Article

Bianchi Type I Cosmological Models with Viscous Fluid and Decaying Vacuum Energy

J. P. Singh, P. S. Baghel[1] and A. Singh
Department of Mathematical Sciences, A. P. S. University, Rewa - 486003, India

Abstract

Cosmological models with viscous media and time-dependent cosmological term Λ have been studied in the background of Bianchi type I space-time. We examine the cosmological senario assuming expansion anisotropy σ/θ (the ratio of shear σ to the volume expansion θ) to be suitable function of spatial volume V. We obtain that models begin with decelerating expansion and enter into accelerating phase at late times. Initial anisotropy of the models dies out asymptotically.

Keywords: Bianchi I; Viscosity; Variable cosmological term; Anisotropy.

1 Introduction

The cosmological term Λ, introduced by Einstein in physics through general relativity has come and gone, only to come back again and again. Observations and theory still disagree on the value of this parameter by 120 orders of magnitude. The cosmological constant can be considered as new kind of world matter and it is natural to identify the cosmological constant with a background vacuum energy density. Huge difference between the observed and calculated values of vacuum energy density constitutes the current cosmological problem.

A number of attempts has been suggested to explain the cosmological constant problem. Some authors suggest that the cosmological constant takes on dynamical attributes . The idea of a decreasing vacuum energy with cosmic expansion has been involved by a number of authors [1-8] in order to understand the phenomenologically the incredibly small value of the cosmological constant. A time-dependent Λ may offer an exit window for the dynamical evolution of Λ from its QFT- calculated value to its presently observed value.

The observed physical phenomena such as large entropy per baryon and remarkable degree of isotropy of the cosmic background radiation, suggest dissipative effects in cosmology. It has been argued for a long time that the dissipation process in the early stages of cosmic expansion may well account for the high degree of isotropy, we observe today. Dissipative effects including coefficients of bulk and shear viscosity are supposed to play very important role in the origin and structure of the universe. Based on this approach Weinberg [9] derived general formulae for bulk and shear viscosities and used these formulae to evaluate the rate of cosmological entropy production. Padmanabhan and Chitre [10] studied that presence of bulk viscosity leads to inflationary like solutions in general relativity. Another peculiar characteristic of bulk viscosity is that it acts like a negative energy field in an expanding universe [11]. The nature of cosmological solutions for homogenous Bianchi type I model was studied by Bilinski and Khalatnikov [12] by taking into account dissipative process due to viscosity. They showed that viscosity cannot remove cosmological singularity but can cause a qualitatively new behavior of the solution near singularity. Bianchi type I solutions in the case of stiff matter with shear viscosity being the power function of energy density were obtained by Banerjee et al.[13] whereas models with bulk viscosity as a power function of energy density and stiff matter were investigated by Huang [14]. The effect of bulk viscosity on the cosmological evolution has been studied by number of workers namely Bali and Pradhan [15], Bali and Kumawat [16], Bali [17], Singh and Baghel [18,19], Peebles [20] and Singh et al.[21].

[1]Correspondence: E-mail: ps.baghel@yahoo.com

In the construction of a cosmological model, assumption of homogeneity and isotropy of the universe are motivated by the cosmological principle and mathematical tractability of the resulting FRW models. However, the observed universe is obviously neither homogeneous nor isotropic. So these symmetries can only be approximate. There are theoretical arguments [22,23] and recent experimental data regarding cosmic background radiation anisotropies which support the existence of an anisotropic phase that approaches an isotropic one [24]. These observations led us to consider general anisotropic cosmologies, whilst retaining the assumption of (large scale) spatial homogeneity. Spatially homogeneous and anisotropic cosmological models which provide a richer structure, both geometrically and physically than the FRW model play significant role in the description of early universe. Bianchi type I models being anisotropic generalization of flat FRW models are interesting to study. These models are favoured by the available evidences for low density universes. Romano and Pavon [25] have studied the evolution of Bianchi type I universe with viscous fluid. Mak and Harko [26] have studied the dynamics of a causal bulk viscous fluid cosmological model with constantly decelerating Bianchi type I space-time. Saha [27-29] in a series of papers discussed Bianchi type I universe with viscous fluid.

In this paper, we investigate Bianchi type I cosmological models with time varying cosmological term Λ for viscous fluid distribution. Exact solutions have been obtained by assuming expansion anisotropy $\frac{\sigma}{\theta}$ in the model to be a suitable function of spatial volume, σ and θ being the shear scalar and expansion scalar of the model. We obtain the cosmological term Λ to be decreasing function of time and the model isotropizes asymptotically. The presence of shear viscosity accelerates the isotropization.

2 Metric and Field Equations

We consider Bianchi type I space-time in form

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2 \tag{2.1}$$

with A, B and C being functions of cosmic time t only. We consider the matter content of source field to be viscous fluid described by the energy-momentum tensor

$$T^M_{ij} = (\rho + \bar{p})v_iv_j + \bar{p}g_{ij} - 2\eta\sigma_{ij} \tag{2.2}$$

where $\bar{p}$ is the effective pressure given by

$$\bar{p} = p - \zeta v^i{}_{;i} \tag{2.3}$$

satisfying linear equation of state

$$p = \omega\rho, \qquad 0 \leq \omega \leq 1. \tag{2.4}$$

Here ρ is the energy density, p, the isotropic pressure, η and ζ are coefficients of shear and bulk viscosity respectively, v^i is the four-velocity vector of the fluid satisfying $v_iv^i = -1$.
Expansion scalar θ, shear tensor σ_{ij} and shear scalar σ are defined by

$$\theta = v^i_{;i}, \tag{2.5}$$

$$\sigma_{ij} = \frac{1}{2}(v_{i;k}h^k_j + v_{j;k}h^k_i) - \frac{1}{3}\theta h_{ij}, \tag{2.6}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij}, \tag{2.7}$$

where $h_{ij} = g_{ij} + v_iv_j$ is the projection tensor and semicolon(;) stands for covariant derivative. We choose gravitational units such that $8\pi G = c = 1$. Since the vacuum has the symmetry of the background, its energy momentum tensor has the form $T^\Lambda_{ij} = \Lambda g_{ij}$, where Λ is a function of time in a homogeneous space. In comoving coordinate system ($v^i = \delta^i_4$), it corresponds to a perfect fluid with energy density $\rho_\Lambda = \Lambda$

and pressure $p_\Lambda = -\Lambda$. The Einstein's field equations with viscous matter and vacuum energy are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T^{total}_{ij}, \tag{2.8}$$

where $T^{total}_{ij} = T^{M}_{ij} + T^{\Lambda}_{ij} = (\rho_t + p_t)v_i v_j + p_t g_{ij}$ with the understanding that $\rho_t = \rho + \Lambda$ and $p_t = p - \Lambda$ are total density and total pressure respectively. The Bianchi identities require that T^{total}_{ij} has vanishing divergence leading to

$$\dot{\rho} + (\rho + \bar{p})\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \dot{\Lambda} = 4\eta\sigma^2 \tag{2.9}$$

