Article

Time-Dependent Λ in C-Field Theory with LRS Bianchi Type III Universe & Barotropic Perfect Fluid

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Abstract

A cosmological model for barotropic fluid distribution in creation field cosmology with varying cosmological constant (Λ) in Bianchi type - III space- time is investigated. To get deterministic model satisfying conservation equation, we assume $\Lambda = 1/R^2$ as considered by Chen and Wu [13] where R is scale factor. We find that creation field (C) increase with time, $\Lambda \sim 1/t^2$, which matches with the result of HN theory. The deceleration parameter $q < 0$ which shows that universe is accelerating. The various special cases of the model (36) viz. Dust filled universe ($\gamma = 0$), radiation dominated universe ($\gamma = 1/3$) and Stiff fluid universe ($\gamma = 1$) are also discussed. The model is free from horizon.

Key words: LRS, Bianchi type III, barotropic, perfect fluid , C-field, variable Λ.

1. Introduction

In the early universe, all the investigations dealing with physical process use a model of the universe, usually called a big-bang model. However, the big-bang model is known to have the short comings in the following aspects:

i. The model has singularity in the past and possibly one in future.
ii. The conservation of energy is violated in the big-bang model.
iii. The big-bang models based on reasonable equations of state lead to a very small particle horizon in the early epochs of the universe. This fact gives rise to the 'Horizon problem'.
iv. No consistent scenario exists within the frame work of big-bang model that explains the origin, evolution and characteristic of structures in the universe at small scales.
v. Flatness problem.

Astronomical observations in the late eighties have revealed that the predictions of Friedmann-Robertson-Walker type models do not always meet our requirements as was believed earlier [1]. So alternative theories were proposed time to time –the most well knowntheory was steady state theory by Bondi and Gold [2]. In this approach, the universe does not have any singular beginning nor at end on the cosmic time scale. To maintain constancy of matter density, they envisaged a very slow but continuous creation of matter in contrast to explosive creation of standard model. But it suffered a serious disqualification by not giving any physical justification

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for continuous creation of matter and principle of conservation of energy was sacrificed in this formalism. To over come this difficulty, Hoyle and Narliker [3] adopted a field theoretic approach introducing a massless and chargeless scalar field in the Einstein-Hilbert action to account for the matter creation. In C-field theory, there is no big-bang type singularity as in steady state theory of Bondi and Gold [2].

If a model explains successfully the creation of positive energy matter without violating the conservation of energy then it is necessary to have some degree of freedom which acts as a negative energy mode. Thus a negative energy field provides a natural way for creation of matter. Narlikar [4] has pointed out that, matter creation is accomplished at the expense of negative energy C-field, thus the introduction of a negative energy field may solve horizon and flatness problem faced by big-bang model. Narlikar and Padmanabhan [5] have obtained the solution of modified Einstein field equation in presence of C-field and they have shown that cosmological model based on this solution is free from singularity, particle horizon and also provides a natural explanation to the flatness problem. Bali and Tikekar [6] have investigated C-field cosmological model for dust distribution in FRW space-time with variable gravitational constant. Recently Bali and Kumawat [7,8] have investigated C-field cosmological models for dust and barotropic fluid distribution in non flat FRW space-time with variable gravitational constant. Adhav et.al.[9,10] studied Bianchi type I cosmological model in string theory. Bali and Saraf [11,12] have investigated Bianchi type I dust field universe with decaying vacuum energy in C- field cosmology.

Motivated by aforesaid, we have investigated a cosmological model for barotropic fluid distribution in C- field cosmology with varying cosmological constant in LRS Bianchi type-III space-time. For deterministic model, we assumed $\Lambda = 1/R^2$, where R is scale factor. We find that creation field (C) increase with time, $\Lambda \sim 1/t^2$. Various special cases of γ are considered. Physical and geometrical aspects of the model are also discussed. These model are free from horizon.

2. The Metric & Field Equation

We consider homogenous LRS Bianchi type-III metric in the form of

$$ds^2 = dt^2 - A^2 dx^2 - B^2(e^{2x} dy^2 + dz^2) \tag{1}$$

where $A\,and\,B$ are functions of t alone.

