Article

Bianchi Type-I Cosmological Model with Varying G & Λ Term in Lyra Geometry & Self-Creation Theory of Gravitation

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Abstract

Bianchi type-I cosmological model with varying G and Λ-term in Lyra geometry in the framework of self-creation theory of gravitation is investigated, with dust as a source of the gravitational field. Various physical and geometrical aspects of the model are also discussed.

Keywords: Bianchi Type-I, cosmological model, varying G. Λ term, Lyra geometry, self-creation theory.

Introduction

Barber [11] proposed two self-creation cosmologies by modifying the scalar tensor theories of gravity which were first proposed by Jordan and then by Brans and Dicke and Dicke as an alternative to Einstein's general theory of gravitation. Brans and Dicke have formulated scalar tensor theory of gravitation which develops Mach's principle in a relativistic framework, assuming that inertial masses of fundamental particles are not constant, but are dependent upon the particles' interaction with some cosmic scalar field coupled to the large scale distribution of matter in motion.

Out of two theories given by Barber, the first self-creation theory is a modified Brans and Dicke theory that is rejected on the grounds of violation of the equivalence principle. The second one is an adoption of general relativity to include continuous creation and is within limits of the observations.

A number of scholars have investigated the Barber's second theory in different contexts. Various aspects of the self-creation theories have been investigated by Pimental [17], Soleng [13] Maharaj and Beesham [19] have obtained various properties of self-creation theory. Pradhan and Vishwakarma [2] have studied LRS Bianchi type-I cosmological models in self-creation theory. Pradhan and Pandey [4] have obtained a class of LRS Bianchi type-I models in Barber's self-

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creation theory in the presence of bulk viscous fluid for constant deceleration parameter. Recently Katore et al.[7] have obtained plane symmetric cosmological models with negative constant deceleration parameter in self-creation theory. Singh et al. [15] have obtained exact solutions of the field equations for orthogonal Bianchi type-I space time self-creation theory of gravitation. Very recently Jain et al. [20] have investigated Bianchi type-I cosmological model with varying Λ term in self-creation theory of gravitation.

Lyra [12] suggested a modification of Riemannian geometry by introducing a gauge function Subsequent investigations were done by Sen [6], Halford [22] , Bhamra [16] with the framework of Lyra geometry. The close connection between these models and general relativistic models has often been noted. Pradhan et al. [1] described isotropic homogeneous universe with a bulk viscous fluid in Lyra geometry. Reddy [8] studied plane symmetric cosmic strings. Reddy and Subba Rao [9] studied axially symmetric cosmic strings and domain walls in this theory. Rahman and Mal [10] studied local cosmic strings in Lyra geometry.

Recently Rao et al. [21] discussed Bianchi type-V cosmological model with perfect fluid using negative constant deceleration parameter in a scalar tensor theory based on Lyra manifold. Bali and Chandnani [18] described Bianchi type-I cosmological model for perfect fluid distribution in Lyra geometry. Singh [14] discussed exact solutions of some cosmological models in Lyra geometry. Very recently Pradhan and Singh [3] explored anisotropic Bianchi type-I string cosmological models in normal gauge for Lyra manifold with constant deceleration parameter.

Metric and Field Equations

We consider the Bianchi type-I metric in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 dy^2 + C^2 dz^2, \tag{1}$$

where A, B, C are metric potentials which are functions of time alone.

The energy momentum tensor has the form

$$T_i^j = (\varepsilon + p) v_i v^j + p g_i^j . \tag{2}$$

In equation (2) ε is energy density, p is the pressure and v^i is the four velocity vector satisfying the relation

$$g_{ij} v^i v^j = -1 . \tag{3}$$

Einstein's field equations in normal gauge for Lyra Manifold in Barber's theory of self-creation are given by

$$R_i^j - \frac{1}{2}Rg_i^j + \frac{3}{2}\alpha_i\alpha^j - \frac{3}{4}\alpha_k\alpha^k g_i^j = -8\pi\phi^{-1}T_i^j - \Lambda g_i^j \ , \tag{4}$$

$$\text{and} \quad \phi_{;k}^{k} = \frac{8\pi}{3}\eta T \ , \tag{5}$$

where $v_i = (0,0,0,-1)$, $v^i v_i = -1$, $\alpha_i = (0,0,0,\beta(t))$, $v_4 = -1$ and β is the gauge function.

