

Exploration**On the Plausibility of Final Unification with Avogadro Number**U. V. S. Seshavatharam^{*1} & S. Lakshminarayana²¹Honorary faculty, I-SERVE, Alakapuri, Hyderabad-35, AP, India.²Dept. of Nuclear Physics, Andhra University, Visakhapatnam-03, AP, India.**Abstract**

Physicists have long suggested that the four observed fundamental forces of nature are separate manifestations of what was once a single force at times close to the Big Bang. If so, magnitude of the unified force can be assumed to be equal to c^4/G . Strength of any interaction can be defined as the ratio of the operating force magnitude and the magnitude of c^4/G . Let the gravitational interaction taking place at black holes be called as ‘Schwarzschild interaction’. If strength of Schwarzschild interaction is unity, then weak interaction strength seems to be N_A^2 times less than the Schwarzschild interaction and strong interaction strength seems to be $N_A^{\sqrt[8]{3}}$ times less than the Schwarzschild interaction. Based on these concepts and considering the Avogadro number as an absolute and discrete number, basics of final unification can be understood. Based on the proposed semi empirical relations it can be suggested that, the magnitude of the gravitational constant may lie in between $(6.65 \times 10^{-11} \text{ \& } 6.71 \times 10^{-11}) \text{ m}^3 \text{ kg}^{-1} \text{ sec}^{-2}$.

Key Words: Avogadro number, Schwarzschild interaction, Final unification, Unified atomic mass unit.

1. Introduction

From final unification point of view, it is very much essential to couple the universal gravitational constant with the elementary physical constants. Then only the essence of unification can be understood. So far scientists proposed several interesting models [1-8]. In this context, important points can be expressed as follows:

- 1) In any inverse square law of force, system is sustained only by means of the central attractive force and it is the root cause of revolving body’s angular momentum. If is confirmed that, revolving body’s angular momentum is discrete, then it is a clear indication of the discrete nature of the central force acting on the revolving body. If one is willing to think in this direction, the historical mystery of Bohr’s discrete atomic structure and discrete angular momentum can be understood.

* Correspondence: Honorary faculty, I-SERVE, Alakapuri, Hyderabad-35, AP, India. E-mail: seshavatharam.uvs@gmail.com

- 2) Note that, as per the basic concepts of final unification, there exists a fundamental unified force from which all the observed forces emerged. If so, magnitude of the unified force can be assumed to be equal to (c^4/G) . Note that, magnitude of the radial inward force acting on any black hole surface [9] is the order of (c^4/G) .
- 3) By considering the squared Avogadro number [10-16] as a characteristic proportionality ratio, the characteristic magnitude of the unified atomic force can be assumed to be $(c^4/N_A^2 G)$.
- 4) Considering the Avogadro number as discrete, $(c^4/N_A^2 G)$ can be assumed to be discrete.

In this paper authors reviewed [10-17] their published concepts on final unification and proposed different relations for connecting and fitting the Avogadro number and the Newtonian gravitational constant in a unified approach.

2. The classical limits of force and power

Without considering the current notion of black hole physics, Schwarzschild radius of black hole [9] can be estimated with the characteristic limiting force of magnitude (c^4/G) . The outstanding problem in particle physics today is the inclusion of gravity in a single, unified quantum theory of all the fundamental interactions. Particle physicists have long suggested that the four observed fundamental forces of nature (the gravitational, electromagnetic, weak nuclear and strong nuclear forces) are separate, low energy manifestations of what was once a single force at times close to the Big Bang. It is postulated that as the universe expanded and cooled, this single force gradually broke down into the four separate interactions as observed today. However, unification theories that seek to unify the force of gravity with all the other forces (Theories of Everything) remain elusive, as the gravitational interaction lacks a quantum formulation.

To unify cosmology, quantum mechanics and the four observed fundamental cosmological interactions – certainly a ‘unified force’ is required. In this connection (c^4/G) can be considered as the classical force limit. Similarly (c^5/G) can be considered as the classical power limit. If it is true that c and G are fundamental physical constants in physics, then (c^4/G) and (c^5/G) can also be considered as fundamental compound physical constants. These classical limits are more powerful than the Uncertainty limit. Note that by considering the classical force limit (c^4/G) , the famous Planck mass can be obtained.

