Article

Bianchi Type-III Inflationary Cosmological Model with Bulk Viscosity in General Relativity

Raj Bali*[1] & Laxmi Poonia[2]

[1]Department of Mathematics, University of Rajasthan, Jaipur-302004, India
[2]Department of Mathematics, Manipal University, Ajmer Road, Jaipur, India

Abstract

Bianchi Type III inflationary cosmological model with bulk viscosity and flat potential is investigated. To get the deterministic model of the universe, we assume the condition that shear (σ) is proportional to expansion (θ) which leads to $B = C^n$ where B and C are metric potentials and n is a constant. We find that spatial volume increases with time exponentially, hence represents inflationary scenario of the universe. The deceleration parameter $q < 0$, thus the model represents accelerating universe. The model in special case, isotropizes. The presence of bulk viscosity leads to inflationary like solution. The Hubble parameter is initially large but tends to finite limit for large values of t. The model has Point Type singularity. The Higgs field (ϕ) evolves slowly but the universe expands in an exponential way due to vacuum field energy.

Keywords: Bianchi Type III, inflationary, cosmological model, bulk viscosity, flat potential.

1. Introduction

Bianchi Type I models are very special subset of spatially homogeneous cosmological models. We, therefore, consider more general Bianchi Type III space-time for the study of inflationary scenario of universe. Inflation means that universe has gone a period of exponential expansion in its early stage of evolution. In modern cosmology, inflation is an essential ingredient. During the inflationary epoch, the scale factor of the universe grows exponentially allowing a small causally coherent region big enough to be identified with the present observable universe. Therefore, the inflationary scenario is the satisfactory solution to some of the conceptual issues in cosmology and is not understood in the standard big bang theory. The inflationary scenario explains several mysteries of modern cosmology like homogeneity, the isotropy and flatness of observed universe. Guth[1] introduced the idea of early inflationary phase in context of grand unified field theories. Wald[2], Gron[3], Barrow[4] discussed inflationary scenario in Friedmann-Robertson-Walker (FRW) model which is already homogeneous and isotropic. Rothman, and Ellis[5] pointed out that we could have the solution of isotropy problem if we work with anisotropic

*Correspondence: Raj Bali, CSIR Emeritus Scientist, Department of Mathematics, University of Rajasthan, Jaipur-302004, India. E-mail: balir5@yahoo.co.in

metric and show that these models can be isotropized. Stein-Schabes[6] has shown that inflationary scenario is possible when potential $V(\phi)$ has flat region and Higgs field (ϕ) evolves slowly but the universe expands in an exponential way due to vacuum field energy. Several authors viz. Bhattacharjee and Barua[7], Bali and Jain[8], Reddy et al.[9], Bali[10,11] have studied the role of self interacting scalar fields with flat potential in inflationary cosmology. Recently Bali and Singh[12] have investigated LRS Bianchi Type I stiff fluid inflationary universe with variable bulk viscosity.

In early stage of evolution of universe, the dissipative process play a significant role for the high degree of isotropy, we observe today. Misner[13] has studied the effect of bulk viscosity on the evolution of cosmological models. Heller and Klimek[14] investigated viscous fluid cosmological models without initial singularity in which they have shown that the introduction of bulk viscosity removes the initial singularity effectively. Belinski and Khalatnikov[15] studied the effect of viscosity in FRW model in which the coefficient of viscosity is assumed as function of energy density. The effect of bulk viscosity in the early stage of evolution of universe has been studied by Murphy[16]. Roy and Prakash[17,18] have investigated some viscous fluid cosmological models in Bianchi Type I space-time in which coefficients of bulk viscosity is assumed as constant. Padmanabhan and Chitre[19] investigated that the presence of bulk viscosity, leads to inflationary like solutions. The effect of bulk viscosity on cosmological evolution has been studied by Zimdahl[20] Sahni and Starobinski[21], Saha[22,23], Singh et al.[24], Peebles[25], Bali et al.[26, 27, 28, 29].

Motivated by the above mentioned studies, we investigate Bianchi Type III inflationary cosmological model with variable bulk viscosity and flat potential. To get the deterministic scenario, we assume that $\zeta\theta$ = constant as supposed by Brevik et al. [30]. The physical aspects of the model related with inflationary scenario are also discussed.

2. Metric and Field Equations

We consider Bianchi Type III space-time in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{-2\alpha x} dy^2 + C^2 dz^2 \tag{1}$$

where A, B, C are metric potentials and are functions of t-alone, α is constant.

