Article

The LRS Bianchi Type-I Strings with Strange Quark Matter coupled with Zero-Mass Scalar Field

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Abstract

In this paper, we have considered the Einstein field equations for LRS Bianchi Type-I space-time in the context of cosmic strings with strange quark matter coupled with zero-mass scalar field. The solutions of the field equations are obtained using the power law solution. Some physical and geometrical properties of the model are also discussed.

Keywords: LRS Bianchi Type-I, cosmic strings, quark matter, massless, scalar field.

1. Introduction

It is believed that the universe had undergone another phase transition viz., Quark Phase to hadron phase after Big-Bang when TC~200MeV. This state is also called Quark-Gluon-Plasma (QGP) state. Witten (1984) and Fahri and Jaffe (1984) have discussed the importance of this state at the early stages of the universe. The importance of Quarks during phase transition was discussed by Gerlach (1968), Ivanenko and Kurdgelze (1969), Bodmer (1971), Itoh (1970). The concept of Quark matter became known from the Quantum – Chromo – Dynamics (QCD). It can also be thought that there is a possibility of so called quark star or compact star which is smaller than the neutron star and is supported by degenerated quark pressure.

Many researchers are believed that the Universe is filled with extended objects like, stars, galaxies etc. The study of cosmic strings throws ample light on the basis of the evolution of the Universe. As a result, in recent years, there has been lot of interest in the study of cosmic strings. The structure formation in the early stages of the universe is explained by cosmic strings which draws the attention of researchers. Cosmic strings may have been created during phase transitions in the early era (Kibble, 1976) and they act as a source of gravitational field (Letelier, 1983). It is also believed that strings may be one of the sources of density perturbations that are required for the formation of large scale structures of the universe.

The energy momentum tensor for a cloud of strings given by Letelier (1983) is

$$\mathrm{T}_{ij} = \rho u_i u_j - \lambda x_i x_j . \tag{1}$$

where ρ is the rest energy density of the cloud of strings with particles attached to them, λ is the tension density of the strings and $\rho = \rho_p + \lambda$, ρ_p being the energy density of the particles. The

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velocity u^i describes the 4 – velocity which has components (1, 0, 0, 0) for a cloud of particles and x^i represents the direction of string which will satisfy

$$u^i u_i = -x^i x_i = 1 \text{ and } u^i x_i = 0. \tag{2}$$

In this paper, an attempt is made to attach strange quark matter to the string cloud. It is quite reasonable to attach strange quark matter to the strings as strings are free to vibrate in different modes and different modes represent different particles. These different modes of vibrations are also observed as different masses and spins. Thus we take quark matter energy density instead of particle energy density in the string cloud. So we get

$$\rho = \rho_p + \lambda + B_C \tag{3}$$

where B_C is the Bag constant.

As per the Bag model, quark density and quark pressure takes the following form

$$\rho_p = \rho_q + B_C \tag{4}$$

$$P_p = P_q - B_C,\ P_q = \frac{\rho_q}{3} \tag{5}$$

Strange quark matter was modeled with an equation of state based on the phenomenological bag model of quark model in which quark confinement was described by an energy term proportional to the volume. In this model, quarks are through as degenerate Fermi gas which exists only in a region of space endowed with a vacuum energy density B_C called Bag constant. In a simplified version of the bag model, it is assumed that quarks are massless and in-interacting.

Mak and Harko (2004) have discussed the cylindrically space time admitting conformal motion for a charged strange quark matter attached to string cloud. Yavuz et al (2005), Yilmaz (2005,2006), Sanjay Oli (2009), Pradhan et al(2009), Mahanta et al (2012), Singh and Sharma (2010), Bali and Pradhan (2007), Ozel et al (2010), Namrata et al(2013) , Sahoo and Mishra (2013) are some of the authors who have studied various aspects of strange quark matter attached to string cloud in general theory of relativity.

The study of interacting fields, one of the fields being a zero-mass scalar field, is basically an attempt to look into the yet unsolved problem of the unification of gravitational and quantum theories. For the last few decades, considerable interest has been focused on a set of field equations representing zero-mass scalar fields coupled with the gravitational field. Bergman and Leipnik (1957), Bramhacary (1960), Das (1960, 1962), Stepehenson (1972), Gautreau (1969), Rao et.al. (1972), Singh (1975), Patel (1975), Reddy (1988), Venkateswarlu and Reddy (1989), Pradhan et al. (2001) and Venkateswarlu and Pavan (2010) are some of the authors who have studied various aspects of interacting fields in the framework of general relativity. Venkateswarlu and satish (2014) and Venkateswarlu and Srinivas (2014) studied the Kantowski-Sacks cosmic strings with bulk viscosity coupled with massless scalar field and anisotropic Bianchi type-I and type-II cosmic strings with bulk viscosity coupled with zero-mass scalar field respectively. Recently, Venkateswarlu et. al (2014) have discussed the cosmic strings with Kaluza-Klien space – time in the frame work of zero-mass scalar fields.