2.1 Simple applications of (c^4/G) can be stated as follows

- a) Magnitude of force of attraction or repulsion between any two charged particles never crosses (c^4/G) .
- b) Magnitude of gravitational force of attraction between any two massive bodies never crosses (c^4/G) .
- c) Magnitude of mechanical force on a revolving/rotating body never crosses (c^4/G) .
- d) Magnitude of electromagnetic force on a revolving body never crosses (c^4/G) .

2.2 Simple applications of (c^5/G) can be stated as follows

- a) Mechanical power never crosses (c^5/G)
- b) Electromagnetic power never crosses (c^5/G)
- c) Thermal radiation power never crosses (c^5/G)
- d) Gravitational radiation power never crosses (c^5/G)

3. Understanding the role of (c^4/G) in Black hole formation and Planck mass generation

3.1 Schwarzschild radius of a black hole

The 4 basic physical properties of a rotating black hole are its mass, size, angular velocity and temperature. Without going deep into the mathematics of black hole physics in this subsection an attempt is made to understand the black hole radius.

In all directions if a force of magnitude (c^4/G) acts on the mass-energy content of the assumed celestial body it approaches a minimum radius of (GM/c^2) in the following way [18,19]. Origin of the force (c^4/G) may be due to self-weight or internal attraction or external compression or a something else.

$$R_{\min} \cong \frac{Mc^2}{(c^4/G)} \cong \frac{GM}{c^2} \tag{1}$$

If no force (of zero magnitude) acts on the mass content M of the assumed massive body, its radius becomes infinity. With reference to the average magnitude of $\left(0, \frac{c^4}{G}\right) \cong \frac{c^4}{2G}$, the presently believed Schwarzschild radius can be obtained as

$$(R)_{ave} \cong \frac{Mc^2}{(c^4/2G)} \cong \frac{2GM}{c^2} \quad (2)$$

This proposal is very simple and seems to be different from the existing concepts of General theory of relativity.

3.2 To derive the Planck mass

So far no theoretical model proposed a derivation for the Planck mass. To derive the Planck mass the following two conditions can be given a chance.

Assuming that gravitational force of attraction between two Planck particles of mass (M_p) separated by a minimum distance (r_{min}) be,

$$\left[\frac{GM_p M_p}{r_{min}^2} \right] \cong \left(\frac{c^4}{G} \right) \quad (3)$$

With reference to wave mechanics, let

$$2\pi.r_{min} \cong \lambda_p = \left[\frac{h}{c.M_p} \right] \quad (4)$$

Here, λ_p represents the wavelength associated with the Planck mass. With these two assumed conditions Planck mass can be obtained as follows.

$$M_p = \sqrt{\frac{hc}{2\pi G}} \cong \sqrt{\frac{\hbar c}{G}} \quad (5)$$

3.3 Understanding the strength of any interaction

From above relations it is reasonable to say that,

- 1) If it is true that c and G are fundamental physical constants, then (c^4/G) can be considered as a fundamental compound constant related to a characteristic limiting force.
- 2) Black holes are the ultimate state of matter's geometric structure.
- 3) Magnitude of the operating force at the black hole surface is the order of (c^4/G) .
- 4) Gravitational interaction taking place at black holes can be called as 'Schwarzschild interaction'.
- 5) Strength of 'Schwarzschild interaction' can be assumed to be unity.
- 6) Strength of any other interaction can be defined as the ratio of operating force magnitude and the classical force magnitude (c^4/G) .

- 7) If one is willing to represent the magnitude of the operating force as a fraction of (c^4/G) i.e X times of (c^4/G) , where $X \ll 1$, then

$$\frac{X \text{ times of } (c^4/G)}{(c^4/G)} \cong X \rightarrow \text{Effective } G \Rightarrow \frac{G}{X} \tag{6}$$

If x is very small, $\frac{1}{x}$ becomes very large. In this way, x can be called as the strength of interaction. Clearly speaking, strength of any interaction is $\frac{1}{X}$ times less than the ‘Schwarzschild interaction’ and effective G becomes $\frac{G}{X}$.

4. Basic concepts on final unification

The following concepts and relations can be given a chance in final unification program.