Here $\phi_{;k}^{k}$ is invariant d'Alembertian and the contracted tensor T is trace of energy momentum tensor describing all non-gravitational and non-scalar field matter and energy. Here η is a coupling constant to be determined from experiments. The measurements of the deflection of light restrict the value of coupling to $|\eta| < 10^{-1}$. In the limit $\eta \to 0$ the Barber's second theory approaches the standard general relativity in every respect. Because of the homogeneity condition imposed by the metric, the scalar field ϕ will be a function of t only. For the line element (1) field equations (4) and (5) lead to the following system of equations:

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC} + \frac{3}{4}\beta^2 = -8\pi G\phi^{-1}p - \Lambda \ , \tag{6}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4C_4}{AC} + \frac{3}{4}\beta^2 = -8\pi G\phi^{-1}p - \Lambda \ , \tag{7}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4B_4}{AB} + \frac{3}{4}\beta^2 = -8\pi G\phi^{-1}p - \Lambda \ , \tag{8}$$

$$\frac{A_4B_4}{AB} + \frac{B_4C_4}{BC} + \frac{A_4C_4}{AC} - \frac{3}{4}\beta^2 = 8\pi G\phi^{-1}\varepsilon - \Lambda \ , \tag{9}$$

$$\phi_{44} + \phi_4\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = \frac{8\pi}{3}\eta(\varepsilon - 3p) \ , \tag{10}$$

where the suffix '4' denotes differentiation with respect to time t. An additional equation for time changes of G and Λ are obtained by the divergence of Einstein's tensor i.e. $(R_i^j - \frac{1}{2}Rg_i^j)_{;j} = 0$ which leads to

$$(8\pi G T_i^j - \Lambda g_i^j)_{;j} = 0 , \tag{11}$$

which yields

$$8\pi G_4\varepsilon + \Lambda_4 + 8\pi G\left[\varepsilon_4 + (\varepsilon + \mathrm{p})\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)\right] = 0 \ . \tag{12}$$

From equation (12) we have

$$\varepsilon_4 + (\varepsilon + p)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0 \ , \tag{13}$$

and

$$8\pi G_4 \varepsilon + \Lambda_4 = 0 \tag{14}$$

From equation (11) one concludes that when Λ is constant or $\Lambda = 0$, G turns out to be constant.

The conservation of left hand side of equation (4) leads to

$$\left(R_i^j - \frac{1}{2} R g_i^j\right)_{;j} + \frac{3}{2}(\alpha_i \alpha^j)_{;j} - \frac{3}{4}(\alpha_k \alpha^k \, g_i^j)_{;j} = 0, \tag{15}$$

which leads to

$$\frac{3}{2}\beta\beta_4 + \frac{3}{2}\beta^2\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0 \ . \tag{16}$$

In general relativity, we define equivalent densities and pressures given by Soleng (1987a,1987b) as

$$\varepsilon_{eq} = \frac{\varepsilon}{\phi} \ , \tag{17}$$

$$p_{eq} = \frac{p}{\phi}, \tag{18}$$

Using equations (17) and (18) in equation (13) we get

$$\left(\frac{\varepsilon}{\phi}\right)_4 + \left(\frac{\varepsilon + p}{\phi}\right)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0 \ , \tag{19}$$

where the sub-indices 4 in A,B and C denotes ordinary differentiation with respect to ‘t’. The scalar expansion θ and components of shear tensor σ_i^j are given by

$$\theta = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}, \tag{20}$$

$$\sigma_1^1 = \frac{1}{3}\left[\frac{2A_4}{A} - \frac{B_4}{B} - \frac{C_4}{C}\right], \tag{21}$$

$$\sigma_2^2 = \frac{1}{3}\left[\frac{2B_4}{B} - \frac{A_4}{A} - \frac{C_4}{C}\right], \tag{22}$$

$$\sigma_3^3 = \frac{1}{3}\left[\frac{2C_4}{C} - \frac{A_4}{A} - \frac{B_4}{B}\right], \tag{23}$$

$$\sigma_4^4 = 0 \ , \tag{24}$$

therefore

$$\sigma^2 = \frac{1}{3}\left[\frac{A_4^2}{A^2} + \frac{B_4^2}{B^2} + \frac{C_4^2}{C^2} - \frac{A_4 B_4}{AB} - \frac{B_4 C_4}{BC} - \frac{C_4 A_4}{AC}\right]. \tag{25}$$