Thus, in this paper, it is proposed to study the LRS Bianchi type-I strings with strange quark matter coupled with zero-mass scalar field.

2. Field equations and Solution

We consider the LRS Bianchi type –I metric as

$$ds^2 = -dt^2 + A^2dx^2 + B^2(dy^2 + dz^2) \tag{6}$$

where A, B are functions of time only .

The Einstein field equations corresponding to interacting zero-mass scalar fields are given by

$$R_{ij} - \frac{1}{2}g_{ij}R = -T_{ij} - \left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) \tag{7}$$

and

$$\phi^{k}_{;k} = 0. \tag{8}$$

The explicit field equations (7) and (8) for the metric (6) are given by

$$2\frac{B_{44}}{B} + \frac{B_4{}^2}{B^2} = -\lambda - \frac{\phi_4^2}{2} \tag{9}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4B_4}{AB} = -\frac{\phi_4^2}{2} \tag{10}$$

$$\frac{2A_4B_4}{AB} + \frac{B_4{}^2}{B^2} = -\rho + \frac{\phi_4^2}{2} \tag{11}$$

$$\phi_{44} + \phi_4\left(\frac{A_4}{A} + \frac{2B_4}{B}\right) = 0 \tag{12}$$

where the subscript 4 denotes ordinary differentiation with respect to t.

We have five unknowns A, B, ϕ, ρ and λ in four equations (9) to (12). For a deterministic solution one extra condition is needed. Now we shall explore physically meaningful solution of the above set of field equations considering the power law solution to the field variable A and B.

Let us assume $A = t^m$, m=constant. Therefore the solution of the field equations is given by

$$A = t^m$$
$$B = t^{\frac{1-m}{2}} \tag{13}$$
$$\phi = \phi_0 \log(t) + \phi_1$$

where ϕ_1 is a constant, $\phi_0 = \sqrt{(3-2m-m^2)/2}$ and $-3 < m < 1$.

Now the LRS Bianchi type-I cosmic string model with strange quark matter coupled with zero-mass scalar field can be expressed as

$$ds^2 = -dt^2 + t^{2m}dx^2 + t^{(1-m)}\left(dy^2 + dz^2\right). \tag{14}$$

Using the values of A , B and ϕ in the field equations (9)-(12), the expressions for the rest energy density of the cloud of strings (ρ), the tension density of the strings (λ), the energy density of the particles (ρ_p), the quark density (ρ_q) and quark pressure (P_q) are as follows:

$$\lambda = \frac{-(m-1)^2}{2t^2} \tag{15}$$

$$\rho = \frac{(m-1)^2}{2t^2} \tag{16}$$

$$\rho_p = \frac{(m-1)^2}{t^2} - B_C \tag{17}$$

$$\rho_q = \frac{(m-1)^2}{t^2} - 2B_C \tag{18}$$

$$P_q = \frac{(m-1)^2}{3t^2} - \frac{2B_C}{3}. \tag{19}$$

The expressions for scalar expansion (θ), shear scalar (σ) and the deceleration parameter (q) are given by

$$\theta = \frac{A_4}{A} + \frac{2B_4}{B} = \frac{1}{t}$$

$$\sigma^2 = \sigma_{ij}\sigma^{ij} = \frac{1}{3}\left(\frac{A_4}{A} - \frac{B_4}{B}\right)^2 = \frac{(3m-1)^2}{4t^2} \tag{20}$$

$$q = -\frac{1}{\theta^2}\left(\theta_4^2 + \theta^2\right) = 2.$$

3. Discussions

From equations (15) and (16), it is observed that $\lambda < 0, \rho > 0$. The rest energy density ρ decreases with time and always positive. The tension density of the strings λ also decreases with time but takes negative values only. It is further observed that the sum of rest energy density ρ and the tension density of the strings is equal to zero. The energy density of the particles ρ_p will be positive if $t < \frac{(m-1)}{\sqrt{B_C}}$. Further it is noted that the scalar expansion, shear scalar, the energy density and pressure tend to zero as t tend to infinity. Since the spatial volume $V = t$, it can be said that the model starts with big-bang. The model leads to anisotropic model for any t as $\frac{\sigma}{\theta} =$ constant. Since the deceleration parameter q is positive, the model as usual decelerates.

4. Conclusions

Here we have studied the LRS Bianchi Type-I string cosmological model with strange quark matter coupled with zero-mass scalar field. It is noted that the rest energy density ρ and the tension density of the strings λ are decreasing functions of cosmic time. Further, the energy density of the particles ρ_p will be positive if $t < \frac{(m-1)}{\sqrt{B_C}}$.

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