- 1) With reference to the elementary charge and with mass similar to the Planck mass, a new mass unit can be constructed in the following way.

$$\left. \begin{aligned} (M_S)^\pm &\cong \sqrt{\frac{e^2}{4\pi\epsilon_0 G}} \cong 1.859272 \times 10^{-9} \text{ kg} \\ M_S c^2 &\cong \sqrt{\frac{e^2 c^4}{4\pi\epsilon_0 G}} \cong 1.042975 \times 10^{18} \text{ GeV} \end{aligned} \right\} \tag{7}$$

It can be called as the Stoney mass [20]. It is well known that e, c, G play a vital role in fundamental physics. With these 3 constants space-time curvature concepts at a charged particle surface can be studied. It was first introduced by the physicist George Johnstone Stoney. He is most famous for introducing the term ‘electron’ as the ‘fundamental unit quantity of electricity’. In unification program, with this mass unit and with a suitable proportionality ratio-characteristic mass of any elementary charge can be generated.

- 2) Avogadro number is an absolute discrete number and it is having no units like ‘per mole’.
 3) Atomic gravitational constant is squared Avogadro number times the Newtonian gravitational constant.
 4) Similar to the classical force limit (c^4/G) , in atomic system there exists a characteristic quantum force of magnitude:

$$F_X \cong (1/N_A^2)(c^4/G) \cong (c^4/N_A^2 G) \tag{8}$$

And its discrete form is:

$$(F_X)_n \cong \left[1/(nN_A)^2 \right] (c^4/G) \cong (1/n^2) (c^4/N_A^2 G) \cong (c^4/n^2 N_A^2 G) \tag{9}$$

where $n = 1, 2, 3, \dots$

- 5) $(F_X)_n$ is responsible for the observed discrete atomic structure and discrete angular momentum of the revolving electron.
- 6) Each proton will attractive the electron with a characteristic discrete force of magnitude $(F_X)_n$ and thus every electron will have total attractive force of $Z(F_X)_n$ where Z is the proton number of the nucleus.
- 7) If $R_c \approx 1.22 \text{ fm}$ is the characteristic nuclear charge radius, qualitatively and quantitatively it is also noticed that,

$$\left(\frac{c^2 R_c}{2N_A^2 G m_e} \right) \frac{\sqrt{m_p m_e} c^2}{2} \cong 27.18 \text{ eV.} \tag{10}$$

- 8) With reference to the discrete potential energy of electron in hydrogen atom [21,22] this observation can be re-written as,

$$(E_{\text{potential}})_n \cong -\sqrt{\frac{m_p}{m_e}} \frac{(F_X)_n R_c}{4} \cong \sqrt{\frac{m_p}{m_e}} \frac{F_X R_c}{4n^2}. \tag{11}$$

- 9) By considering the centripetal force on the electron, discrete Bohr radii can be expressed as,

$$(a_B)_n \cong 4 \sqrt{\frac{m_e}{m_p}} \left(\frac{e^2}{4\pi\epsilon_0 (F_X)_n R_c} \right) \cong 4n^2 \sqrt{\frac{m_e}{m_p}} \left(\frac{e^2}{4\pi\epsilon_0 F_X R_c} \right) \tag{12}$$

and it is possible to show that,

$$\hbar \cong 2 \left(\frac{m_e}{m_p} \right)^{\frac{1}{4}} \sqrt{\frac{m_e c^2}{F_X R_c}} \left(\frac{e^2}{4\pi\epsilon_0 c} \right) \cong \left(\frac{m_e}{m_p} \right)^{\frac{1}{4}} \sqrt{\frac{4N_A^2 G m_e}{c^2 R_c}} \left(\frac{e^2}{4\pi\epsilon_0 c} \right) \tag{13}$$

$$\left(\frac{1}{\alpha} \right) \cong 2 \left(\frac{m_e}{m_p} \right)^{\frac{1}{4}} \sqrt{\frac{m_e c^2}{F_X R_c}} \cong \left(\frac{m_e}{m_p} \right)^{\frac{1}{4}} \sqrt{\frac{4N_A^2 G m_e}{c^2 R_c}} \tag{14}$$

$$n\hbar \cong 2 \left(\frac{m_e}{m_p} \right)^{\frac{1}{4}} \sqrt{\frac{m_e c^2}{(F_X)_n R_c}} \left(\frac{e^2}{4\pi\epsilon_0 c} \right) \cong 2n \left(\frac{m_e}{m_p} \right)^{\frac{1}{4}} \sqrt{\frac{m_e c^2}{F_X R_c}} \left(\frac{e^2}{4\pi\epsilon_0 c} \right) \tag{15}$$

$$\frac{n\hbar}{2} \cong \left(\frac{m_e}{m_p}\right)^{\frac{1}{4}} \sqrt{\frac{m_e c^2}{(F_X)_n R_c} \left(\frac{e^2}{4\pi\epsilon_0 c}\right)} \cong n \left(\frac{m_e}{m_p}\right)^{\frac{1}{4}} \sqrt{\frac{m_e c^2}{F_X R_c} \left(\frac{e^2}{4\pi\epsilon_0 c}\right)} \quad (16)$$