The Einstein's field equations for the line element (2.1) reduce to

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - 2\eta\frac{\dot{A}}{A} = -p + (\zeta - \frac{2}{3}\eta)\theta + \Lambda, \tag{2.10}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} - 2\eta\frac{\dot{B}}{B} = -p + (\zeta - \frac{2}{3}\eta)\theta + \Lambda, \tag{2.11}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - 2\eta\frac{\dot{C}}{C} = -p + (\zeta - \frac{2}{3}\eta)\theta + \Lambda, \tag{2.12}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} = \rho + \Lambda, \tag{2.13}$$

where an overhead dot (.) denotes ordinary differentiation with respect to t.
We define average scale factor S as

$$S^3 = ABC \tag{2.14}$$

In analogy with FRW universe, we define a generalized Hubble parameter H and generalized deceleration parameter q as

$$H = \frac{\dot{S}}{S} = \frac{1}{3}(H_1 + H_2 + H_3), \tag{2.15}$$

$$q = -\frac{\dot{H}}{H^2} - 1 = 2 - 3\frac{V\ddot{V}}{\dot{V}^2}, \tag{2.16}$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are directional Hubble's factors along x, y and z directions respectively, V is the spatial volume given by

$$V = S^3. \tag{2.17}$$

Expansion scalar θ and components of shear tensor σ_i^j for the metric (2.1) are given by

$$\theta = 3H, \tag{2.18}$$

$$\sigma_1^1 = H_1 - H, \sigma_2^2 = H_2 - H, \sigma_3^3 = H_3 - H, \sigma_4^4 = 0. \tag{2.19}$$

Shear scalar σ assumes the form

$$\sigma^2 = \frac{1}{6}\left[\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right)^2 + \left(\frac{\dot{C}}{C} - \frac{\dot{A}}{A}\right)^2\right]. \tag{2.20}$$

We define the average expansion anisotropy of space by[30]

$$\bar{A} = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2. \tag{2.21}$$

With the help of equations (2.18) and (2.20), we obtain

$$\bar{A} = 6\frac{\sigma^2}{\theta^2}. \tag{2.22}$$

From equations (2.10)-(2.13), we have

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)\left(\frac{\dot{C}}{C} + 2\eta\right) = 0, \tag{2.23}$$

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right)\left(\frac{\dot{A}}{A} + 2\eta\right) = 0. \tag{2.24}$$

On integration, equations (2.23) and (2.24) lead to

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{k_1}{S^3}e^{-2\int \eta dt}, \tag{2.25}$$

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{k_2}{S^3}e^{-2\int \eta dt}, \tag{2.26}$$

where k_1 and k_2 are constants of integration.
From equations (2.15), (2.25) and (2.26), we get

$$\frac{\dot{A}}{A} = \frac{\dot{S}}{S} + \frac{k_2 + 2k_1}{3S^3}e^{-2\int \eta dt}, \tag{2.27}$$

$$\frac{\dot{B}}{B} = \frac{\dot{S}}{S} + \frac{k_2 - k_1}{3S^3}e^{-2\int \eta dt}, \tag{2.28}$$

$$\frac{\dot{C}}{C} = \frac{\dot{S}}{S} - \frac{2k_2 + k_1}{3S^3}e^{-2\int \eta dt}. \tag{2.29}$$

We propose the form of shear and bulk viscosity as considered by Saha[31], Mostafapoor and Grϕn[32] as:

$$\eta = 3\eta_0\frac{\dot{S}}{S} \tag{2.30}$$

and

$$\zeta = \zeta_0 + \zeta_1\frac{\dot{S}}{S} + \zeta_2\frac{\ddot{S}}{S}, \tag{2.31}$$

where η_0, ζ_0, ζ_1 and ζ_2 are constants.
Equations (2.27), (2.28) and (2.29) together with (2.30) reduce to

$$\frac{\dot{A}}{A} = \frac{\dot{S}}{S} + \frac{k_2 + 2k_1}{3S^{3+6\eta_0}}, \tag{2.32}$$

$$\frac{\dot{B}}{B} = \frac{\dot{S}}{S} + \frac{k_2 - k_1}{3S^{3+6\eta_0}}, \tag{2.33}$$

$$\frac{\dot{C}}{C} = \frac{\dot{S}}{S} - \frac{2k_2 + k_1}{3S^{3+6\eta_0}}. \tag{2.34}$$

We obtain

$$\sigma_1^1 = \frac{k_2 + 2k_1}{3S^{3+6\eta_0}}, \tag{2.35}$$

$$\sigma_2^2 = \frac{k_2 - k_1}{3S^{3+6\eta_0}}, \tag{2.36}$$

$$\sigma_3^3 = -\frac{2k_2 + k_1}{3S^{3+6\eta_0}}, \tag{2.37}$$

$$\sigma_4^4 = 0. \tag{2.38}$$

Therefore,

$$\sigma = \frac{k}{S^{3+6\eta_0}}, \tag{2.39}$$

where $3k^2 = k_1^2 + k_2^2 + k_1 k_2$.
Equations (2.10)- (2.13) and (2.9) can be written in terms of H, σ and q as

$$\bar{p} - \Lambda = (2q - 1)H^2 - \sigma^2, \tag{2.40}$$

$$\rho + \Lambda = 3H^2 - \sigma^2, \tag{2.41}$$

$$\dot{\rho} + 3(\rho + \bar{p})H + \dot{\Lambda} = 12\eta_0 H\sigma^2. \tag{2.42}$$

From equations (2.40) and (2.41), we get

$$\dot{H} = -3H^2 + \frac{1}{2}(\rho - p) + \frac{3}{2}\zeta H + \Lambda, \tag{2.43}$$

which is the Raychaudhuri equation for the present models. This equation shows that the shear viscosity does not affect the mean expansion rate but bulk viscosity and positive Λ increase the rate of expansion. From equations (2.40), (2.41) and (2.43), we get

$$\dot{H} + \frac{3}{2}(1+\omega)H^2 - \frac{3}{2}\zeta H - \frac{(1+\omega)}{2}\Lambda = \frac{3(\omega - 1)}{4}\bar{A}H^2 \tag{2.44}$$

Thus, the anisotropy does not influence the mean expansion rate of a flat unverse for Zel'dovich fluid ($\rho = p$). Also $\dot{\sigma} = -(3 + 6\eta_0)\sigma H$. Thus the energy density associated with anisotropy σ decays due to expansion by converting into photons and presence of shear viscosity accelerates this decay.

