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Two Fluids Viscous Dark Energy Cosmological Models with Linearly Varying Deceleration Parameter in Self Creation Cosmology

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Abstract

We present a class of solutions of Barber field equations describing two-fluid universe for the new class of Bianchi type model. In this model, one fluid is the barotropic fluid and other bulk viscous. The exact solutions of the field equations are obtained by considering a Linearly Varying Deceleration Parameter. The cosmological parameters have been discussed in detail during the evolution of the universe.

Keywords: self-creation cosmology, dark energy, viscous fluid.

1. Introduction

The origin of accelerating expansion is regarded that the universe is dominated by an exotic component with the negative pressure called ”dark energy” which constitutes 70 percent of the energy density of the universe and dark matter about 26 percent. There are several candidates for dark energy: first is the cosmological constant [1], and the second is the so-called dynamic candidates such as: Phantom [2], quintessence [3], K-essence [4] and quintom [5]. The scalar field is postulated to couple to the trace of the energy-momentum tensor. The consistency of Barber’s second theory motivates us to study cosmological model in this theory. Various aspects of the self-creation theories have been investigated by Pimentel [6], Soleng [7, 8] and Reddy et al. [9]. Singh [10], Maharaj et al. [11] Venkateswarlu et al. [12-14] Reddy [15], Pradhan et al.[16,18] and katore et. al. [19-23].

The introduction of viscosity into cosmology has been investigated from different view points Grøn [24]; Padmanabhan et al. [25]; Barrow [26]; Zimdahl [27]; Farzin et al. [28]. Misner [29, 30] noted that the “measurement of the isotropy of the cosmic background radiation represents the most accurate observational datum in cosmology”. The astrophysical observations also indicate some evidences that cosmic media is not a perfect fluid Jaffe et al. [31]. The viscosity effect could be concerned in the evolution of the universe Brevik et al. [32, 33] and Cataldo et al.

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[34]. Several authors (Singh and Kale [35] and Yadav [36] and references therein) have investigated cosmological models with dissipative effect, also Pradhan [37- 41], Rao et al. [42], Singh et al. [43], Bali and Pradhan [44] have studied bulk viscous fluid cosmological models in different physical context.

Bianchi type cosmological models are important in the sense that these are homogeneous and anisotropic. Bianchi type space times provide spatially homogeneous and anisotropic models of the universe as compared to the homogeneous and isotropic FRW models. From which the process of isotropization of the universe is studied through the passage of time. The simplicity of the field equations and relative ease of the solutions of Bianchi space times are useful in constructing models of spatially homogeneous and anisotropic cosmologies. Chakraborty et al. [45], Kalligas et al. [46], Arbab [47], Beesham et al. [48] and Kilinc [49] obtained solutions of the Bianchi type cosmological models.

Singh and Chaubey [50] have studied interacting dark energy in Bianchi type I space-time. The viscous dark tachyon cosmology in interacting and non-interacting cases in non-flat FRW Universe was studied by Setare et al. [51]. Two-Fluid Cosmological Models in Bianchi Type-III and V Space-Time has been investigated by Adhav et al. [52,53], Bianchi type-VI_0 model with a two-fluid source has been investigated by Coley et al.[54]. Pant et al.[55] examined two fluid cosmological models using Bianchi type-II space-time. Two fluid Bianchi type-I models are studied by Oli [56]. Amirhashchi et al. [57, 58] have studied an interacting and non-interacting two-fluid scenario for dark energy models in FRW universe. Two-Fluid Cosmological Models in Bianchi Type-V Space-Time in Higher Dimensions examined by Mete et al.[59].

2. Metric and field equations

Consider the general class of Bianchi cosmological models as

$$ds^2 = dt^2 - A^2 dx^2 - B^2 e^{-2x} dy^2 - C^2 e^{-2mx} dz^2 \tag{1}$$

We have the additional classes of Bianchi models as follows: type III corresponds to $m = 0$, type-V corresponds to $m = 1$, type VI_0 corresponds to $m = -1$, and all other m give VI_h, where $m = h - 1$. A, B and C are the functions of t only and m is constant.

The Einstein field equation in Barbers second self creation theory of Gravitation is

$$G_{ij} = R_{ij} - \frac{1}{2} R g_{ij} = -8\pi\phi^{-1} T_{ij} \, , \tag{2}$$

where G_{ij} is the Einstein tensor and T_{ij} is the energy-momentum tensor.

The scalar field equation is defined as

$$\Box\phi = \frac{8\pi}{3}\lambda T \ , \tag{3}$$

where $\Box\phi = \phi;^k_k$ the invariant d'Alembertion and T is the trace of the energy momentum tensor that describes all non Gravitational non scalar field theory. Barber scalar field ϕ be the function of time t.