Solution of Field Equations

It can be easily seen that we have six independent equations (6)-(10) and (14) with nine unknowns $A, B, C, \phi, \beta, \varepsilon, \mathrm{p}, \Lambda$ and G. For complete determinacy of the system one extra condition is needed. For this we consider dust filled model

which gives

$$\mathrm{p} = 0 \tag{26}$$

Now equation (19) becomes.

$$\left(\frac{\varepsilon}{\phi}\right)_4 + \left(\frac{\varepsilon}{\phi}\right)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0 \ , \tag{27}$$

which on integration gives,

$$\left(\frac{\varepsilon}{\phi}\right) = \frac{K_1}{ABC}, \tag{28}$$

where K_1 is constant of integration.

Now adding equations (6), (7) and (8), we get

$$\frac{2A_{44}}{A} + \frac{2B_{44}}{B} + \frac{2C_{44}}{C} + \frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{A_4 C_4}{AC} + \frac{9}{4}\beta^2 = -24\pi\phi^{-1} p - 3\Lambda \ . \tag{29}$$

Adding three times of equation (9) in (29), we get

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{2A_4B_4}{AB}+\frac{2B_4C_4}{BC}+\frac{2A_4C_4}{AC}=-12\pi\phi^{-1}\left(p-\varepsilon\right)-3\Lambda \quad . \tag{30}$$

For dust filled model (p=0) equation (30) reduces to

$$\frac{\left(ABC\right)_{44}}{ABC}=12\pi\left(\frac{\varepsilon}{\phi}\right)-3\Lambda \quad . \tag{31}$$

Using equation (28) in (31), we have

$$\left(ABC\right)_{44}=12\pi K_1-3\Lambda\left(ABC\right) . \tag{32}$$

Now assuming

$$\Lambda=\frac{K_2}{ABC} \tag{33}$$

Integrating equation (32) two times, we have

$$\left(ABC\right)=\left(6\pi K_1-\frac{3}{2}K_2\right)t^2+K_3t+K_4 \quad , \tag{34}$$

where K_3 and K_4 are constants of integration .

Equation (34) becomes

$$ABC=\left(d_1t+d_2\right)^2 \quad , \tag{35}$$

where $\left(6\pi K_1-\frac{3}{2}K_2\right)=d_1^2, K_3=2d_1d_2, K_4=d_2^2,$

Hence the values of metric potentials are calculated as

$$A=p_0\left(d_1t+d_2\right)^{a_1}, \tag{36}$$

$$B=q_0\left(d_1t+d_2\right)^{a_2}, \tag{37}$$

$$C=r_0\left(d_1t+d_2\right)^{a_3} \quad , \tag{38}$$

where the constants $a_1, a_{2,}, a_3$ and p_0, q_0, r_0 satisfy the relations

$$a_1 + a_2 + a_3 = 2 \quad \text{and} \quad p_0 q_0 r_0 = 1 \,. \tag{39}$$

Now using equation (28) in (10), we get

$$\left(ABC\phi_4\right)_4 = \frac{8\pi}{3}\eta K_1 \varepsilon \,. \tag{40}$$

For deterministic solution we can take $K_1 = 0$.

Using K_1=0 in equation (40) and integrating twice, we get

$$\phi = K_6 - \frac{2K_5}{d_1(d_1 t + d_2)} \,, \tag{41}$$

Where K_5 and K_6 are constants of integration.

Equation (16) gives either β=0 or $\frac{\beta_4}{\beta} + \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0$,

Therefore taking

$$\frac{\beta_4}{\beta} + \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0 \,, \tag{42}$$

which on integration leads to

$$\beta = \frac{K_7}{ABC} \,, \tag{43}$$

Where K_7 is constant of integration.