The Einstein's field equation (C = 1)by introduction of C-field is modified by Hoyle- Narlikar [3] with time dependent cosmological term is given by

$$R_j^i - \frac{1}{2} R g_j^i = -8\pi G\left[T_j^i + T_j^i\right] - \Lambda g_j^i \tag{2}$$

The energy-momentum tensor $_{(m)}T_j^i$ for perfect fluid and creation field $_{(c)}T_j^i$ are given by

$$ {}_{(m)}T_j^i = (\rho + p)v_j v^i - p g_j^i \qquad (3)$$

$$ {}_{(c)}T_j^i = -f\left(c_j c^i - \frac{1}{2} g_j^i c_\alpha c^\alpha\right) \qquad (4)$$

where f > 0 is coupling constant between the matter and creation field and $C_i = \frac{dc}{dx^i}$.

The coordinate are chosen to co-moving such that $v^i = (0, 0, 0, 1)$.
which gives

$$T_1^1 = -p = T_2^2 = T_3^3\,,\ T_4^4 = \rho \qquad \text{for matter} \qquad (5)$$

$$T_1^1 = \frac{1}{2} f\dot{c}^2 = T_2^2 = T_3^3\,, T_4^4 = -\frac{1}{2} f\dot{c}^2 \quad \text{for creation field} \qquad (6)$$

Thus,

$$T_1^1 = (-p + \frac{1}{2} f\dot{c}^2) = T_2^2 = T_3^3, T_4^4 = (\rho - \frac{1}{2} f\dot{c}^2) \qquad (7)$$

Hence the Einstein field equation (2) for the metric (1) and EMT (3), (4) takes the form

$$2\,\frac{B_{44}}{B} + \frac{B_4^2}{B^2} = 8\pi G\,(-p + \frac{1}{2} f\dot{c}^2) + \Lambda \qquad (8)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} = 8\pi G\,(-p + \frac{1}{2} f\dot{c}^2) + \Lambda \qquad (9)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{1}{A^2} = 8\pi G\,(-p + \frac{1}{2} f\dot{c}^2) + \Lambda \qquad (10)$$

$$2\frac{A_4 B_4}{AB} + \frac{B_4^2}{B^2} - \frac{1}{A^2} = 8\pi G\,(\rho - \frac{1}{2} f\dot{c}^2) + \Lambda \qquad (11)$$

$$2\left(\frac{A_4}{A} - \frac{B_4}{B}\right) = 0 \qquad (12)$$

The Suffix 4 by the symbols A and B denotes, a differentiation with respect to *t*.

3. Solution of Field Equations

The conservation equation of EMT

$$\left(8\pi G T_j^i + \Lambda g_j^i\right)_{;i} = 0 \qquad (13)$$

which leads to

$$8\pi G\left[\dot{\rho} - f\dot{c}\ddot{c} + \{(\rho + p) - f\dot{c}^2\}\left(\frac{A_4}{A} + 2\frac{B_4}{B}\right)\right] + \dot{\Lambda} = 0 \qquad (14)$$

Following Hoyle and Narlikar [3], the source equation of C-field :$c^{i}_{;i} = n/f$leads to c = t for large r. thus $\dot{c} = 1$.

Equation (12) leads to

$$A = lB \tag{15}$$

Without loss of generality ,we assume $l = 1$, using equation (15) into (8) - (11) leads to

$$2\frac{B_{44}}{B} + \frac{B_4^2}{B^2} = 8\pi G\,(-p+\frac{1}{2}f\dot{c}^2) + \Lambda \tag{16}$$

$$2\frac{B_{44}}{B} + \frac{B_4^2}{B^2} - \frac{1}{B^2} = 8\pi G\,(-p+\frac{1}{2}f\dot{c}^2) + \Lambda \tag{17}$$

$$3\frac{B_4^2}{B^2} - \frac{1}{B^2} = 8\pi G\,(\rho-\frac{1}{2}f\dot{c}^2) + \Lambda \tag{18}$$