3 Solution and Discussion

Observations of the temperature of Cosmic Microwave Background Radiation(CMBR) suggest that the universe is expanding quite isotropically since the epoch in which it became definitively transparent to radiation. Recent history of our observed universe is well represented by spatially flat Friedmann-Robertson-Walker models which are both homogeneous and isotropic. The assumption of space isotropy seems to be very artificial at early stages of the cosmological expansion. Instability of homogeneous isotropic cosmological models against perturbations near the singularity indicates that FRW models in some sense are good global approximation of the present day universe but they are not likely to describe the early universe suitably. Therefore, it would be more appropriate to study cosmological models in which anisotropy of space existing at the beginning of universe are damped out in the course of evolution leading to isotropic phase. Average expansion anisotropy of space is given by

$$\bar{A} = 6\frac{\sigma^2}{\theta^2}. \tag{3.1}$$

A possible way to obtain a cosmological model described above, we consider $\frac{\sigma}{\theta}$ to be a function of S so that anisotropy $\frac{\sigma}{\theta}$ decreases and eventually goes over to isotropic phase with negligible $\frac{\sigma}{\theta}$ at late times. For the Bianchi type I space-time, we obtain from (2.18) and (2.39)

$$\frac{\sigma}{\theta} = \frac{k}{V^{2\eta_0}\dot{V}} = f(V). \tag{3.2}$$

We observe that when $\frac{\sigma}{\theta}$ is constant, equation (3.2) becomes

$$\frac{k}{V^{2\eta_0}\dot{V}} = b(constant), \tag{3.3}$$

which integrates to yield

$$S^3 = V = (k_0 t + k_1)^{\frac{1}{(2\eta_0+1)}}. \tag{3.4}$$

where $k_0 = \frac{(2\eta_0+1)k}{b}$ and k_1 is constant of integration.
For this assumption $\theta \propto t^{-1}$ and $q = 2$. In this case, the model does not approach isotropy and represents decelerating expansion throughout the evolution of the universe.
Integrating equation (3.2), we obtain

$$k(t + t_0) = \int V^{2\eta_0} f(V) dV, \tag{3.5}$$

where t_0 is a constant of integration.

The choice of the function $f(V)$ is quite arbitrary. To obtain a mathematically tractable and physically viable model of the universe consistent with observations, we consider two simple cases. We propose the function $f(V)$ as

$$f(V) = \begin{cases} \frac{1}{V^{2\eta_0}(a+V)^m} & ; \\ \frac{1}{V^{2\eta_0}(a^2+V^2)^{\frac{1}{2}}}, & \end{cases} \tag{3.6}$$

where $a(> 0)$ and $m(> 0)$ are constants. For these choices, we obtain $\frac{\sigma}{\theta} \neq 0$ in the beginning of universe $(i.e. S \approx 0)$ whereas at late times $(i.e. S \to \infty)$, $\frac{\sigma}{\theta} \to 0$.

3.1 Case 1

$$f(V) = \frac{1}{V^{2\eta_0}(a+V)^m}; \quad a(> 0), m(> 0). \tag{3.7}$$

For this choice of $f(V)$, the Hubble parameter H and deceleration parameter q come out to be

$$H = \frac{k(a+S^3)^m}{3S^3}, \tag{3.8}$$

$$q = 2 - \frac{3mS^3}{a+S^3}. \tag{3.9}$$

We assume that $S = 0$ for $t = 0$. We observe that, when $S \approx 0$, $q \approx 2$; $q = 0$ for $S^3 = \frac{2a}{3m-2}$ and for $S^3 > \frac{2a}{2m-2}$, $q < 0$. Therefore, the range of q is $2 - 3m \leq q \leq 2$. For $m > \frac{2}{3}$, q may attain negative value and for $m = 1$, $-1 \leq q \leq 2$. Thus universe begins with a decelerating expansion and the expansion changes from decelerating phase to an accelerating one. This cosmological scenario is in agreement with $SNeIa$ astronomical observations [33-37] and it presents a unified description of the evolution of the universe.

From (2.40), (2.41), (3.8) and (3.9), we obtain expressions for pressure p and matter density ρ as:

$$p = \frac{k^2(a+S^3)^{2m-1}}{3S^6}\{a + S^3(1-2m)\} - \frac{k^2}{S^{6+12\eta_0}} + 3\zeta H + \Lambda \tag{3.10}$$

i.e.

$$\begin{aligned} p =& \frac{k^2(a+S^3)^{2m-1}}{3S^6}\{a + S^3(1-2m)\} - \frac{k^2}{S^{6+12\eta_0}} \\ &+ \frac{\zeta_0 k(a+S^3)^m}{S^3} + \frac{\zeta_1 k^2(a+S^3)^{2m}}{3S^6} \\ &+ \frac{\zeta_2 k^3(a+S^3)^{2m-1}}{9S^9}[(3m-2)S^3 - 2a] + \Lambda \end{aligned} \tag{3.11}$$

and

$$\rho = \frac{k^2(a+S^3)^{2m}}{3S^6} - \frac{k^2}{S^{6+12\eta_0}} - \Lambda. \tag{3.12}$$

We observe that the model has singularity at $t = 0$ ($i.e. S = 0$). For $m < 1$ and large values of S, we get $p = -\rho = \Lambda$, i.e. the universe is dominated by vacuum energy at late times. For $m = 1$, equations (3.8) and (3.9) give $H^{-1} = \frac{2-q}{k}$. Thus H^{-1} increases as q decreases being maximum ($= \frac{3}{k}$) for $q_{min} = -1$. From equations (3.5) and (3.7), we get

$$k(t+t_0) = \int \frac{1}{(a+V)^m} dV. \tag{3.13}$$

We discuss the solution of (3.13) for $m = 1$ and $m \neq 1$.

3.1.1 Subcase-I

For $m = 1$, equation (3.13) integrates to give

$$V = e^{k(t+t_0)} - a. \tag{3.14}$$

Assuming $S = 0$ for $t = 0$, we get $t_0 = \frac{1}{k}\log(a)$.
Therefore

$$V = S^3 = \{a(e^{kt} - 1)\}. \tag{3.15}$$

Matter density ρ and cosmological term Λ for the model are given by

$$\begin{aligned}\rho =& \frac{1}{\omega+1}\left[\frac{2k^2e^{-kt}}{3(1-e^{-kt})^2} - \frac{2k^2}{\{a(e^{kt}-1)\}^{4\eta_0+2}}\right] \\ &+ \frac{1}{\omega+1}\left[\frac{\zeta_0 k}{(1-e^{-kt})} + \frac{\zeta_1 k^2}{3(1-e^{-kt})^2} + \frac{\zeta_2 k^3(1-3e^{-kt})}{9(1-e^{-kt})^3}\right],\end{aligned} \tag{3.16}$$

$$\begin{aligned}\Lambda =& \frac{(\omega - 2e^{-kt} + 1)k^2}{3(\omega+1)(1-e^{-kt})^2} + \frac{(1-\omega)k^2}{(\omega+1)\{a(e^{kt}-1)\}^{4\eta_0+2}} \\ &- \frac{1}{\omega+1}\left[\frac{\zeta_0 k}{(1-e^{-kt})} + \frac{\zeta_1 k^2}{3(1-e^{-kt})^2} + \frac{\zeta_2 k^3(1-3e^{-kt})}{9(1-e^{-kt})^3}\right].\end{aligned} \tag{3.17}$$

Expansion scaler θ, shear scalar σ, deceleration parameter q, coefficients of shear viscosity η and bulk viscosity ζ for the model are obtained as

$$\theta = \frac{k}{1-e^{-kt}}, \tag{3.18}$$

$$\sigma = \frac{k}{\{a(e^{kt}-1)\}^{2\eta_0+1}}, \tag{3.19}$$

$$q = 3e^{-kt} - 1, \tag{3.20}$$

$$\eta = \frac{\eta_0 k}{1-e^{-kt}}, \tag{3.21}$$

$$\zeta = \zeta_0 + \frac{\zeta_1 k}{3(1-e^{-kt})} + \frac{\zeta_2 k^2(1-3e^{-kt})}{9(1-e^{-kt})^2}. \tag{3.22}$$