Hence

$$\beta = \frac{K_7}{\left(d_1 t + d_2\right)^2} \,. \tag{44}$$

Hence metric (1) reduces to

$$ds^2 = -dt^2 + p_0^2 \left(d_1 t + d_2\right)^{2a_1} dx^2 + q_0^2 \left(d_1 t + d_2\right)^{2a_2} dy^2 + r_0^2 \left(d_1 t + d_2\right)^{2a_3} dz^2 \,, \tag{45}$$

Which can be transformed through a proper choice of coordinates to the form

$$ds^2 = -\frac{dT^2}{d_1^2} + T^{2a_1} dX^2 + T^{2a_2} dY^2 + T^{2a_3} dZ^2 \,. \tag{46}$$

Some Physical and Kinematic Properties of the Model

Using equations (35) and (41) in equation (27) we get

$$\varepsilon = \frac{K_1(K_6 T - K_8)}{T^3} \tag{47}$$

where $K_8 = \frac{2K_5}{d_1}$.

Scalar field ϕ is given by

$$\phi = \left(K_6 - \frac{K_8}{T} \right), \tag{48}$$

which is not defined at $T = 0$.

The scalar expansion (θ) is evaluated as

$$\theta = \frac{K_9}{T}, \tag{49}$$

Where $K_9 = a_1 d_1 + a_2 d_1 + a_3 d_1$.

The behaviour of expansion (θ) with time (T) is shown in the figure given below.