In case of gravity minimally coupled to a scalar field $V(\phi)$, we have

$$S = \int \sqrt{-g}\left[R - \frac{1}{2} g^{ij}\partial_i\phi\partial_j\phi - V(\phi)\right] d^4x \tag{2}$$

which on variation of S with respect to dynamical fields leads to Einstein field equation

$$R_{ij} - \frac{1}{2} R\, g_{ij} = -8\pi T_{ij} \tag{3}$$

where energy momentum tensor (T_{ij}) for scalar field is given by Guth[1] in presence of viscosity is given by

$$T_{ij} = \partial_i \phi \partial_j \phi - \left[\frac{1}{2} \partial_\ell \phi \partial^\ell \phi + V(\phi) \right] g_{ij} - \zeta\theta (g_{ij} + v_i v_j) \tag{4}$$

and

$$\frac{1}{\sqrt{-g}} \partial_i [\sqrt{-g}\, \partial^i \phi] = -\frac{dV}{d\phi} \tag{5}$$

where V is the effective potential, ϕ the Higgs field, ζ the coefficient of bulk viscosity and θ the expansion in the model. We assume the coordinates to be comoving so that

$$v^1 = 0 = v^2 = v^3, v^4 = 1.$$

The Einstein field equation (3) for the metric (1) leads to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} = -8\pi \left[\frac{\dot{\phi}^2}{2} - K - \beta \right] \tag{6}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} = -8\pi \left[\frac{\dot{\phi}^2}{2} - K - \beta \right] \tag{7}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{\alpha^2}{A^2} = -8\pi \left[\frac{1}{2} \dot{\phi}^2 - K - \beta \right] \tag{8}$$

$$\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{A_4 C_4}{AC} - \frac{\alpha^2}{A^2} = 8\pi \left[\frac{\dot{\phi}^2}{2} + K \right] \tag{9}$$

$$\alpha \left(\frac{B_4}{B} - \frac{A_4}{A} \right) = 0 \tag{10}$$

For deterministic model of inflationary universe, we have assumed that $V(\phi) = K$ (constant) and $\zeta\theta = \beta$ (constant)

We use the ansatz $\zeta\theta$ = constant because it has significant role to connect with occurrence of Little Rip (LR) cosmology using FRW metric as given by Brevik et al. [30]
Equations (5) and (10) lead to

$$\ddot{\phi} + \left(2\frac{B_4}{B} + \frac{C_4}{C} \right) \dot{\phi} = 0 \tag{11}$$

and

$$A = k\,B \tag{12}$$

Equation (11) leads to

$$\dot{\phi} = \frac{\gamma}{B^2 C} \tag{13}$$

where γ is a constant. Equations (13) and (6) lead to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} = -8\pi\left[\frac{\gamma^2}{2B^4C^2} - K - \beta\right] \tag{14}$$

Equation (14) is one equation in two unknown B and C. To get the deterministic solution, we assume that shear (σ) is proportional to expansion (θ) which leads to

$$B = C^n \tag{15}$$

where n is a constant. Equation (14) and (15) lead to

$$C_{44} + \frac{n^2}{n+1}\frac{C_4^2}{C} = -\frac{8\pi\gamma^2 C^{-(4n+2)+1}}{2(n+1)} + \frac{8\pi C}{n+1}(K+\beta) \tag{16}$$

which leads to

$$2C_{44} + \frac{2n^2}{n+1}\frac{C_4^2}{C} = -\delta\gamma^2 C^{-(4n+1)} + 2\delta\, C(K+\beta) \tag{17}$$

where

$$\frac{8\pi}{n+1} = \delta\,(\text{constant}) \tag{18}$$

Equation (17) leads to

$$f^2 = a\,C^{-4n} + b\,C^2 \tag{19}$$

where

$$C_4 = f(C),\ a = \frac{\delta\gamma^2(n+1)}{2n(n+2)},\ b = \frac{2\delta(n+1)(K+\beta)}{(2n^2+2n+2)}$$

Now equation (19) leads to

$$\frac{C^{2n}\,dC}{\sqrt{a + b\,C^{2(2n+1)}}} = dt \tag{20}$$

which leads to

$$C^{2n+1} = \ell\,\sinh mT \tag{21}$$

where

$$\ell=\sqrt{\frac{a}{b}},(2n+1)\sqrt{b}=m,\ t+t_0=T \tag{22}$$

Thus

$$B=C^n=\ell^{\frac{n}{2n+1}}\sinh^{\frac{n}{2n+1}} mT \tag{23}$$

$$A=kB=k\,\ell^{\frac{n}{2n+1}}\sinh^{\frac{n}{2n+1}} mT \tag{24}$$

Therefore, the metric (1) leads to

$$ds^2=-dT^2+k^2\ell^{\frac{2n}{2n+1}}\sinh^{\frac{2n}{2n+1}}(mT)\,dx^2+\ell^{\frac{2n}{2n+1}}\sinh^{\frac{2n}{2n+1}}(mT)e^{-2\alpha x}\,dy^2$$
$$+\ell^{\frac{2}{2n+1}}\sinh^{\frac{2}{2n+1}} mT\,dz^2 \tag{25}$$