We observe that the model has singularity at $t = 0$. The model starts with big-bang from its singular state at $t = 0$. Expansion in the model decreases and spatial volume of the model increases as time increases. For large t (i.e. $t \to \infty$), expansion in the model becomes finite. The physical quantities ρ, σ, Λ, η and ζ are infinite and $q = 2$ for $t = 0$. In limit of large times ($i.e. t \to \infty$), $\rho \to \frac{1}{\omega+1}\left[\zeta_0 k + \frac{\zeta_1 k^2}{3} + \frac{\zeta_2 k^3}{9}\right]$, $\sigma \to 0$,

$\Lambda \to \frac{k^2}{3} - \frac{1}{\omega+1}\left[\zeta_0 k + \frac{\zeta_1 k^2}{3} + \frac{\zeta_2 k^3}{9}\right]$, $\eta \to \eta_0 k$, $\zeta \to \zeta_0 + \frac{\zeta_1 k}{3} + \frac{\zeta_2 k^2}{9}$ and $q \to -1$. From (3.20) we conclude that the time $t_q = \frac{1}{k}\ln 3$, when the expansion changes from the decelerating phase to an accelerating one. We observe that the presence of bulk viscosity does not allow the matter density to become zero in infinitely far future. Also, at late times cosmological term Λ tends to a genuine cosmological constant. The nature of decaying vacuum energy density $\Lambda(t)$ in our derived model is supported by recent cosmological observations. Coefficients of shear and bulk viscosity tend to genuine constants for large values of t. We also find that the presence of bulk viscosity is to increase the value of matter density ρ and to decrease the value of vacuum energy density Λ. In the absence of bulk viscosity, the model results in a de Sitter universe for large values of t with $H = \sqrt{\frac{\Lambda}{3}} = \frac{k}{3}$ and matter density becomes negligible asymptotically. Presence of bulk viscosity prevents the model to reduce to de Sitter universe asymptotically.
For the model expansion anisotropy

$$\frac{\sigma}{\theta} = \frac{1-e^{-kt}}{\{a(e^{kt}-1)\}^{2\eta_0+1}} \tag{3.23}$$

We observe that in the absence of shear viscosity (i.e.$\eta_0 = 0$), $\frac{\sigma}{\theta}$ is non-zero finite at $t = 0$ whereas in the presence of shear viscosity, $\frac{\sigma}{\theta}$ becomes infinite at $t = 0$. For large values of t, $\frac{\sigma}{\theta}$ tends to zero. Therefore, the model approaches isotropy at late times. The shear viscosity is found to be responsible for faster removal of initial anisotrpies in the universe. Variation of ρ, Λ, σ/θ and q with cosmic time t are shown in Figs. [1-4].

3.1.2 Subcase-II

When $m \neq 1$, integrating equation (3.13), we obtain

$$V = \{a^{1-m} + (1-m)kt\}^{\frac{1}{1-m}} - a \tag{3.24}$$

Therefore

$$S = [\{a^{1-m} + (1-m)kt\}^{\frac{1}{1-m}} - a]^{\frac{1}{3}} \tag{3.25}$$

Thus scale factor S is increasing function of time for $0 < m < 1$. The expressions for matter density ρ and cosmological term Λ for the model are obtained as

$$\begin{aligned}
\rho = & \frac{2k^2\left[a^{1-m} + (1-m)kt\right]^{\frac{2m}{1-m}}}{3(\omega+1)\left[\{a^{1-m} + (1-m)kt\}^{\frac{1}{1-m}} - a\right]^2}. \\
& \left[1 - m\{1 - a(a^{1-m} + (1-m)kt)^{\frac{-1}{1-m}}\}\right] \\
& - \frac{2k^2}{(\omega+1)\left[\{a^{1-m} + (1-m)kt\}^{\frac{1}{1-m}} - a\right]^{4\eta_0+2}} \\
& + \frac{\zeta_0 k\{a^{1-m} + (1-m)kt\}^{\frac{m}{(1-m)}}}{(\omega+1)\left[\{a^{1-m} + (1-m)kt\}^{\frac{1}{(1-m)}} - a\right]} \\
& + \frac{\zeta_1 k^2\{a^{1-m} + (1-m)kt\}^{\frac{2m}{1-m}}}{3(\omega+1)\left[\{a^{1-m} + (1-m)kt\}^{\frac{1}{1-m}} - a\right]^2} \\
& + \frac{\zeta_2 k^3\{a^{1-m} + (1-m)kt\}^{\frac{3m}{1-m}}}{9(\omega+1)\left[\{a^{1-m} + (1-m)kt\}^{\frac{1}{(1-m)}} - a\right]^3}. \\
& \left[(3\omega-2) - 3ma\{a^{1-m} + (1-m)kt\}^{\frac{-1}{1-m}}\right],
\end{aligned} \tag{3.26}$$

$$\begin{aligned}\Lambda =&\frac{k^2\{a^{1-m}+(1-m)kt\}^{\frac{2m}{1-m}}}{3(\omega+1)\left[\{a^{1-m}+(1-m)kt\}^{\frac{1}{1-m}}-a\right]^2}.\\ &\left[\omega-1+2m\{1-a(a^{1-m}+(1-m)kt)^{\frac{-1}{1-m}}\}\right]\\ &+\frac{(1-\omega)k^2}{(1+\omega)\left[\{a^{1-m}+(1-m)kt\}^{\frac{1}{1-m}}-a\right]^{4\eta_0+2}}\\ &-\frac{\zeta_0 k\{a^{1-m}+(1-m)kt\}^{\frac{m}{1-m}}}{(\omega+1)\left[\{a^{1-m}+(1-m)kt\}^{\frac{1}{1-m}}-a\right]}\\ &-\frac{\zeta_1 k^2\{a^{1-m}+(1-m)kt\}^{\frac{2m}{1-m}}}{3(\omega+1)\left[\{a^{1-m}+(1-m)kt\}^{\frac{1}{1-m}}-a\right]^2}\\ &-\frac{\zeta_2 k^3\{a^{1-m}+(1-m)kt\}^{\frac{3m}{1-m}}}{9(\omega+1)\left[\{a^{1-m}+(1-m)kt\}^{\frac{1}{1-m}}-a\right]^3}.\\ &\left[(3\omega-2)-3ma\{a^{1-m}+(1-m)kt\}^{\frac{-1}{1-m}}\right].\end{aligned} \tag{3.27}$$