The energy-momentum tensor for two fluid sources is given by

$$T^i_j = (\rho + \bar{p})U_i U^i - \bar{p} g^i_j \ , \tag{4}$$

where T^i_j represent two fluid energy momentum tensor of bulk viscous dark energy and barotropic fluid, together with

$$U_i U^i = 1 \text{ and} \tag{5}$$

$$\bar{p} = p - \eta U;^i_i \ , \tag{6}$$

here ρ is the energy density, p is the isotropic pressure, $\bar{p}$ is the effective pressure, η is the bulk viscosity coefficient and U^i is the four-velocity vector of the matter distribution defined as

$$U^i = \delta^i_4, i = 1,2,3,4. \tag{7}$$

We assume that the co-ordinate to be commoving so that $U^i = (0,0,0,1)$. Since the bulk viscous pressure represents only a small correction to the thermo dynamical pressure, it is a reasonable assumption that the inclusion of viscous term in the energy-momentum tensor does not change fundamentally the dynamics of the cosmic evolution. In general, η is a function of time t. a comma (,) and a semi-colon (;) denotes ordinary and covariant differentiation with respect to t respectively.

The universe field with bulk viscous fluid, form (4) one find

$$T^1_1 = T^2_2 = T^3_3 = -\bar{p}, T^4_4 = \rho \text{ and } T = \rho - 3\bar{p} \ . \tag{8}$$

In a commoving co-ordinate system, the field equations for the metric (1) with the help of equations (2) and (8) can be written as

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} - \frac{m}{A^2} = -8\pi\phi^{-1}\bar{p} \ , \tag{9}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} - \frac{m^2}{A^2} = -8\pi\phi^{-1}\bar{p} \ , \tag{10}$$

$$\frac{B_{44}}{B} + \frac{A_{44}}{A} + \frac{B_4 A_4}{BA} - \frac{1}{A^2} = -8\pi\phi^{-1}\bar{p} \ , \tag{11}$$

$$\frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} + \frac{B_4 C_4}{BC} - \frac{(m^2 + m + 1)}{A^2} = 8\pi\phi^{-1}\rho \ , \tag{12}$$

$$(m+1)\frac{A_4}{A} - \frac{B_4}{B} - m\frac{C_4}{C} = 0 \ , \tag{13}$$

$$\phi_{44}+\phi_4\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)=\frac{8\pi\eta}{3}\left(\rho-3\bar{p}\right). \tag{14}$$

We would be sure that the fluid satisfy the energy momentum conservation law $T^{ij}_{;j}=0$. Furthermore, we should require positivity of the energy density of the co-moving fluid $\rho \geq 0$ to be satisfied as a condition for our fluid to be a realistic energy momentum source, the conservation equation yield

$$\rho_4+\left(\rho+\bar{p}\right)\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)=0\,, \tag{15}$$

where

$$\bar{p}=p_m+\bar{p}_D \text{ and } \rho=\rho_m+\rho_D\,. \tag{16}$$

Here p_m & ρ_m are the pressure and density of barotropic fluid, and $\bar{p}_D$ & ρ_D are the pressure and density of dark fluid respectively.

The equation of state (EoS) for barotropic w_m fluid and dark fluid w_D are respectively given by

$$w_m=\frac{p_m}{\rho_m}\,, \tag{17}$$

$$w_D=\frac{\bar{p}_D}{\rho_D}\,. \tag{18}$$

Now we define some parameters for the general class of Bianchi cosmological model which are important in cosmological observations. The average scale factor and spatial volume are

$$a=\left(ABC\right)^{\frac{1}{3}}\,, \tag{19}$$

$$V=a^3=\left(ABC\right). \tag{20}$$

The anisotropy parameter of the expansion is expressed in terms of mean and directional Hubble parameters as

$$A_m=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2\,, \tag{21}$$

where

$$H=\left(\ln a\right)^{\bullet}=\frac{\dot{a}}{a}=\frac{1}{3}\left(H_1+H_2+H_3\right). \tag{22}$$

where $H_1=\dfrac{A_4}{A}$, $H_2=\dfrac{B_4}{B}$ and $H_3=\dfrac{C_4}{C}$ are the directional Hubble parameters of *x*, *y* and *z* axes respectively.

