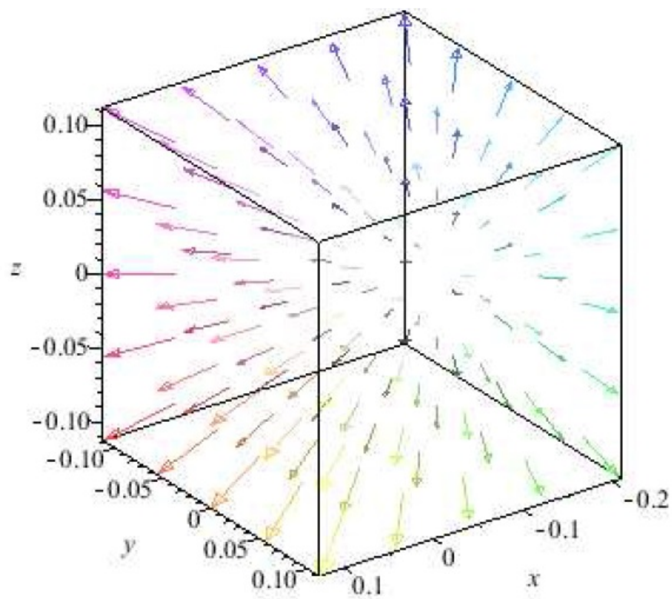

Figure 3: Electric field of the moving charge with the velocity v=c/2 in the positive direction of the axes X.

integrand only

$$
\begin{aligned}
I_1 = & -\frac{1}{5}\frac{c^4 t^8 F_{23}([-5/2,1/2],[-3/2,1,1],-\frac{Q^2\alpha^2}{2})}{Q^5} \\
& +\frac{2}{3}\frac{c^2 t^6 F_{23}([-3/2,1/2],[-1/2,1,1],-\frac{Q^2\alpha^2}{2})}{Q^3} \\
& +\frac{1}{4}\alpha\sqrt{\frac{2}{\pi}}[-\frac{2\sqrt{2\pi}(1+Q^2\alpha^2)(J_0(1/2\sqrt{2}\alpha Q))^2}{Q\alpha} \\
& +4\sqrt{\pi}J_0(\frac{\alpha Q}{\sqrt{2}})J_1(\frac{\alpha Q}{\sqrt{2}})-2\sqrt{2\pi}\alpha Q(J_1(\frac{\alpha Q}{\sqrt{2}}))^2].
\end{aligned} \tag{4.9}
$$

All hypergeometric functions involved in the full antiderivative $I_{total} = \sum_{k=1}^{7} I_k$ converge anywhere in the complex plane of Q since $p = 2 < q = 3$. The upper limit is zero $I_{upper} = I_{total}(Q = \infty) = 0$. This antiderivative contains the time coordinate t as a parameter. The substitution $Q = ct$ assumes the lower limit $r = 0$. Formally this limit is zero for $t = 0$. But more realistic estimation of the charge density under $Q = ct\sqrt{1 - \frac{r^2}{c^2t^2}} = 0$ should be realized on the light cone $\frac{r^2}{c^2t^2} = 1$, i.e. $t = r/c$. Thus total integral of the density with changing sign leads to finite but oscillating value. The approximate calculation gives for $t = 1700s$ the value $U = \frac{\epsilon_0 E^2}{2} = 8.85 \times 10^{-12}[\frac{C^2}{Jm}]E^2[\frac{J^2}{C^2m^2}] \approx 81 \times 10^{-15}[\frac{J}{m^3}]$ the value that should be if $t = 10^{-8}s$. Probably it is another evidence of the too primitive choice of the scalar model of the electron's intrinsic content (4.1).

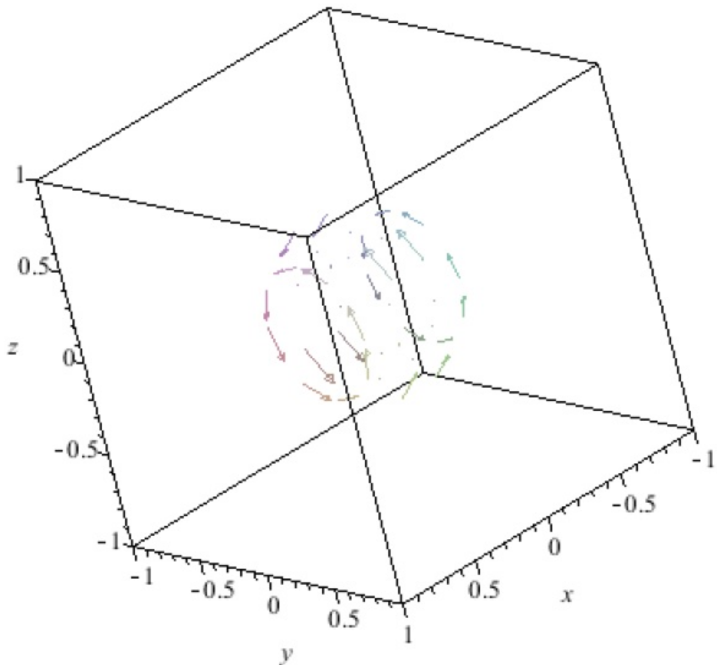

Figure 4: Magnetic field of the moving charge with the velocity v=c/2 in the positive direction of the axes X.