In this way the historical mystery of the discrete nature of electron’s angular momentum can be understood in a unified way.

10) Modified super symmetric fermion-boson mass ratio [13-17] is close to $\Psi \cong 2.254$. Presently believed charged W boson is nothing but the top quark boson and there exists a charged weak boson of rest energy close 45600 MeV. Pair of 45600 MeV generates the presently believed Z boson and 45600 MeV and 80400 MeV combine to form a neutral boson of rest energy 126 GeV [13-17].

4.1 Understanding and fitting the characteristic nuclear radii

It is noticed that,

$$\frac{2(N_A^2 G)m_p}{c^2} \cong 9.0 \times 10^{-7} \text{ m} \cong R_1 \text{ (say)} \quad (17)$$

$$\frac{2Gm_p}{c^2} \cong 2.484 \times 10^{-54} \text{ m} \cong R_2 \text{ (say)} \quad (18)$$

With trial-error method it is noticed that, observed characteristic nuclear radii lie in between

$$(R_1^4 R_2)^{\frac{1}{5}} \cong 2.79 \times 10^{-16} \text{ m} \text{ and } (R_1^5 R_2)^{\frac{1}{6}} \cong 1.073 \times 10^{-14} \text{ m} \quad (19)$$

Thus, to a very good accuracy, it is noticed that,

$$N_A^{\sqrt[3]{8}} \left(\frac{2Gm_p}{c^2}\right) \cong 1.688 \times 10^{-15} \text{ m} \quad (20)$$

This can be compared with the presently believed ‘strong interaction range’. A very surprising observation is that,

$$N_A^{\sqrt[3]{8}} \left(\frac{Gm_p}{c^2}\right) \cong 0.844 \times 10^{-15} \text{ m} \quad (21)$$

This can be compared with the presently believed ‘rms radius’ of proton. This is a discovery and needs further study. From this relation and with reference to the new experimental rms radius of proton [23,24,25,26] $R_{rms} \cong 0.84184(67)\text{fm}$, magnitude of G can be estimated as follows.

$$G \cong N_A^{-\sqrt[8]{3}} \left(\frac{c^2 R_{rms}}{m_p} \right) \cong 6.65742 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ sec}^{-2} \tag{22}$$

Geometric mean of the above two characteristic lengths can be expressed as follows.

$$R_c \cong N_A^{\sqrt[8]{3}} \left(\frac{\sqrt{2} G m_p}{c^2} \right) \cong 1.194 \times 10^{-15} \text{ m} \approx 1.2 \text{ fm} \tag{23}$$

This length can be compared with the currently believed ‘nuclear charge radius’ 1.2 fermi.

If strength of Schwarzschild interaction is unity, then weak interaction strength can be considered as N_A^{-2} and strong interaction strength can be considered as $N_A^{-\sqrt[8]{3}}$. With this proposal the currently believed strong coupling constant α_s can be fitted as follows [26, 27].

$$\left(\frac{1}{\alpha_s} + 1 \right)^2 \cong \ln \left(N_A^{\sqrt[8]{3}} \right) \rightarrow \frac{1}{\alpha_s} \cong 8.45592 \Rightarrow \alpha_s \cong 0.11826 \tag{24}$$

4.2 Understanding and fitting the nucleon rest masses, nuclear stability and nuclear binding energy

Let us guess that N_A^2 plays a characteristic role in generating the rest mass of electron in the following way.

$$\text{let, } m_e c^2 \cong \beta \sqrt{\frac{e^2 c^4}{4\pi\epsilon_0 N_A^2 G}} \cong \beta \sqrt{\frac{e^2 F_X}{4\pi\epsilon_0}} \tag{25}$$

where β can be called as the electron mass index. It can be estimated as:

$$\beta \cong \sqrt{\frac{4\pi\epsilon_0 N_A^2 G m_e^2}{e^2}} \cong 295.0509223 \tag{26}$$