3. Some Physical and Geometrical Aspects

Equation (13) leads to

$$\phi=\frac{\delta}{\ell}\log\tanh\left(\frac{mT}{2}\right)+N \tag{26}$$

The spatial volume (R^3), the expansion (θ), the shear (σ) the decelerating parameter (q), the Hubble parameter (H) for the model (25) are given by

$$R^3=ABC=k\ell\sinh mT \tag{27}$$

$$\theta=\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}=m\coth mT \tag{28}$$

$$\sigma=\frac{1}{\sqrt{3}}\left(\frac{2B_4}{B}-\frac{C_4}{C}\right)=\frac{n-1}{\sqrt{3}}\frac{C_4}{C}=\frac{m(n-1)}{(2n+1)\sqrt{3}}\coth m\,T \tag{29}$$

$$q=-\frac{\ddot{R}/R}{\dot{R}^2/R^2}=2-3\tan h^2 mT \tag{30}$$

$$H=\frac{m}{3}\coth mT \tag{31}$$

The spatial volume (R^3) for the model (25) increases with time exponentially, hence inflationary scenario exists in the model. The deceleration parameter q < 0, thus the model represents accelerating universe. Since $\frac{\sigma}{\theta} \neq 0$, thus the model in general represents anisotropic phase of the universe. However, the model isotropizes for n = 1. The Hubble parameter (H) is initially large but tends to a finite limit for large values of T. The model has Point Type singularity at T = 0 (MacCallum[31]). Equation (27) shows that presence of bulk viscosity leads to inflationary like solution. Higgs field evolves slowly but the universe expands in an exponential way due to vacuum field energy.

References

[1] Guth AH 1981 Phys. Rev. D**23** 347
[2] Wald RM 1983 Phys. Rev. D**28** 2118
[3] Gron O 1985 Phys. Rev. D**32** 2522
[4] Barrow JD 1988 Nucl. Phys. B **296** 697
[5] Rothman T and Ellis GFR 1986 Phys. Lett. B **108** 19
[6] Stein-Schabes JA 1987 Phys. Rev. D **35** 2345
[7] Bhattacharjee R and Barua KK 2001 Ind. J. Pure & Appl. Math. **32** 47
[8] Bali R and Jain VC 2002 Pramana – J. Phys. **58** 1
[9] Reddy DRK Adhav KS Katore SD and Wankhade KS 2009 Int. J. Theor. Phys. **48**, 2884.
[10] Bali R 2011 Int. J. Theor. Phys. **50** 3043.
[11] Bali R 2012 Mod. Phys. Lett. A **27** 1250049
[12] Bali R and Singh S 2014 Canad. J. Phys. **92** 1.
[13] Misner CW 1908 Astrophys. J. **185** 105
[14] Heller N and Klimek Z 1975 Astrophys. and Space-Science **33** 237
[15] Belinski VA and Khalatnikov IM 1976 Sov. Phys. JETP **42** 205
[16] Murphy GL 1977 Phys. Lett. A **62** 75
[17] Roy SR and Prakash S 1976 J. Phys. A – Mathematical and General **9** 261
[18] Roy SR and Prakash S 1977 Ind. J. Pure & Appl. Math. **8** 723
[19] Padmanabhan T and Chitre SM 1987 Phys. Lett. A **120** 433
[20] Zimdahl W 1996 Phys. Rev. D **53** 5483
[21] Sahni V and Starobinski A 2000 Int. J. Mod. Phys. D **9** 373
[22] Saha B 2005 Phys. Lett. A **20** 2127
[23] Saha B 2007 J. Phys. A : Mathematical theory **40** 14011
[24] Singh CP Kumar S and Pradhan A 2007 Class. Quant. Gravity **24** 455
[25] Peebles PJE 2003 Rev. Mod. Phys. **75** 559
[26] Bali R and Sharma K 2003 Astrophys. and Space-Science **283** 11
[27] Bali R and Kumawat P 2008 Phys. Lett. B **665** 332
[28] Bali R and Singh JP 2008 Int. J. Theor. Phys. **47** 3288
[29] Bali R and Singh P 2012 Int. J. Theor. Phys. **51** 772
[30] Brevik E Elizalde S and Odintsov SD 2011 Phys. Rev. D **84** 103508
[31] MacCallum MAH 1971 Comm. Theor. Phys. **20** 57