

String Cosmological Models in Cylindrically Symmetric Inhomogeneous Universe with Electromagnetic Field

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Abstract

Cylindrically symmetric inhomogeneous string cosmological models in the presence of electromagnetic field are investigated. We have assumed that F_{12} is the only non-vanishing component of F_{ij} . The Maxwell's equations require that F_{12} is the function of x and t both and the magnetic permeability is the function of x and t both. To get the deterministic solution, it has been assumed that the expansion (θ) in the model is proportional to the eigen value σ_1^1 of the shear tensor σ_j^i . The derived model represents the inflationary scenario as the proper volume increases exponentially with cosmic time. It is observed that the model has a point type singularity. The physical and geometric aspects of the model are also discussed.

Keywords: cosmic string, electromagnetic field, inhomogeneous universe.

1. Introduction

Cosmic strings play an important role in the study of the early universe. These strings arise during the phase transition after the big bang explosion as the temperature goes down below some critical temperature as predicted by grand unified theories [1] – [5]. It is believed that cosmic strings give rise to density perturbations which lead to formation of galaxies [6]. These cosmic strings have stress energy and couple to the gravitational field. Therefore, it is interesting to study the gravitational effect which arises from strings. The general treatment of strings was initiated by Letelier [7,8] and Stachel [9]. The occurrence of magnetic fields on galactic scale is well-established fact today, and their importance for a variety of astrophysical phenomena is generally acknowledged as pointed out Zel'dovich [10]. Also Harrison [11] has suggested that magnetic field could have a cosmological origin.

As a natural consequence, we should include magnetic field in the energy-momentum tensor of the early universe. The choice of anisotropic cosmological models in Einstein system of field equations leads to the cosmological models more general than Robertson-Walker model [12]. The presence of primordial magnetic fields in the early stages of the evolution of the universe

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has been discussed by several authors (Misner, Thorne and Wheeler [13]; Asseo and Sol [14]; Pudritz and Silk [15]; Kim, Tribble, and Kronberg [16]; Perley and Taylor [17]; Kronberg, Perry and Zukowski [18]; Wolfe, Lanzetta and Oren [19]; Kulsrud, Cen, Ostriker and Ryu [20]; Barrow [21]). Melvin [22], in his cosmological solution for dust and electromagnetic field suggested that during the evolution of the universe, the matter was in a highly ionized state and was smoothly coupled with the field, subsequently forming neutral matter as a result of universe expansion. Hence the presence of magnetic field in string dust universe is not unrealistic.

Banerjee et al. [23] have investigated an axially symmetric Bianchi type I string dust cosmological model in presence and absence of magnetic field using a supplementary condition $\alpha = a\beta$ between metric potential where $\alpha = \alpha(t)$ and $\beta = \beta(t)$ and, a is a constant. The string cosmological models with a magnetic field are also discussed by Chakraborty [24], Tikekar and Patel [25,26]. Patel and Maharaj [27] investigated stationary rotating world model with magnetic field. Ram and Singh [28] obtained some new exact solution of string cosmology with and without a source free magnetic field for Bianchi type I space-time in the different basic form considered by Carminati and McIntosh [29]. Singh and Singh [30] investigated string cosmological models with magnetic field in the context of space-time with G_3 symmetry. Singh [31] has studied string cosmology with electromagnetic fields in Bianchi type-II, -VIII and -IX space-times. Lidsey, Wands and Copeland [32] have reviewed aspects of super string cosmology with the emphasis on the cosmological implications of duality symmetries in the theory. Bali et al. [33-38] have investigated some string cosmological models in different Bianchi space-times with and without magnetic field.

Cylindrically symmetric space-times play an important role in the study of the universe on a scale in which anisotropy and inhomogeneity are not ignored. Inhomogeneous cylindrically symmetric cosmological models have significant contribution in understanding some essential features of the universe such as the formation of galaxies during the early stages of their evolution. Bali and Tyagi [39] and Pradhan et al. [40,41] have investigated cylindrically symmetric inhomogeneous cosmological models in presence of electromagnetic field. Barrow and Kunze [42,43] found a wide class of exact cylindrically symmetric flat and open inhomogeneous string universes. In their solutions all physical quantities depend on at most one space coordinate and the time. The case of cylindrical symmetry is natural because of the mathematical simplicity of the field equations whenever there exists a direction in which the pressure equal to energy density.

Baysal et al. [44] have investigated some string cosmological models in cylindrically symmetric inhomogeneous universe. Motivated by the situation discussed above, in this paper, we have generalized these solutions by including electromagnetic field tensor. We have taken string and electromagnetic field together as the source gravitational field and magnetic field are anisotropic stress source and low strings are one of anisotropic stress source as well. This paper is organized as follows: The metric and field equations are presented in Section 2. In Section 3, we deal with the solution of the field equations in presence of electromagnetic field with perfect fluid distribution. In Section 4, physical and geometrical aspects and in Section 5, we have given the concluding remarks.