5 Discussion

Solitons and instantons are solutions of the very narrow class of nonlinear PDE's. I present some attempt to get localizable solutions of the quasi-linear PDE's obtained for quantum self-interacting electron. This lump-like solutions represent extended electron together with electromagnetic-like quasi-classical field without singularities. This means that spin/charge degrees of freedom dissolved into the smooth vector field over $CP(3)$ replacing Dirac's matrices may induce "from inside" the oscillating ball of the scalar vacuum. Detailed dynamics of such modes may really lead to the fermion properties and it should be investigated in future. This requires explicit metric structure of the DST with found quantum boosts and rotations parameter $\vec{a}_Q, \quad \vec{\omega}_Q$. But a long time it is known that extended classical objects should be quantized as fermions (see [20] and references therein).

References

[1] P. Leifer, T. Massalha, Field-shell of the self-interacting quantum electron, *Annales de la Fondation Louis de Broglie*, Volume 36, 29-51 (2011).

[2] P. Leifer, "The relativistic non-linear quantum dynamics from the $CP(N-1)$ geometry" in Prospects of Differential Geometry and Its Related Fields: pp. 63-82, Proceedings of the 3rd International Colloquium on Differential Geometry and Its Related Fields, Veliko Tarnovo, Bulgaria, 3 7 September 2012, World Scientific, New Jersey, London, Singapore, Beijing, Shanghai, Hong Kong, Taipei, Chennai, 228 pp (2013).

[3] P.A.M. Dirac, An extensible model of the electron, Proc. Roy. Soc. **268**, 57 (1962).

[4] Gerard A. Mourou, Toshiki Tajima, Sergei V. Bulanov, Optics in the relativistic regime, Reviews of Modern Physics, V. 78, April-June 2006.

[5] E.Yu. Petrov, A.V. Kudrin, Exact self-semilar solution in Born-Infeld theory, arXiv:1302.4363v1, [physics.class-ph].

[6] R.G. Littlejohn and M. Reinsch, Gauge fields in the separation of rotations and internal motions in the n-body problem, Rev. Mod. Physics, **69**, No.1, 213-275 (1997).

[7] Misner C.W., Thorne K.S., Wheeler J.A. *Gravitation*, W.H.Freeman and Company, San Francisco, 1279 (1973).

[8] Courant R., Hilbert D. *Methods of Mathematical Physics*, **2**, Partial Differential Equations, Wiley, 830 (1989).

[9] Kobayashi S. & Nomizu K. *Foundations of Differential Geometry*, V. II, Interscience Publishers, New York-London-Sydney, 414 (1969).

[10] P. Leifer, State-dependent dynamical variables in quantum theory, *JETP Letters*, **80**, (5) 367-370 (2004).

[11] P. Leifer, An affine gauge theory of elementary particles, *Found.Phys.Lett.*, **18**, (2) 195-204 (2005).

[12] R.P. Feynman, The theory of fundamental processes, CIT, W.A. Benjamin, inc. NY (1961).

[13] P. Leifer, The role of the $CP(N-1)$ geometry in the intrinsic unification of the general relativity and QFT, arXiv:1209.6389v2,[physics.gen-ph].

[14] I.G. Petrovskii, Lectures in PDE's, GIFML, Moscow, 400 pp, (1961), In Russian.

[15] L.R. Walker, J.Appl.Phys., **29**, 318 (1958).

[16] S. Coleman, Phys. Rev. **D15**, 2929 (1977).

[17] P. Leifer, Superrelativity as an Element of the Final Theory, Found. Phys., **27**, (2) 261-285 (1997).

[18] Liang-Cheng Tu and Jun Luo, Experimental tests of Coulombs Law and the photon rest mass, Metrologia **41**, S136 - S146 (2004).

[19] L. de Broglie, On the Theory of Quanta, A translation of "RECHERCHES SUR LA THEORIE DES QUANTA" (Ann. de Phys., 10e série, t. III (Janvier-Février 1925). by: A. F. Kracklauer (2004).

[20] I.J.R. Aitchison, Berry phases, magnetic monopoles, and Wess-Zumino terms or how the skyrmion got its spin, lecture given at the XXVI Cracow School of Theoretical Physics, Zakopane, Poland, 1=13 June, 1986.