The barotropic fluid condition leads to

$$p = \gamma\rho, \text{where } 0 \leq \gamma \leq 1 \tag{19}$$

From equation (17) and (19) together leads to

$$2\frac{B_{44}}{B} + \frac{B_4^2}{B^2} - \frac{1}{B^2} = 8\pi G\,(-\gamma\rho+\frac{1}{2}f\dot{c}^2) + \Lambda \tag{20}$$

equation (18) and (20) together with $\dot{C} = 1$leads to

$$2\frac{B_{44}}{B} + (1+3\gamma)\frac{B_4^2}{B^2} - (1+\gamma)\frac{1}{B^2} = 4\pi G f(1-\gamma) + (1+\gamma)\Lambda \tag{21}$$

To get deterministic solution of equation (21) we assume

$$\Lambda = \frac{1}{R^2} = \frac{1}{B^2} \tag{22}$$

equation (21) and (22) together leads to

$$2\frac{B_{44}}{B} + (1+3\gamma)\frac{B_4^2}{B^2} = 4\pi G f(1-\gamma) + 2(1+\gamma)\frac{1}{B^2} \tag{23}$$

Let $B_4 = f(B)$ which leads to $B_{44} = ff'$

Equation (23) leads to

$$\frac{df^2}{dB} + \frac{(1+3\gamma)}{B}f^2 = 4\pi G f(1-\gamma)B + 2(1+\gamma)\frac{1}{B} \tag{24}$$

The constant of integration has been taken zero for simplicity.

Equation (24) leads to

$$f^2 = \left(\frac{dB}{dt}\right)^2 = \alpha B^2 + \beta \tag{25}$$

Where $\alpha = \frac{4\pi f G(1-\gamma)}{3(1+\gamma)}$ and $\beta = \frac{2(1+\gamma)}{(1+3\gamma)}$

Equation (25) leads to

$$\frac{dB}{\sqrt{\alpha B^2 + \beta}} = dt \tag{26}$$

Hence the metric (1) takes the form

$$ds^2 = dt^2 - \left(\frac{\beta}{\alpha} sinh^2\sqrt{\alpha}t\right)[dx^2 + e^{2x}dy^2 + dz^2] \tag{27}$$

for the metric (27) the physical and kinematical parameters are given by

$$\mathrm{R} = \mathrm{B} = \sqrt{\frac{\beta}{\alpha}} sinh\sqrt{\alpha}t \tag{28}$$

$$\Lambda = \frac{1}{R^2} = \frac{1}{B^2} = \frac{\alpha}{\beta} cosech^2\sqrt{\alpha}t \tag{29}$$

$$8\pi G\rho = cosech^2\sqrt{\alpha}t\left(3\alpha - 2\frac{\alpha}{\beta}\right) + 3\alpha + 4\pi f G \tag{30}$$

Now using $p = \gamma\rho$, eq. (14) leads to

$$8\pi G\left[\dot{\rho} - f\dot{c}\ddot{c} + \{(1+\gamma)\rho - f\dot{c}^2\}\left(3\frac{B_4}{B}\right)\right] + \dot{\Lambda} = 0 \tag{31}$$

Equation (31) leads to

$$\frac{d\dot{C}^2}{dt} + \left(6\sqrt{\alpha}coth\sqrt{\alpha}t\right)\dot{C}^2 = \left(6\sqrt{\alpha}coth\sqrt{\alpha}t\right)$$
$$\times\left[cosech^2\sqrt{\alpha}t\left(3\alpha - 2\frac{\alpha}{\beta}\right) + 3\alpha + 4\pi f G\right]\frac{(1+\gamma)}{8\pi G f} +$$
$$\frac{-\sqrt{\alpha}cosech^2\sqrt{\alpha}t coth\sqrt{\alpha}t\left(3\alpha - 2\frac{\alpha}{\beta}\right)}{4\pi G f} - 2\frac{\alpha\sqrt{\alpha}}{\beta}\frac{cosech^2\sqrt{\alpha}t coth\sqrt{\alpha}t}{4\pi G f} \tag{32}$$