With this number β , electron, muon and tau rest masses can be fitted with the semi empirical relation.

$$\begin{aligned} (m_{lepton})_n c^2 &\cong \left[\beta^3 + (n^2 \beta)^n \sqrt{N_A} \right]^{\frac{1}{3}} \sqrt{\frac{e^2 F_X}{4\pi\epsilon_0}} \\ &\cong \left[\beta^3 + (n^2 \beta)^n \sqrt{N_A} \right]^{\frac{1}{3}} 0.001731 \text{ MeV} \end{aligned} \tag{27}$$

where $n = 0, 1, 2$. Obtained rest energies are 0.511 MeV, 105.95 MeV and 1777.4 MeV respectively [27]. New heavy charged lepton at $n = 3$ may be predicted close to 42262 MeV.

Proceeding further let us define another interesting number in the following way.

$$\gamma \cong \beta - \frac{1}{\alpha} \cong 158.0149232 \tag{28}$$

where γ can be called as the nucleon mass index. The two numbers $\left(\gamma, \frac{1}{\alpha}\right)$ can be fitted with the following semi empirical relation.

$$\left. \begin{aligned} \left(\gamma, \frac{1}{\alpha}\right) &\cong \left(\frac{\beta}{2}\right) \pm 2 \ln\left(\frac{\beta}{2}\right) \pm \frac{1}{2} \\ \gamma &\cong \left(\frac{\beta}{2}\right) + 2 \ln\left(\frac{\beta}{2}\right) + \frac{1}{2} \cong 158.01345 \\ \frac{1}{\alpha} &\cong \left(\frac{\beta}{2}\right) - 2 \ln\left(\frac{\beta}{2}\right) - \frac{1}{2} \cong 137.03746 \end{aligned} \right\} \tag{29}$$

If so, with trial-error it is noticed that [26],

$$\left. \begin{aligned} \left(\frac{m_n - m_p}{m_e}\right) &\cong \ln \sqrt{\gamma} \text{ and } \left(\frac{m_n + m_p}{2m_e}\right) \cong \sqrt{\frac{1}{\gamma} \cdot \frac{N_A^2}{N_A^{8/3}}} \\ \Rightarrow m_n c^2 &\cong 939.80 \text{ MeV and } m_p c^2 \cong 938.51 \text{ MeV} \end{aligned} \right\} \tag{30}$$

Interesting observation connected with proton-nucleon stability can be expressed as follows.

$$A_s \cong 2Z + \frac{Z^2}{\gamma} \cong 2Z + 0.00633Z^2 \tag{31}$$

where Z and A_s represent the ‘proton number and ‘stable nucleon number’ of the atomic nucleus respectively. This is a direct relation compared to the existing stability relation [28],

$$Z \cong \frac{A}{2 + (a_c/2a_a) A^{2/3}} \tag{32}$$

where Z is the estimated stable proton number and A is the assumed stable mass number.

See the following data. $Z = 21, A_s \cong 44.79$; $Z = 29, A_s \cong 63.32$; $Z = 47, A_s \cong 107.98$; $Z = 67, A_s \cong 168.13$; $Z = 83, A_s \cong 209.60$; $Z = 92, A_s \cong 237.56$ and so on. Super heavy stable isotopes can also be predicted with relation (31) directly.

With (γ, α) , the semi empirical mass formula energy coefficients [28,29] can be fitted in the following way.

$$\left. \begin{aligned}
 &\text{Pairing energy coefficient, } a_p \cong \frac{2m_p c^2}{\gamma} \cong 11.876 \text{ MeV} \\
 &\text{Asymmetry energy coefficient, } a_a \cong 2a_p \cong 23.752 \text{ MeV} \\
 &\text{Coulombic energy coefficient, } a_c \cong \alpha \sqrt{(m_p c^2)} (a_p) \cong 0.77 \text{ MeV} \\
 &\text{Surface energy coefficient, } a_s \cong \left(\frac{a_p + a_a}{2} \right) + 2a_c \cong 19.354 \text{ MeV} \\
 &\text{Volume energy coefficient, } a_v \cong \left(\frac{a_p + a_a}{2} \right) - 2a_c \cong 16.274 \text{ MeV} \\
 &\rightarrow (a_s, a_v) \cong \left(\frac{a_p + a_a}{2} \right) \pm 2a_c \cong \left(\frac{3a_p}{2} \right) \pm 2a_c \\
 &\text{thus, } a_p + a_a \cong a_s + a_v \cong 3a_p \cong 35.627 \text{ MeV}
 \end{aligned} \right\} \tag{33}$$

See table-1 for the existing and proposed binding energy coefficients.