Expansion scalar θ, shear scalar σ, deceleration parameter q, coefficients of shear viscosity η and bulk viscosity ζ of the model are given by

$$\theta=\frac{k\{a^{1-m}+(1-m)kt\}^{\frac{m}{1-m}}}{\left[\{a^{1-m}+(1-m)kt\}^{\frac{1}{1-m}}-a\right]}, \tag{3.28}$$

$$\sigma=\frac{k}{\left[\{a^{1-m}+(1-m)kt\}^{\frac{1}{1-m}}-a\right]^{2\eta_0+1}}, \tag{3.29}$$

$$q=2-3m\left[1-a\{a^{1-m}+(1-m)kt\}^{\frac{-1}{1-m}}\right], \tag{3.30}$$

$$\eta=\frac{\eta_0 k\{a^{1-m}+(1-m)kt\}^{\frac{m}{1-m}}}{\left[\{a^{1-m}+(1-m)kt\}^{\frac{1}{1-m}}-a\right]}, \tag{3.31}$$

$$\begin{aligned}\zeta =&\zeta_0+\frac{k\zeta_1\{a^{1-m}+(1-m)kt\}^{\frac{m}{1-m}}}{3\left[\{a^{1-m}+(1-m)kt\}^{\frac{1}{1-m}}-a\right]}\\ &+\frac{\zeta_2 k^2\{a^{1-m}+(1-m)kt\}^{\frac{2m}{1-m}}}{\left[\{a^{1-m}+(1-m)kt\}^{\frac{1}{(1-m)}}-a\right]^2}.\\ &\left[(3m-2)-3am\{a^{1-m}+(1-m)kt\}^{\frac{-1}{1-m}}\right].\end{aligned} \tag{3.32}$$

We observe that the model starts with big-bang from its singular state at $t=0$ and the expansion in the model decreases with increase of time whereas spatial volume of the model increases as time increases. At $t=0$, ρ, Λ, θ, σ, η and ζ all diverge for $0<m<1$. These quantities become negligible for large values of t but $\zeta\to\zeta_0$ for $t\to\infty$. At $t=0$, $q=2$ and for $t\to\infty$, $q=2-3m$. Thus model represents initially decelerating and late time accelerating expansion for $\frac{2}{3}<m<1$.
For the model

$$\frac{\sigma}{\theta}=\frac{1}{\left[\{a^{1-m}+(1-m)kt\}^{\frac{m}{1-m}}\right]\left[\{a^{1-m}+(1-m)kt\}^{\frac{1}{1-m}}-a\right]^{2\eta_0}} \tag{3.33}$$

We notice that $\frac{\sigma}{\theta}$ is a non-zero finite quantity at $t = 0$ in the absence of shear viscosity and presence of shear viscosity makes $\frac{\sigma}{\theta}$ infinite at $t = 0$. For large t, $\frac{\sigma}{\theta} \to 0$ implying that the model approaches isotropy. We observe that the presence of shear viscosity accelerates the process of isotropization. Variation of ρ, Λ, σ/θ and q with cosmic time t are shown in Figs. [5-8].

3.2 Case 2

$$f(V) = \frac{1}{V^{2\eta_0}(a^2 + V^2)^{\frac{1}{2}}}. \tag{3.34}$$

For this assumption of $f(V)$, the Hubble parameter H and deceleration parameter q turn out to be

$$H = \frac{k(a^2 + S^6)^{\frac{1}{2}}}{3S^3}, \tag{3.35}$$

$$q = 2 - \frac{3S^6}{(a^2 + S^6)}. \tag{3.36}$$

We observe that at $S = 0$, $q = 2$ and for $S \geq (2a^2)^{\frac{1}{6}}$, $q \leq 0$. Therefore model starts with decelerating expansion at $S = 0$. For $S = (2a^2)^{\frac{1}{6}}$, expansion in the model changes from decelerating phase to an accelerating one.
From equations (2.40), (2.41), (3.35) and (3.36), pressure p and matter density ρ are given by

$$p = \frac{k^2(a^2 - S^6)}{3S^6} - \frac{k^2}{S^{6+12\eta_0}} + 3\zeta H + \Lambda \tag{3.37}$$

i.e.

$$\begin{aligned} p =& \frac{k^2(a^2 - S^6)}{3S^6} - \frac{k^2}{S^{6+12\eta_0}} \\ &+ \frac{\zeta_0 k(a^2 + S^6)^{\frac{1}{2}}}{S^3} + \frac{\zeta_1 k^2(a^2 + S^6)}{3S^6} \\ &+ \frac{\zeta_2 k^3(a^2 + S^6)^{\frac{1}{2}}(S^8 + S^2a^2 - 3a^2)}{9S^{11}} + \Lambda, \end{aligned} \tag{3.38}$$

$$\rho = \frac{k^2(a^2 + S^6)}{3S^6} - \frac{k^2}{S^{6+12\eta_0}} - \Lambda. \tag{3.39}$$

We observe that model has singularity at $S = 0$. For $S \to \infty$, $p = -\rho + k\zeta_0 + \frac{k^2}{3}\zeta_1 + \frac{k^3}{9}\zeta_2$. Equation (3.5) together with (3.34) gives

$$k(t + t_0) = \int \frac{dV}{(a^2 + V^2)^{\frac{1}{2}}}. \tag{3.40}$$

Integrating (3.40), we get

$$V = a \sinh kt \tag{3.41}$$

Therefore

$$S = (a \sinh kt)^{\frac{1}{3}} \tag{3.42}$$

The expressions for matter density ρ and cosmological term Λ are given by

$$\begin{aligned} \rho = \frac{1}{\omega + 1}\left[\frac{6}{\cosh^2 kt} - \frac{2k^2}{(a \sinh kt)^{4\eta_0 + 2}} - \right] + \frac{k}{\omega + 1}\cdot \\ \left[\zeta_0 \coth kt + \frac{\zeta_1 k \coth^2 kt}{3} + \frac{\zeta_2 k^2 \coth kt \csc h^2 kt(\cosh^2 kt - 3)}{9}\right], \end{aligned} \tag{3.43}$$

$$\begin{aligned}\Lambda = &\left(\omega+1-\frac{2}{\cosh^2 kt}\right)\frac{k^2\coth^2 kt}{3(\omega+1)}\\ &+\frac{(1-\omega)k^2}{(1+\omega)(a\sinh kt)^{4\eta_0+2}}-\frac{k}{\omega+1}\cdot\\ &\left[\zeta_0\coth kt+\frac{\zeta_1 k}{3}\coth^2 kt+\frac{\zeta_2 k^2}{9}\coth kt\csc h^2 kt(\cosh^2 kt-3)\right].\end{aligned} \tag{3.44}$$

Expansion scalar θ, shear scalar σ, deceleration parameter q, coefficients of shear viscosity and bulk viscosity of the model are given by

$$\theta = k\coth kt, \tag{3.45}$$

$$\sigma = \frac{k}{(a\sinh kt)^{2\eta_0+1}}, \tag{3.46}$$

$$q = \frac{3}{\cosh^2 kt}-1, \tag{3.47}$$

$$\eta = \eta_0 k\coth kt, \tag{3.48}$$

$$\zeta = \zeta_0+\frac{\zeta_1 k}{3}\coth kt+\frac{\zeta_2 k^2}{9}\csc h^2 kt(\cosh^2 kt-3). \tag{3.49}$$