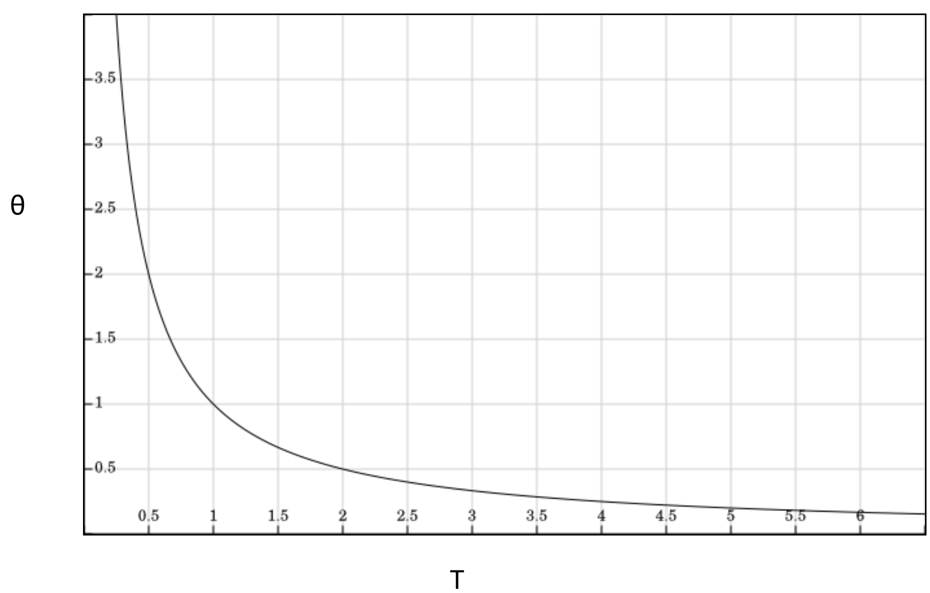

Figure 1: variation of scalar expansion with time T when K_9=1

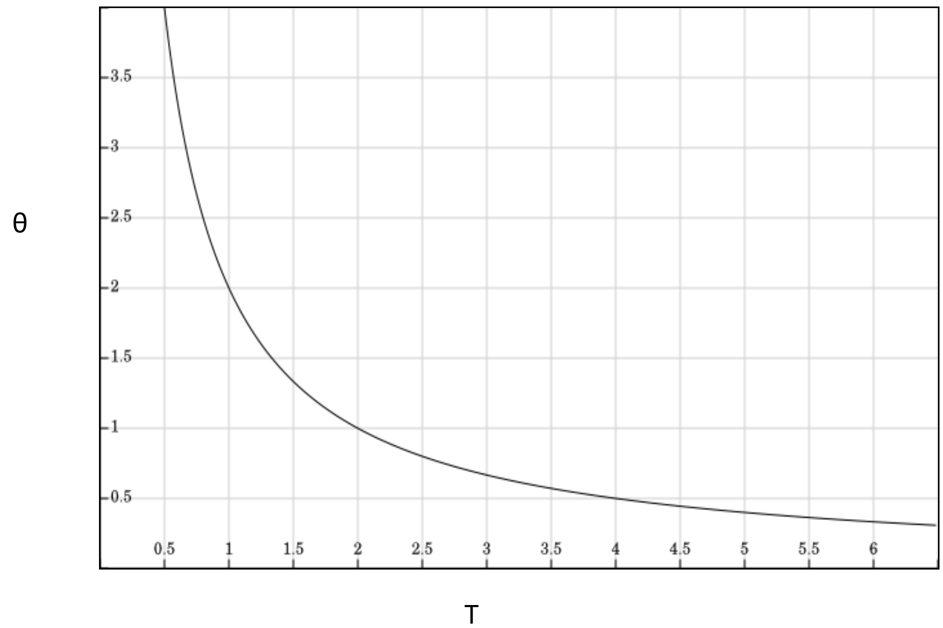

Figure 2: variation of scalar expansion with time T when K_9=2

The graphs 1 and 2 show that scalar expansion (θ) increases rapidly at initial time and slows down when time increases, θ stops at $T \to \infty$, thus the model starts with a Big-Bang at T=0.

The components of shear are calculated as

$$\sigma_1^1 = \frac{K_{10}}{T} \ , \tag{50}$$

where $K_{10} = \left(\frac{2a_1 - a_2 - a_3}{3}\right) d_1$

$$\sigma_2^2 = \frac{K_{11}}{T} \ , \tag{51}$$

where $K_{11} = 2a_2 - a_1 - a_3$

$$\sigma_3^3 = \frac{K_{12}}{T} , \tag{52}$$

where $K_{12} = \frac{d_1}{3}\left[2a_3 - a_1 - a_2\right]$

and $$\sigma_4^4 = 0 \ ,$$
(53)

hence the shear (σ) is given by

$$\sigma^2 = \frac{K_{13}}{T^2} \quad ,$$

(54)

where

$$\left[\frac{K_{10}^2 + K_{11}^2 + K_{12}^2}{2}\right] = K_{13} \, .$$

The behaviour of shear (σ) with time (T) is shown in the figure given below.