See table-2 for the calculated nuclear binding energy based on the standard semi empirical mass formula,

$$B = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} \pm \frac{a_p}{\sqrt{A}} \tag{34}$$

In table-2, column 3 data represents the calculated binding energy and column 4 data represents the experimental binding energy [30].

Table-1: Existing and proposed SEMF binding energy coefficients

Existing energy coefficients	Proposed energy coefficients
$a_v \cong 15.78 \text{ MeV}$	$a_v \cong 16.274 \text{ MeV}$
$a_s \cong 18.34 \text{ MeV}$	$a_s \cong 19.354 \text{ MeV}$
$a_c \cong 0.71 \text{ MeV}$	$a_c \cong 0.77 \text{ MeV}$
$a_a \cong 23.21 \text{ MeV}$	$a_a \cong 23.752 \text{ MeV}$
$a_p \cong 12.0 \text{ MeV}$	$a_p \cong 11.876 \text{ MeV}$

Table-2: To fit the SEMF binding energy with the proposed energy coefficients

Z	A	$(BE)_{cal}$ in MeV	$(BE)_{meas}$ in MeV
26	56	492.04	492.254
28	62	546.45	545.259
34	84	727.44	727.341
50	118	1005.98	1004.950
60	142	1181.69	1185.145
79	197	1551.89	1559.40
82	208	1622.13	1636.44
92	238	1799.46	1801.693

5. To fit the rest energy of weakly interacting particles

If $R_c \approx 1.22 fm$ it is noticed that,

$$\left(\frac{m_e c^2}{F_X R_c}\right) m_e c^2 \cong 102750 \text{ MeV} \tag{35}$$

It is possible to guess that, there exists a charged weakly interacting fermion of mass 102750 MeV. Based on the modified Super symmetry concepts, in the earlier published papers the authors suggested that fermion-boson mass ratio is 2.254 [17]. If so, bosonic form of 102750 MeV can be expressed as follows. If $R_c \approx 1.22 fm$

$$\left(\frac{m_e c^2}{F_X R_c}\right) \left(\frac{m_e c^2}{2.254}\right) \cong 45586 \text{ MeV} \tag{36}$$

Two such oppositely charged bosons combine together to form a neutral boson of rest energy 91172 MeV. This can be compared with the presently believed neutral Z boson. In the published paper authors suggested that, presently believed charged W boson can be considered as the super symmetric boson of Top quark. Another interesting observation is that, 45586 MeV boson and Top quark boson combine together to form the observed 126 GeV boson. For details see references [12-17],[26,27].

6. Discussion

Considering the proposed concepts and relations accurate values of Gravitational constant [31,32,33] and Avogadro number can be estimated from elementary atomic physical constants. For the time being (i.e until a perfect model is developed), if one is willing to consider the revolving electron’s angular momentum as a compound physical constant and depends on the proton-electron rest masses, characteristic nuclear charge radius and the proposed discrete force $(c^4/N_A^2 G)$, it paves a path for coupling and interconnecting the micro-macro elementary physical constants in a consistent manner.

Method-1: From relations (30, 29, 28 and 26), within 4 steps, magnitudes of (N_A, G) can be fitted in the following way.