We observe that model has singularity at $t=0$. The model begins with big-bang start from its singular state. Spatial volume of model increases with increase in time. At $t=0$, ρ, Λ, θ, σ, η and ζ all diverge. In the limit of large times ($i.e. t\to\infty$), $\rho\to\frac{1}{\omega+1}[\zeta_0 k+\frac{\zeta_1 k^2}{3}+\frac{\zeta_2 k^3}{9}]$, $\Lambda=\frac{k^2}{3}-\frac{1}{\omega+1}[\zeta_0+\frac{\zeta_1 k}{3}+\frac{\zeta_2 k^3}{9}]$, $\theta\to k$, $\sigma\to 0$, $\eta=\eta_0 k$ and $\zeta\to\zeta_0+\frac{\zeta_1 k}{3}+\frac{\zeta_2 k^2}{9}$. At $t=0$, $q=2$ and $q=-1$ for $t=\infty$. We observe that the presence of bulk viscosity increases the value of ρ and decreases the value of Λ. In the absence of bulk viscosity, we recover de Sitter universe at late times. Thus the presence of bulk viscosity prevents the model to tend to a de Sitter universe asymptotically.
For the model

$$\frac{\sigma}{\theta}=\frac{1}{a\cosh t(a\sinh t)^{2\eta_0}}. \tag{3.50}$$

We observe that in the absence of shear viscosity i.e. $\eta_0=0$, $\frac{\sigma}{\theta}=\frac{1}{a}$ at $t=0$ whereas in presence of shear viscosity, $\frac{\sigma}{\theta}$ becomes infinite at $t=0$. For $t\to\infty$, $\frac{\sigma}{\theta}\to 0$. Therefore, the model approaches isotropy at late times. We also observe that the presence of shear viscosity accelerates the process of isotropization. The evolution of cosmological parameters against cosmic time t are shown in the Figs. [9-12].

4 Conclusion

In this article, Bianchi type I cosmological models are investigated in the presence of viscous fluid source and time dependent cosmological term Λ. Exact solutions of Einstein field equations have been obtained by assuming expansion anisotropy $\frac{\sigma}{\theta}$ in the models to be a suitable function of spatial volume V. The resulting models begin with a decelerating expansion and the expansion changes from decelerating phase to an accelerating one. This cosmological scenario is in agreement with $SNeIa$ astronomical observations and it presents a unified description of the evolution of the universe. We have found that cosmological term Λ being very large at initial times relaxes to a genuine cosmological constant at late times which is in agreement with observations. The presence of bulk viscosity is to increase the value of matter density ρ and to decrease the value of vacuum energy density Λ. Presence of bulk viscosity prevents the model to reduce to a de Sitter universe asymptotically except in one subcase $m\neq 1$. The models being anisotropic initially reduce to isotropic universe for large values of t. The shear viscosity is found to be responsible for the faster removal of initial anisotropies in the universe. We observe that the proposed variation law for expansion anisotropy $\frac{\sigma}{\theta}$ provides an alternative approach to obtain exact solutions of Einstein's field equations. It may remove the discrepancies of the models obtained by assuming $\frac{\sigma}{\theta}$ to be constant.

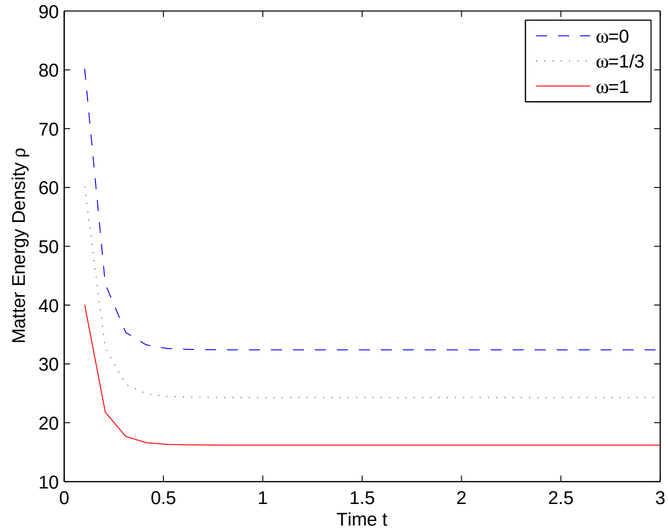

Figure 1: Variation of matter energy density ρ with cosmic time t in subcase m=1.

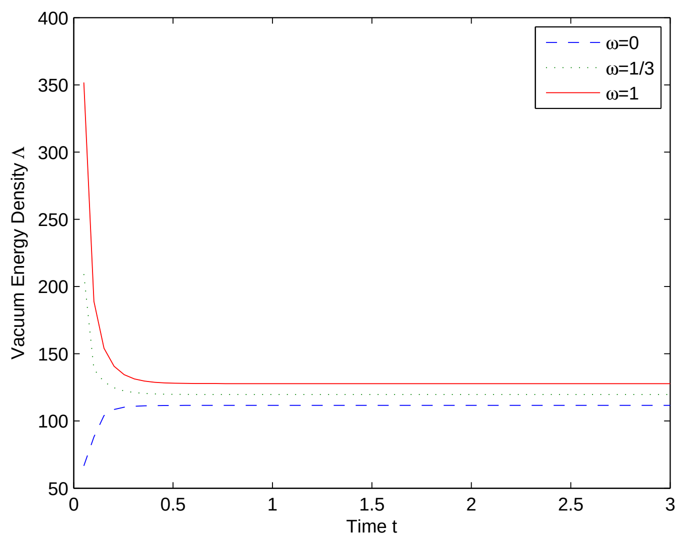

Figure 2: Variation of vacuum energy density Λ with cosmic time t in subcase m=1.

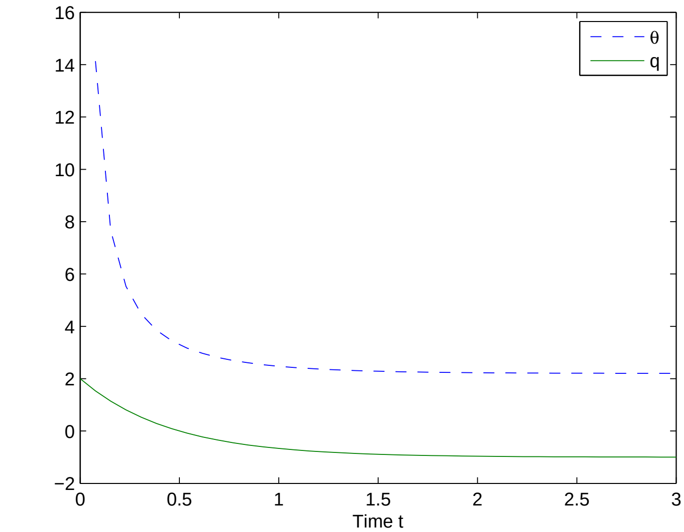

Figure 3: Variation of expansion scalar θ and deceleration parameter q with cosmic time t in subcase m=1.

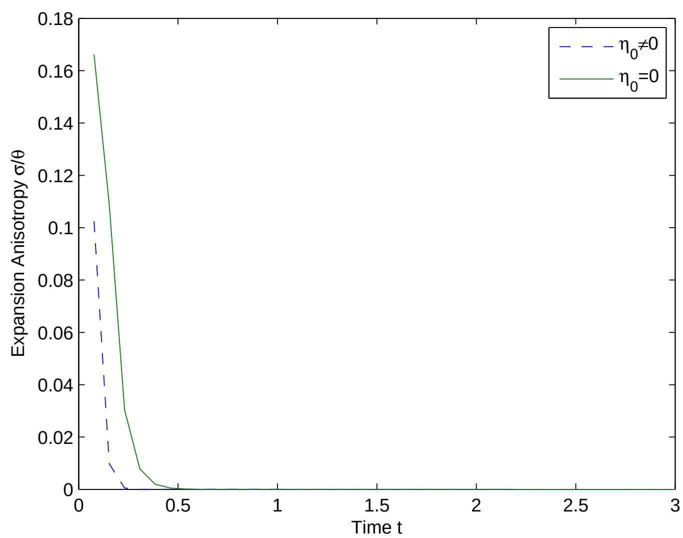

Figure 4: Variation of expansion anisotropy $\frac{\sigma}{\theta}$ with cosmic time t in subcase m=1.