The physical parameters which are observational interest in cosmology are the expansion scalar θ, the shear scalar σ^2, and the deceleration parameter q, defined as

$$\theta=u^m_{;m}=\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right), \tag{23}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\,\sigma^{ij} = \frac{1}{2}\left\{\left(\frac{A_4}{A}\right)^2 + \left(\frac{B_4}{B}\right)^2 + \left(\frac{C_4}{C}\right)^2\right\} - \frac{\theta^2}{6}\,, \tag{24}$$

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = \frac{d}{dt}\left(\frac{1}{H}\right) - 1\,. \tag{25}$$

The field equations (9) - (12) can be expressed in terms of q, H, σ^2 as

$$\bar{p} = H^2(2q-1) - \sigma^2 + \frac{(m^2+m+1)}{3A^2}\,, \tag{26}$$

$$\rho = H^2 - \sigma^2 - \frac{(m^2+m+1)}{A^2}\,. \tag{27}$$

3. Exact solution of field equation

Berman [60] and Berman and Gomide [61] proposed in the 1980's a law of variation for Hubble parameter within the context of RW space times in general relativity that yields constant deceleration parameter ($q = n-1$, where $n \geq 0$ is a constant). In Berman's law the deceleration parameter can get values $q \geq -1$, and since $-1 \leq q < 0$ corresponds to the accelerating expansion, many authors have studied cosmological models using this law in the context of dark energy. In this paper, we propose a generalized linearly varying deceleration parameter,

$$q = \frac{-a\ddot{a}}{\dot{a}^2} = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = -kt + n - 1\,. \tag{28}$$

We consider the solution for $k > 0$ and $n > 0$. The reason for considering the solution only for $k > 0$ and n> 0 is not only for simplicity but also for compatibility with the observed universe. $k > 0$ means we are dealing with increasing acceleration $(\dot{q} = -k < 0)$ and $k > 0$ is the only way to shift the deceleration parameter to values higher than (−1).
From equation (28) one obtains solutions for the scale factor:

$$a = a_1 e^{\frac{2}{n} tamh^{-1}\left(\frac{kt}{n} - 1\right)}\,, \tag{29}$$

where a_1 be the constant of integration.

After solving equations (13), (19) and (28) we get,

$$A = \alpha_3 e^{\left\{\frac{1+mb-b}{n(m+2)}\right\} tamh^{-1}\left(\frac{kt}{n} - 1\right)}\,,$$

$$B = \alpha_2 e^{\left(\frac{6(m-2mb-b+1)}{n(m+2)}\right) tamh^{-1}\left(\frac{kt}{n} - 1\right)}\,,$$

$$C = \alpha_1 e^{\left(\frac{6b}{n}\right) tamh^{-1}\left(\frac{kt}{n} - 1\right)}\,.$$

Now, using the above result, the Hubble parameter of the universe is obtained as follows:

$$H = \frac{2}{\left(2nt - kt^2\right)}. \tag{30}$$

Also from the equations (24) and (25) we get,

$$\sigma^2 = \frac{3\left(n^2\beta_1^2 + \beta_2^2 + \beta_3^2\right) - \left(n\beta_1 + \beta_2 + \beta_3\right)^2}{6\left(2nt - kt^2\right)^2}, \tag{31}$$

$$q = -kt + n - 1. \tag{32}$$

where $\beta_1 = \frac{6mb(m+2) + 6(m - 2mb - b + 1)}{n(m+1)(m+2)}$, $\beta_2 = \frac{6(m - 2mb - b + 1)}{(m+2)}$, $\beta_3 = 6b$.

Here we consider two cases:

i) Non-interacting model and ii) interacting model.

Case i. For non-interacting two fluid models:

In this case we assume that two fluids do not interact. Therefore, the general form of conservation equation (15) leads for barotropic and dark fluid separately as

$$(\rho_m)_4 + (\rho_m + p_m)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0, \tag{33}$$

$$(\rho_D)_4 + (\rho_D + p_D)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0. \tag{34}$$

Here is, of course, a structural difference between equations (33) and (34), because equation (33) is in the form of w_m which is constant and hence equation (34) is integrable. But equation (34) is a function of w_D which is an unknown time dependent parameter. Using equations (17) and (33), we get density and pressure of barotropic fluid as

$$\rho_m = \alpha_2 e^{\frac{-6(1+w_m)}{n} tamh^{-1}\left(\frac{kt}{n} - 1\right)}, \tag{35}$$

$$p_m = \alpha_2 w_m e^{\frac{-6(1+w_m)}{n} tamh^{-1}\left(\frac{kt}{n} - 1\right)}, \tag{36}$$

where ρ_0 be the constant of integration.