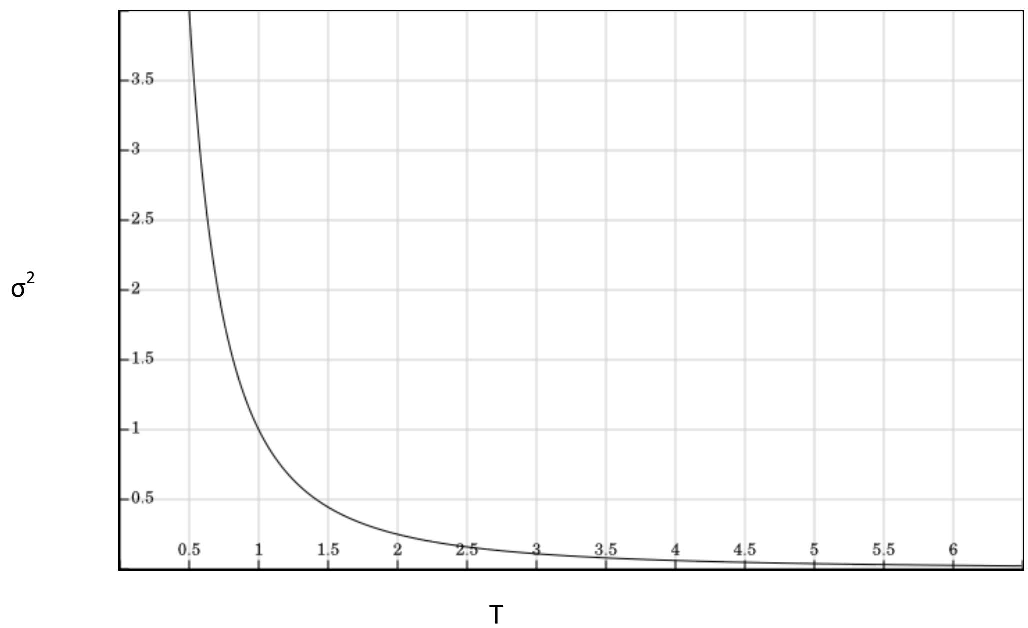

Figure 3 variation of shear with time when K_{13}=1

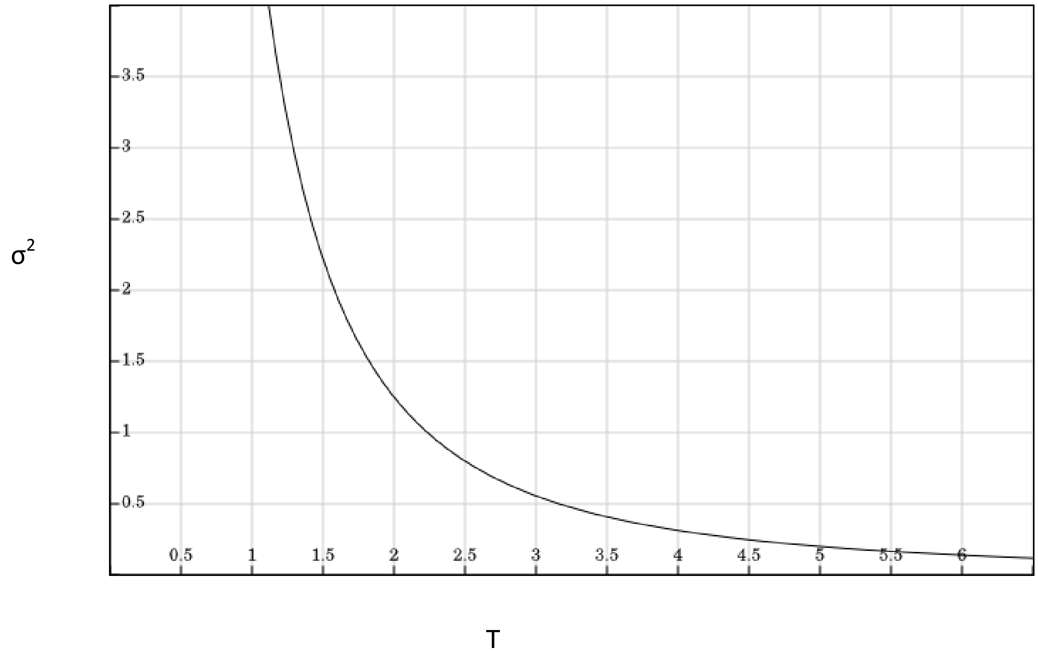

Figure 4 variation of shear with time when K_{13}=5

The graphs 3 and 4 show that shear (σ) increases rapidly at initial time and slows down when time increases, θ stops at $T \to \infty$.

The ratio $\frac{\sigma}{\theta}$ is evaluated as

$$\frac{\sigma}{\theta} = \frac{\sqrt{K_{13}}}{K_9} \ , \tag{55}$$

which is constant.

The spatial volume (V) is given by

$$V^3 = ABC = T^2$$

(56)

The variation of spatial volume (V) with T is shown in the graph below.