$$\left. \begin{aligned}
 \text{Step-1: } \gamma &\cong \left\{ \exp\left(\frac{m_n c^2 - m_p c^2}{m_e c^2}\right) \right\}^2 \cong 157.9021274 \\
 \text{Step-2: } \frac{N_A^2}{N_A^{\sqrt[8]{3}}} &\cong \gamma \left(\frac{m_n c^2 + m_p c^2}{2m_e c^2}\right)^2 \\
 \Rightarrow N_A &\cong \left\{ \gamma \left(\frac{m_n c^2 + m_p c^2}{2m_e c^2}\right)^2 \right\}^{1/x} \cong 6.002254694 \times 10^{23} \\
 \text{where } x &\cong (2 - \sqrt{8/3}) \cong 0.367006838 \\
 \text{Step-3: } \left(\frac{\beta}{2}\right) + 2\ln\left(\frac{\beta}{2}\right) + \frac{1}{2} - \gamma &\cong 0 \Rightarrow \text{with trial-error, } \beta \cong 294.8312312 \\
 \text{Step-4: } G &\cong \beta^2 \left(\frac{e^2}{4\pi\epsilon_0 N_A^2 m_e^2}\right) \cong 6.708143922 \times 10^{-11} \text{ m}^3 \text{kg}^{-1} \text{sec}^{-2}
 \end{aligned} \right\} \quad (37)$$

Method-2: From relations (30 and 22), within 3 steps, magnitudes of (N_A, G) can be fitted in the following way.

$$\left. \begin{aligned}
 \text{Step-1: } \gamma &\cong \left\{ \exp\left(\frac{m_n c^2 - m_p c^2}{m_e c^2}\right) \right\}^2 \cong 157.9021274 \\
 \text{Step-2: } \frac{N_A^2}{N_A^{\sqrt[8]{3}}} &\cong \gamma \left(\frac{m_n c^2 + m_p c^2}{2m_e c^2}\right)^2 \\
 \Rightarrow N_A &\cong \left\{ \gamma \left(\frac{m_n c^2 + m_p c^2}{2m_e c^2}\right)^2 \right\}^{1/x} \cong 6.002254694 \times 10^{23} \\
 \text{where } x &\cong (2 - \sqrt{8/3}) \cong 0.367006838 \\
 \text{Step-3: If } R_{rms} &\cong 0.84184(67) \text{ fm} \\
 G &\cong \frac{c^2 R_{rms}}{N_A^{\sqrt[8]{3}} m_p} \cong 6.69347776 \times 10^{-11} \text{ m}^3 \text{kg}^{-1} \text{sec}^{-2}
 \end{aligned} \right\} \quad (38)$$

These obtained values may not be absolute but can be given some consideration in unification program for further analysis. From all of the above semi empirical relations it can be suggested that, the magnitude of the gravitational constant may lie in between $(6.65 \times 10^{-11} \text{ and } 6.71 \times 10^{-11}) \text{ m}^3 \text{kg}^{-1} \text{sec}^{-2}$.

Now the key problem is to understand and fit the ‘unified atomic mass unit’. Independent of system of units and without considering the Avogadro number, unified atomic mass unit can be fitted as follows.

$$m_u c^2 \cong (\sqrt{m_n m_p} c^2 - B_a) + m_e c^2 \tag{39}$$

where m_u is the unified atomic mass unit and B_a is the ‘average binding energy per nucleon’. If $B_a \cong 8$ MeV, obtained $m_u \cong 931.42949$ MeV/c². Clearly speaking accuracy of m_u depends on the accurate ‘average binding energy per nucleon’. For the most tightly bound Isotope ${}^{62}_{28}\text{Ni}$, $B_a \cong 8.7946$ MeV, and for the stable ${}^4_2\text{He}$, $B_a \cong 7.1$ MeV. If so it is noticed that,

$$\left. \begin{aligned} &\sqrt{(B_a \text{ of } {}^4_2\text{He})(B_a \text{ of } {}^{62}_{28}\text{Ni})} \cong 7.9 \text{ MeV} \\ &\frac{(B_a \text{ of } {}^4_2\text{He}) + (B_a \text{ of } {}^{62}_{28}\text{Ni})}{2} \cong 7.9473 \text{ MeV} \\ &\sqrt{\frac{(B_a \text{ of } {}^4_2\text{He})^2 + (B_a \text{ of } {}^{62}_{28}\text{Ni})^2}{2}} \cong 7.99234 \text{ MeV} \end{aligned} \right\} \tag{40}$$

Considering any of the above values as the ‘average binding energy per nucleon’, unified atomic mass unit can be defined.

7. Conclusion

So far no model has been succeeded in coupling and understanding the unified concepts of gravity and atomic interactions. Considering the proposed concepts and relations accurate values of Gravitational constant and Avogadro number can be estimated from elementary atomic physical constants. With further research and analysis, different models of final unification can be developed with different proportionality ratios and finally a unified model can be standardized with a refined proportionality ratio.

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