From equations (26) and (27), we get effective pressure and energy density as

$$8\pi\bar{p} = a_1^2 e^{2tamh^{-1}\left(\frac{kt}{n} - 1\right)}\left\{X_1 + \frac{\left(m^2 + m + 1\right)}{3\alpha_3^2 e^{2\left\{\frac{1+mb-b}{n(m+2)}\right\} tamh^{-1}\left(\frac{kt}{n} - 1\right)}}\right\}, \tag{37}$$

$$8\pi\rho = a_1^2 e^{2tamh^{-1}\left(\frac{kt}{n}-1\right)}\left\{X_2 - \frac{\left(m^2+m+1\right)}{\alpha_3^2 e^{2\left\{\frac{1+mb-b}{n(m+2)}\right\}tamh^{-1}\left(\frac{kt}{n}-1\right)}}\right\}. \tag{38}$$

Also we obtain pressure and density for dark fluid as

$$(8\pi)\rho_D = a_1^2 e^{2tamh^{-1}\left(\frac{kt}{n}-1\right)}\left\{X_2 - \frac{\left(m^2+m+1\right)}{\alpha_3^2 e^{2\left\{\frac{1+mb-b}{n(m+2)}\right\}tamh^{-1}\left(\frac{kt}{n}-1\right)}}\right\} - (8\pi\alpha_2) e^{\frac{-6(1+w_m)}{n}tamh^{-1}\left(\frac{kt}{n}-1\right)}, \tag{39}$$

$$(8\pi)\bar{p}_D = a_1^2 e^{2tamh^{-1}\left(\frac{kt}{n}-1\right)}\left\{X_1 + \frac{\left(m^2+m+1\right)}{3\alpha_3^2 e^{2\left\{\frac{1+mb-b}{n(m+2)}\right\}tamh^{-1}\left(\frac{kt}{n}-1\right)}}\right\} - (8\pi\alpha_2 w_m) e^{\frac{-6(1+w_m)}{n}tamh^{-1}\left(\frac{kt}{n}-1\right)}. \tag{40}$$

The equation of state (EoS) parameter for dark fluid is

$$w_D = \frac{a_1^2 e^{2tamh^{-1}\left(\frac{kt}{n}-1\right)}\left\{X_1 + \frac{\left(m^2+m+1\right)}{3\alpha_3^2 e^{2\left\{\frac{1+mb-b}{n(m+2)}\right\}tamh^{-1}\left(\frac{kt}{n}-1\right)}}\right\} - (8\pi\alpha_2 w_m) e^{\frac{-6(1+w_m)}{n}tamh^{-1}\left(\frac{kt}{n}-1\right)}}{a_1^2 e^{2tamh^{-1}\left(\frac{kt}{n}-1\right)}\left\{X_2 + \frac{\left(m^2+m+1\right)}{\alpha_3^2 e^{2\left\{\frac{1+mb-b}{n(m+2)}\right\}tamh^{-1}\left(\frac{kt}{n}-1\right)}}\right\} - (8\pi\alpha_2) e^{\frac{-6(1+w_m)}{n}tamh^{-1}\left(\frac{kt}{n}-1\right)}}. \tag{41}$$

Case ii. For interacting two fluid models:

In this case we consider the interaction between dark viscous and barotropic fluids. For this purpose we can write the continuity equation as [57].

$$(\rho_m)_4 + (\rho_m + p_m)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = Q, \tag{42}$$

$$(\rho_D)_4 + (\rho_D + p_D)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = -Q. \tag{43}$$

where the quantity Q expresses the interaction between the fluids. Since we are interested in an energy transfer from the dark energy to dark matter, we consider $Q > 0$. $Q > 0$ ensures that the second law of thermodynamics is fulfilling [64]. Here we emphasize that the continuity equations (42) and (43) imply that the interaction term Q should be proportional to a quantity with units of inverse of time i.e. $Q \propto 1/t$. Therefore, a first and natural candidate can be the Hubble factor H multiplied with the energy density. Generally, Q could be taken in any of the following forms: (i) $Q \propto H\rho_m$ (ii) $Q \propto H\rho_D$, or (iii) $Q \propto H(\rho_m + \rho_D)$ [62,66]. It is worth to mention that because of our lack of knowledge of the nature of dark energy and dark matter, one can choose any of the above specific form of the interaction term Q. Moreover, as noted in ref [24], describing the interaction between the dark components of the universe through a microphysical model is not available today. In our study, we consider Q of the form Hassan Amirhashchi et al.[67], Amendola et al. [62] and Gou et al. [63] as

$$Q = 3H\sigma\rho_m . \tag{44}$$

where σ is coupling constant.