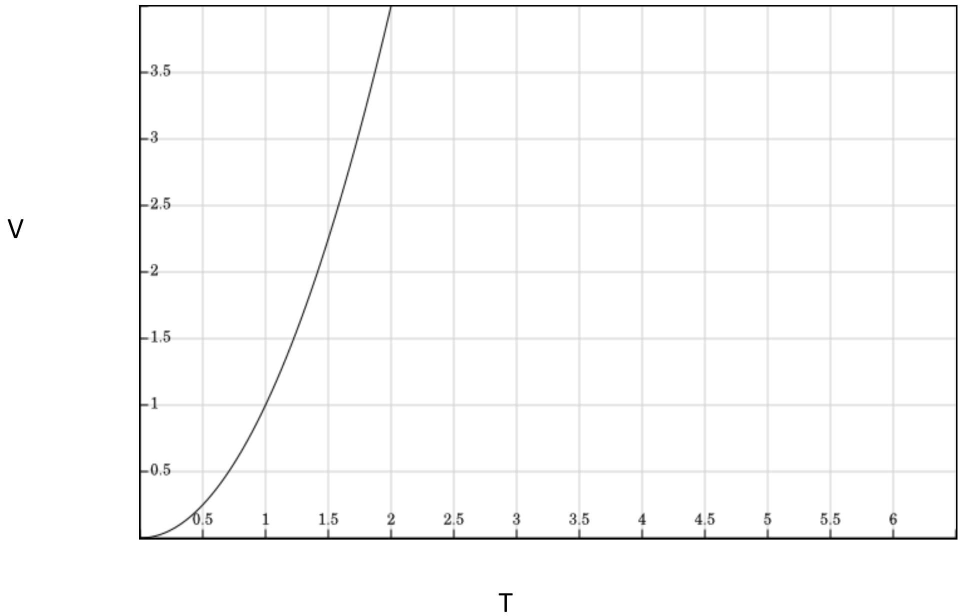

Figure 5 variation of spatial volume with time

Hence we can observe that spatial volume (V) is zero at initial time and increases as time increases i.e. $V \to 0 \ as T \to 0$ and $V \to \infty \ \ as \ T \to \infty$.

Now since

$$\int_{T_0}^{T} \frac{dt}{V(t)} = \frac{3}{d_1}\left[\left(d_1 t + d_2\right)^{1/3}\right]_{T_0}^{T}, \tag{57}$$

which is a convergent integral, so the particle horizon exists.

The deceleration parameter q is given by

$$q = -\frac{RR_{44}}{R_4^2}, \tag{58}$$

where R is a scalar factor given by $R^3 = ABC$. For our model the deceleration parameter is evaluated as

$$q = \frac{1}{2}, \tag{59}$$

which is positive hence the model (46) decelerates.

The cosmological constant (Λ) is given by,

$$\Lambda = \frac{K_2}{T^2}, \tag{60}$$

Using equations (47) and (60) in the equation (14), we get

$$G_4 = \frac{2K_2 d_1}{8\pi K_1 (\mathrm{K}_6 \mathrm{T} - \mathrm{K}_8)} . \tag{61}$$

Which on integration leads to

$$G = \frac{2K_2 d_1}{8\pi K_1 K_6} \log(\mathrm{K}_6 \mathrm{T} - \mathrm{K}_8) + K_{14}$$

(62)

Where K_{14} is constant of integration.

Clearly $T \to 0$ gives $\Lambda \to \infty$ and $T \to \infty$ gives $\Lambda \to 0$.

When T is finite Λ becomes a constant. It is worth showing that Λ is inversely proportional to the square of time T. The value of cosmological constant Λ is in an excellent agreement with observation of type Ia Supernovae (SNe).The main conclusion of these observations is that the expansion of the universe is accelerating and the cosmological term was very large at initial time which relaxes to a genuine cosmological constant with due course of time.

Components of Hubble parameter are given by

$$H_1 = \frac{A_4}{A} = \frac{a_1 d_1}{T} \tag{63}$$

$$H_2 = \frac{B_4}{B} = \frac{a_2 d_1}{T} \tag{64}$$

$$H_3 = \frac{C_4}{C} = \frac{a_3 d_1}{T} \tag{65}$$

Hence the Hubble parameter H is given by

$$H = \frac{2d_1}{3T} \tag{66}$$

The anisotropy parameter $\overline{A}$ is given by

$$\overline{A} = \frac{1}{3}\sum_{i=1}^{n}\left(\frac{\Delta H_i}{H}\right), \ where \ \Delta H_i = H_i - H$$

For our model $\bar{A}$ is evaluated as

$$\bar{A} = \frac{3}{4}\left[\left(a_1 - \frac{2}{3}\right)^2 + \left(a_2 - \frac{2}{3}\right)^2 + \left(a_3 - \frac{2}{3}\right)^2\right]$$

(67)

The expansion velocity R_4 is given by

$$R_4 = \frac{2d_1}{3} T^{-1/3} \tag{68}$$

Thus R_4 diverges as $T \to 0$.