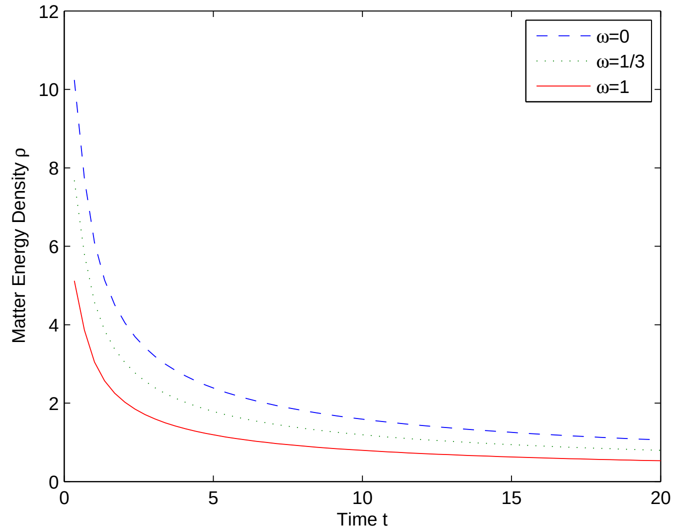

Figure 5: Variation of matter energy density ρ with cosmic time t in subcase $m \neq 1$.

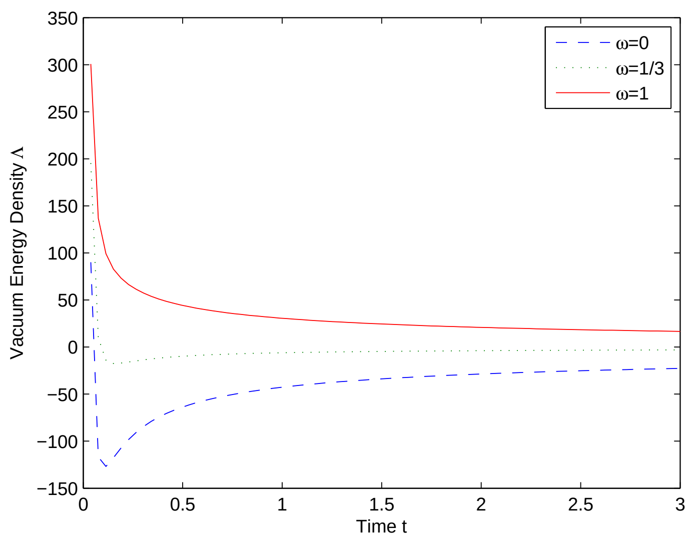

Figure 6: Variation of vacuum energy density Λ with cosmic time t in subcase $m \neq 1$.

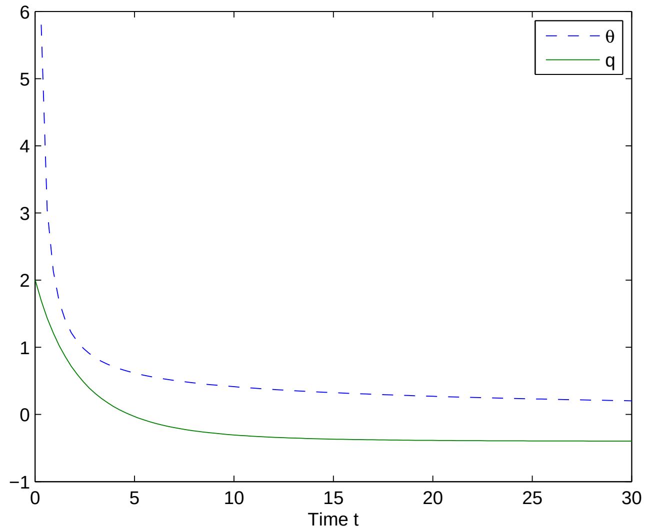

Figure 7: Variation of expansion scalar θ and deceleration parameter q with cosmic time t in subcase $m \neq 1$.

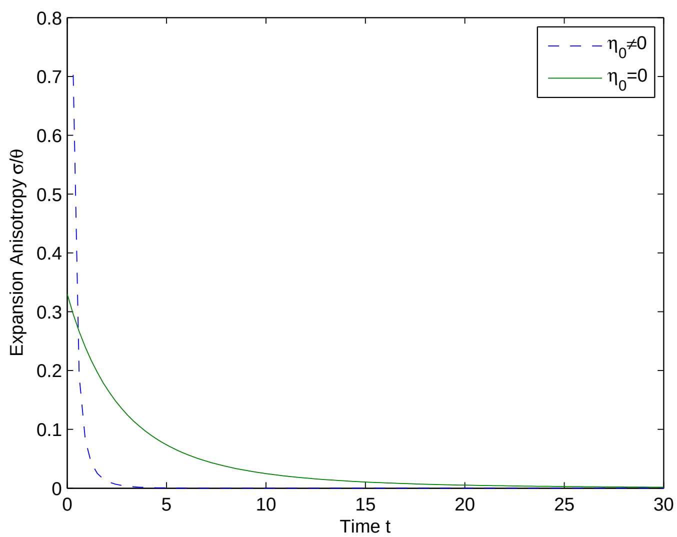

Figure 8: Variation of expansion anisotropy $\frac{\sigma}{\theta}$ with cosmic time t in subcase $m \neq 1$.

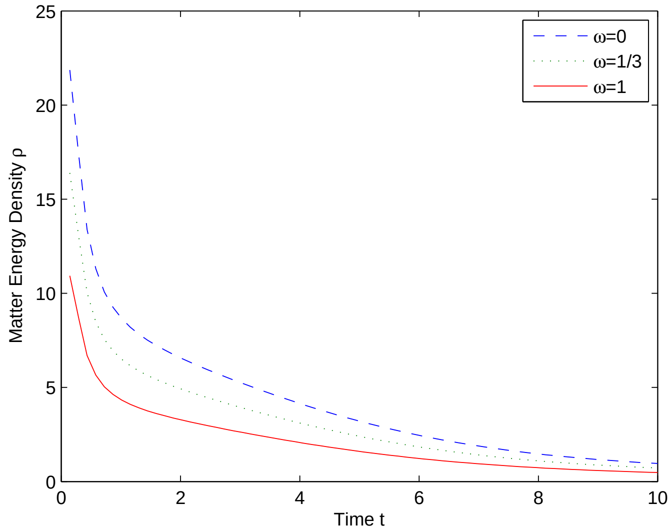

Figure 9: Variation of matter energy density ρ with cosmic time t in case 2.