To get deterministic solution of equation (32) we assume α = 1

Equation (32) leads to

$$\frac{d\dot{c}^2}{dt} + 6\,(coth t)\dot{C}^2 = 6\,(coth t) \tag{33}$$

equation (33) leads to

$$\dot{C}^2 = 1 \tag{34}$$

Thus, we have

$$\dot{C} = 1 \tag{35}$$

which agrees with the value used in the source equation. Thus creation field is proportional to time t and the metric (1) for constraints mentioned above, leads to

$$ds^2 = dt^2 - \left(\beta sinh^2\sqrt{\alpha}t\right)[dx^2 + e^{2x}dy^2 + dz^2] \tag{36}$$

4. Physical & Geometrical Aspects

The homogeneous mass density ρ, the cosmological constant Λ, the scale factor R and deceleration parameter q for the model (36) are given by

$$8\pi G\rho = cosech^2t\left(3 - \frac{2}{\beta}\right) + 3 + 4\pi fG \tag{37}$$

$$R = B = \sqrt{\beta}sinht \tag{38}$$

$$\Lambda = \frac{1}{R^2} = \frac{1}{B^2} = \frac{1}{\beta}cosech^2t \tag{39}$$

$$q = -tanht \tag{40}$$

Special Cases I: γ = 0 (Dust Universe)

Equation (26) leads to

$$\frac{dB}{\sqrt{a^2B^2 + 2}} = dt \text{ where} a^2 = \frac{4\pi fG}{3} \tag{41}$$

$$R = B = \frac{\sqrt{2}}{a}sinh[a(t + t_0)] \tag{42}$$

$$\Lambda = \frac{1}{R^2} = \frac{1}{B^2} = \frac{a^2}{2}cosech^2[a(t + t_0)] \tag{43}$$

$$q = -tanh^2[a(t + t_0)] \tag{44}$$

Special Cases II: γ = 1 (Stiff fluid Universe)

$$R = B = (t + t_0) \tag{45}$$

$$\Lambda = \frac{1}{R^2} = \frac{1}{B^2} = \frac{1}{(t+t_0)^2} \tag{46}$$

$$q = 0 \tag{47}$$

Special Cases III: γ = 1/3 (Radiation dominated Universe)

$$R = B = \frac{2\sqrt{2}}{a\sqrt{3}} sinh\left[\frac{\sqrt{2}}{a}(t + t_0)\right] \tag{48}$$

$$\Lambda = \frac{1}{R^2} = \frac{1}{B^2} = \frac{3a^2}{8} cosech^2\left[\frac{\sqrt{2}}{a}(t + t_0)\right] \tag{49}$$

$$q = -tanh^2\left[\frac{\sqrt{2}}{a}(t + t_0)\right] \tag{50}$$

5. Conclusions

The scale factor R increases with time.The cosmological constant Λ decreases as time increases. Since the deceleration parameter q < 0, hence the model (36) represents an accelerating universe.Also the coordinate distance (γ_H) to the horizon is maximum distance a null ray could have travelled at time t starting from infinite past i.e.

$$\gamma_H(t) = \int_{-\infty}^{t} \frac{dt}{R(t)}$$

We could extent the proper time t to the past because of the non-singular nature of the space-time:

$$\gamma_H(t) = \int_{0}^{t} \frac{dt}{R(t)} = \int_{0}^{t} \frac{dt}{\sqrt{\beta} sinht}$$

The integral diverges at lower limit showing that the model (21) is free from particle horizon, thus creation field cosmology solves one of the outstanding problem (Horizon problem) faced by Big-Bang cosmology. Further for all special cases viz. dust filled universe (γ = 0), radiation dominated universe (γ = 1/3) and Stiff fluid universe (γ = 1) scale factor increases and cosmological constant decreases as time increase, for dust filled and radiation dominated universe q < 0 shows accelerating universe. In case of stiff fluid, model has uniform motion.

Acknowledgement: The authors are thankful to Prof. Raj Bali, Emeritus Scientist CSIR, Department of Mathematics, University of Rajasthan, Jaipur, India for valuable discussions and Suggestions.

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