Hence the expansion of the universe is infinite as we approach towards $t \to -d_2 / d_1$.

Now

$$\frac{\varepsilon}{\theta^2} = \frac{\left(K_6 - \frac{K_8}{T}\right)K_1}{K_9^2} \tag{69}$$

If $K_8 = 0$ or $T \to \infty$ then $\frac{\varepsilon}{\theta^2}$ is constant which means energy density (ε) is proportional to the square of scalar expansion(θ^2).Thus the model approaches homogeneity and matter is dynamically negligible near the origin. This behaviour of the model is similar to the results given by Collins(1977).

Also

$$\beta^2 > \frac{\left(\frac{2KK_5}{d_1 T^3} - \frac{KK_6}{T^2}\right)}{\left(\frac{3}{2}\frac{K_5}{d_1}\frac{1}{T} - \frac{3}{4}K_6\right)}$$

(70)

It is possible to discuss entropy. To solve entropy we have $ds > 0$ necessarily.

The conservation equation $T^j_{i;j} = 0$ for the metric (1) is

$$\varepsilon_4 + (\varepsilon + p)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) + \frac{3}{2}\beta\beta_4 + \frac{3}{2}\beta^2\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0 \tag{71}$$

In our case $S^3 = ABC$

$$\text{Since } \varepsilon_4 + (\varepsilon + p)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) > 0 \tag{72}$$

It is evident that

$$\frac{3}{2}\beta\beta_4 + \frac{3}{2}\beta^2\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) < 0 \tag{73}$$

Which leads to $\beta < 0$.

Thus the displacement vector β affects entropy because for entropy $ds > 0$ which leads to $\beta < 0$.

Conclusion

The model (46) starts with a big bang at $T = 0$ and the expansion in the model decreases as time increases. It ceases out when $T \to \infty$. It is evident from the above physical quantities that the fluid has non-zero expansion(i.e. $\theta \neq 0$). It is also observed that when $T \to 0$ i.e. $t = -d_2 / \mathrm{d}_1$, the physical quantities for the model (46) like expansion scalar(θ), shear scalar(σ), cosmological constant(Λ) and Hubble parameter (H) diverge. Initially at T=0 the energy density ε is infinite. The anisotropy parameter $\bar{A}$ is constant for any T. Shear(σ) also tends to infinity at initial epoch T=0 and it vanishes at $T \to \infty$.

As t increases, the spatial volume V increases but the expansion scalar θ decreases. Thus the expansion rate decreases as time increases. As $T \to \infty$ the spatial volume V becomes infinitely large and the expansion in the model stops. Since the physical parameters $\varepsilon, \sigma, H_1, H_2, H_3$ *and* H tend to zero when $T \to \infty$, therefore the model (46) essentially gives an empty universe for large values of T. All physical quantities remain finite and physically significant at finite region of the universe. Also clearly the scalar field remains finite through the evolution of universe. In case $\eta \to 0$ the solution approach Einstein's general theory of relativity in all respect and the model represents shearing, non-rotating and expanding universe with a big bang start.

The space time exhibits POINT TYPE singularity at T=0 i.e. $t = -d_2 / \mathrm{d}_1$, with $a_1>0$, $a_2>0$, $a_3>0$. The model (46) has CIGAR TYPE Singularity when T=0 with either $a_1>0$, $a_2>0$, $a_3<0$ or $a_1>0$, $a_2<0$, $a_3>0$ or $a_1<0$, $a_2>0$, $a_3>0$. Since $\lim_{n\to\infty}\left(\frac{\sigma}{\theta}\right) = \frac{\sqrt{K_{13}}}{K_9}$ which is non-zero constant, hence the model does not approach isotropy for large values of T but if $K_{13} = 0$ then the model approaches isotropy.

The displacement vector (β) decreases as time increases. The displacement vector (β) also affects entropy because for entropy $ds > 0$ which leads to $\beta < 0$.When $\beta \to 0$ our model reduces to the model obtained by Jain et al. (2009).

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