From equations (42) and (44), we get.

$$\rho_m = \alpha_4 e^{\frac{6(\sigma-1-w_m)}{n} tamh^{-1}\left(\frac{kt}{n}-1\right)} . \tag{45}$$

The density for dark fluid as

$$(8\pi)\rho_D = a_1^2 e^{2tamh^{-1}\left(\frac{kt}{n}-1\right)} \left\{ X_2 - \frac{(m^2+m+1)}{\alpha_3^2 e^{2\left\{\frac{1+mb-b}{n(m+2)}\right\} tamh^{-1}\left(\frac{kt}{n}-1\right)}} \right\} - (8\pi\alpha_4) e^{\frac{6(\sigma-1-w_m)}{n} tamh^{-1}\left(\frac{kt}{n}-1\right)} . \tag{46}$$

The effective pressure as

$$(8\pi)\bar{p}_D = a_1^2 e^{2tamh^{-1}\left(\frac{kt}{n}-1\right)} \left\{ X_1 + \frac{(m^2+m+1)}{3\alpha_3^2 e^{2\left\{\frac{1+mb-b}{n(m+2)}\right\} tamh^{-1}\left(\frac{kt}{n}-1\right)}} \right\} - (8\pi\alpha_4) e^{\frac{6(\sigma-1-w_m)}{n} tamh^{-1}\left(\frac{kt}{n}-1\right)} . \tag{47}$$

The effective EoS parameter as

$$w_D^{eff} = \frac{a_1^2 e^{2tamh^{-1}\left(\frac{kt}{n}-1\right)} \left\{ X_1 + \frac{(m^2+m+1)}{3\alpha_3^2 e^{2\left\{\frac{1+mb-b}{n(m+2)}\right\} tamh^{-1}\left(\frac{kt}{n}-1\right)}} \right\} - (8\pi\alpha_2 w_m) e^{\frac{-6(1+w_m)}{n} tamh^{-1}\left(\frac{kt}{n}-1\right)} - \frac{6\zeta}{(2nt-kt^2)}}{a_1^2 e^{2tamh^{-1}\left(\frac{kt}{n}-1\right)} \left\{ X_2 + \frac{(m^2+m+1)}{\alpha_3^2 e^{2\left\{\frac{1+mb-b}{n(m+2)}\right\} tamh^{-1}\left(\frac{kt}{n}-1\right)}} \right\} - (8\pi\alpha_2) e^{\frac{-6(1+w_m)}{n} tamh^{-1}\left(\frac{kt}{n}-1\right)}} . \tag{48}$$

where

$$X_1 = \frac{(n\beta_1+\beta_2+\beta_3)^2(2n-2kt-3)}{9(2nt-kt^2)^2} - \frac{3(n^2\beta_1^2+\beta_2^2+\beta_3^2)-(n\beta_1+\beta_2+\beta_3)^2}{6(2nt-kt^2)^2}$$

$$X_2 = \frac{(n\beta_1+\beta_2+\beta_3)^2}{9(2nt-kt^2)^2} - \frac{3(n^2\beta_1^2+\beta_2^2+\beta_3^2)-(n\beta_1+\beta_2+\beta_3)^2}{6(2nt-kt^2)^2} .$$

4. Some Physical and Geometrical Properties of the Model

The physical quantities that are important in cosmology are average scale factor, spatial volume, the expansion scalar and the hubble's parameter H is

1. Average scale factor: $a = a_1 e^{\frac{2}{n} tamh^{-1}\left(\frac{kt}{n} - 1\right)}$.
2. Spatial volume : $V = a_1^2 e^{\frac{4}{n} tamh^{-1}\left(\frac{kt}{n} - 1\right)}$.
3. Expansion scalar: $\theta = \frac{6}{\left(2nt - kt^2\right)}$.
4. Hubble's parameter : $H = \frac{2}{\left(2nt - kt^2\right)}$.

In our model the universe has finite lifetime. It starts with a big bang at $t = 0$ and ends at $t = m/k$, Both of the spatial volume of the fluid and the scale factor diverge in finite time, where as the expansion scalar and Hubble parameter tends to zero as $t \to \infty$.If we consider the ratio $\lim_{t\to\infty} \theta/\sigma = cons\tan t$ hence our models does not approaches isotropically.

5. Conclusion

We propose a special law for the deceleration parameter that is linear in time with a negative slope. The law we suggest gives the opportunity to generalize many of these models with better consistency with the cosmological observations. The law we have proposed can lead to many applications in cosmology; not only in the context of general relativity but also in generalized gravity theories such as the *f (R)* theory of gravity. In the derived models, *w* is obtained as time varying which is consistent with recent observations (Knop et al. [68]; Tegmark et al. [69]).

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