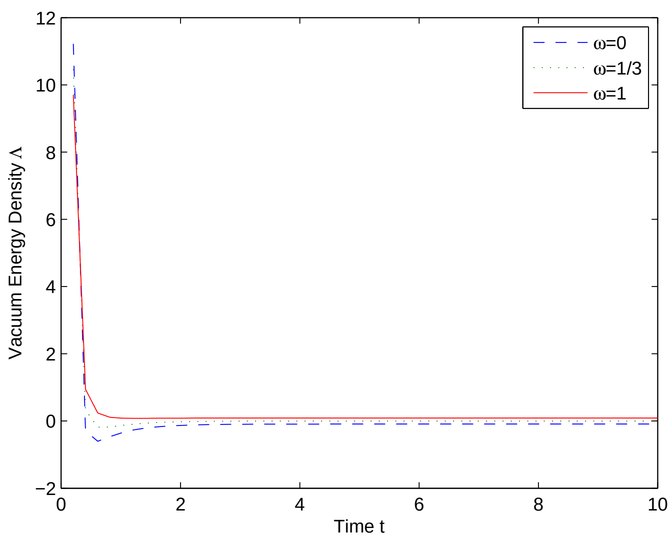

Figure 10: Variation of vacuum energy density Λ with cosmic time t in case 2.

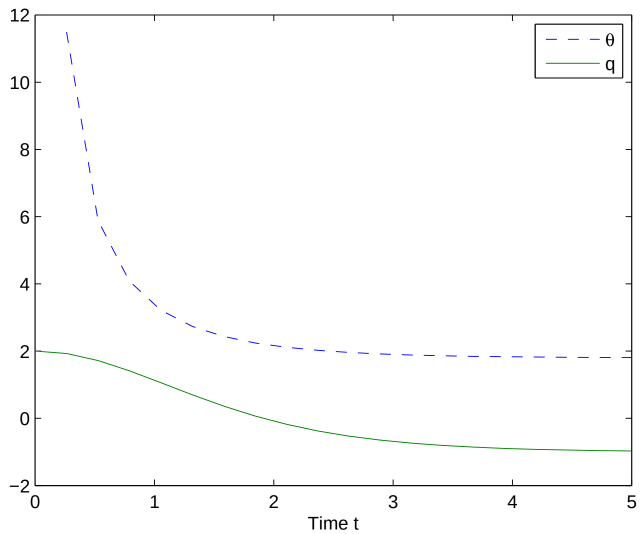

Figure 11: Variation of expansion scalar θ and deceleration parameter q with cosmic time t in case 2.

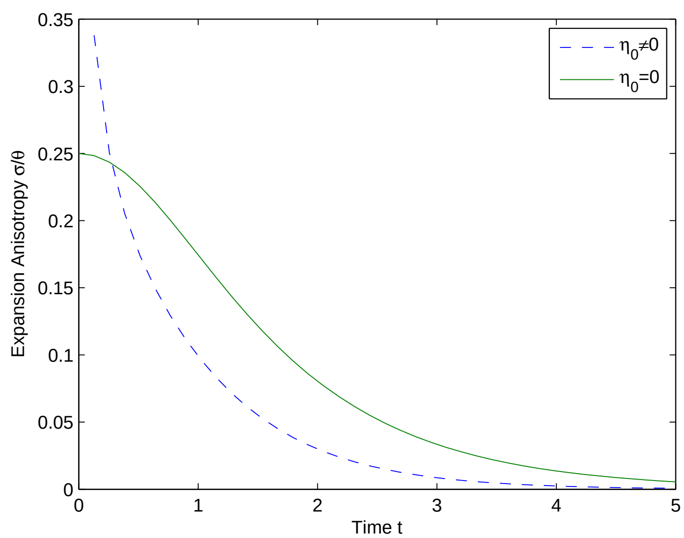

Figure 12: Variation of expansion anisotropy $\frac{\sigma}{\theta}$ with cosmic time t in case 2.

References

[1] J. C. Carvalho, J. A. S. Lima and I. Waga: Phy. Rev. D46, 2406 (1992)

[2] R. G. Vishwakarma: Class. Quant. Grav. 19, 4747 (2002)

[3] S. B. Duttachoudhary and A. Sil: Astrophys. Space Sci. 301, 61 (2006)

[4] A. I. Arbab: Gen. Relativ. Gravit. 29, 61 (1997)

[5] Abdussattar and R. G. Vishwakarma: Class. Quantum Grav. 14, 945 (1997)

[6] R. Bali and S. Tinker: Chin. Phys. Lett. 26, 029802 (2009)

[7] R. Bali and J. P. Singh: Int. J. Theor. Phys. 47, 3288 (2008)

[8] J. P. Singh and R. K. Tiwari: Pramana, 70, 565 (2008)

[9] S. Weinberg: Astophys. J. 168, 175 (1971)

[10] T. Padmanabhan and S. M. Chitre: Phys. Lett. A120, 433 (1987)

[11] V. B. Johari and R. Sudarshan: World. Scientific Singapore (1988)

[12] V. A. Belinski and I. M. Khalatnikov: JETP, 69, 401 (1975)

[13] A. Banerjee, S. B. Duttachoudhary and A. K. Sanyal: J. Math. Phys. 26, 3010 (1985)

[14] W. Huang: J. Math. Phys. 31, 1456 (1990)

[15] R. Bali and A. Pradhan: Chin. Phys. Lett. 24, 585 (2007)

[16] R. Bali and P. Kumawat: Phys. Lett. B665, 332 (2008)

[17] R. Bali: Int. J. Theor. Phys. 48, 476 (2009)

[18] J. P. Singh and P. S. Baghel: Int. J. Theor. Phys. 49, 2734 (2010)

[19] J. P. Singh and P. S. Baghel: Electron. J. Theor. Phys. 6, 85 (2009)

[20] P. J. E. Peebles: Rev. Mod. Phys. 75, 559 (2003)

[21] C. P. Singh, S. Kumar and A. Pradhan: Class. Quant. Grav. 24, 455 (2007)

[22] L. P. Chimento: Phys. Rev. D69, 123517 (2004)

[23] C. W. Misner: Astrophys. J. 151, 431 (1968)

[24] L. Land and J. Magueijo: Phys. Rev. Lett. 95, 071301 (2005)

[25] V. Romano and D. Pavon: Phys. Rev. D 47, 1396 (1993)

[26] M. K. Mak and T. Harko: Int. J. Mod. Phys. D11, 893 (2002)

[27] B. Saha: Mod. Phys. Lett. A20, 2127 (2005)

[28] B. Saha: Phys. Rev. D 219, 1685 (2006)

[29] B. Saha: J. Phys. A. Math. Theor. 40, 14011 (2007)

[30] N. Caderni and R. Fabbri: Nuovo Cimento 44B, 228 (1978)

[31] B. Saha: Mod. Phys. Lett. A20, 2117 (2005)

[32] N. Mostafapoor and O.Grøn, Astrophys. Space Sci. 330, 151 (2011)

[33] S. Perlmutter et al.: Astrophys. J. 517, 565 (1999)

[34] A. G. Riess et al.: Astrophys. J. 116, 1009 (1998)

[35] D. N. Spergel et al.: Astrophys. J. Suppl. 148, 175 (2003)

[36] D. N. Spergel et al.: Astrophys. J. Suppl. 170, 377 (2007)

[37] R. A. Knop et al.: Astron. J. 598, 102 (1003)