2. The Metric and Field Equations

We consider the cylindrically symmetric metric in the form

$$ds^2 = A^2(dx^2 - dt^2) + B^2dy^2 + C^2dz^2, \tag{1}$$

where A, B and C are functions of x and t. The energy momentum tensor for the string with electromagnetic field has the form

$$T_i^j = \rho u_i u^j - \lambda x_i x^j + E_i^j, \tag{2}$$

where u_i and x_i satisfy conditions

$$u^i u_i = -x^i x_i = -1 \text{ and } u^i x_i = 0. \tag{3}$$

Here ρ being the rest energy density of the system of strings, λ the tension density of the strings, x^i is a unit space-like vector representing the direction of strings so that $x^2 = 0 = x^3 = x^4$ and $x^1 = \neq 0$, and u^i is the four velocity vector satisfying the following conditions

$$g_{ij} u^i u^j = -1. \tag{4}$$

In Equation (2), E_i^j is the electromagnetic field given by Lichnerowicz [45]

$$E_i^j = \bar{\mu} \left[h_\ell h^\ell \left(u_i u^j + \frac{1}{2} g_i^j \right) - h_i h^j \right], \tag{5}$$

where $\bar{\mu}$ is the magnetic permeability and h_i the magnetic flux vector defined by

$$h_i = \frac{1}{\mu} *F_{ji} u^j, \tag{6}$$

where the dual electromagnetic field tensor $*F_{ij}$ is defined by Synge[46]

$$*F_{ij} = \frac{\sqrt{-g}}{2} \epsilon_{ijkl} F^{kl}. \tag{7}$$

Here F_{ij} is the electromagnetic field tensor and ϵ_{ijkl} is the Levi-Civita tensor density.

The components of electromagnetic field are obtained as

$$E_1^1 = E_2^2 = E_4^4 = \frac{F_{12}^2}{2\bar{\mu}A^2B^2},$$

$$E_3^3 = -\frac{F_{12}^2}{2\bar{\mu}A^2B^2}. \tag{8}$$

In the present scenario, the comoving coordinates are taken as

$$u^i = \left(0, 0, 0, \frac{1}{A} \right). \tag{9}$$

We choose the direction of string parallel to x-axis so that

$$x^i = \left(\frac{1}{A}, 0, 0, 0 \right). \tag{10}$$

We consider that F_{12} is the only non-vanishing component of F_{ij} so that $h_3 \neq 0$. Maxwell's equations

$$F_{[ij;k]} = 0, \tag{11}$$

$$\left[\frac{1}{\bar{\mu}} F^{ij} \right]_{;j} = 0, \tag{12}$$

require that F_{12} is the function of x and t both and the magnetic permeability is also the functions of x and t both. The semicolon represents a covariant differentiation.

The Einstein's field equations (with $\frac{8\pi G}{c^4} = 1$)

$$R_i^j - \frac{1}{2} R g_i^j = -T_i^j, \tag{13}$$

for the line-element (1) lead to the following system of equations;

$$\begin{aligned} \frac{B_{44}}{B} + \frac{C_{44}}{C} - \frac{A_4}{A} \left(\frac{B_4}{B} + \frac{C_4}{C} \right) - \frac{A_1}{A} \left(\frac{B_1}{B} + \frac{C_1}{C} \right) - \frac{B_1 C_1}{BC} + \frac{B_4 C_4}{BC} \\ = \left[\lambda - \frac{F_{12}^2}{2\bar{\mu}A^2B^2} \right] A^2, \end{aligned} \tag{14}$$

$$\left(\frac{A_4}{A} \right)_4 - \left(\frac{A_1}{A} \right)_1 + \frac{C_{44}}{C} - \frac{C_{11}}{C} = - \left[\frac{F_{12}^2}{2\bar{\mu}A^2B^2} \right] A^2, \tag{15}$$

$$\left(\frac{A_4}{A} \right)_4 - \left(\frac{A_1}{A} \right)_1 + \frac{B_{44}}{B} - \frac{B_{11}}{B} = \left[\frac{F_{12}^2}{2\bar{\mu}A^2B^2} \right] A^2, \tag{16}$$

$$\begin{aligned} -\frac{B_{11}}{B} - \frac{C_{11}}{C} + \frac{A_1}{A} \left(\frac{B_1}{B} + \frac{C_1}{C} \right) + \frac{A_4}{A} \left(\frac{B_4}{B} + \frac{C_4}{C} \right) - \frac{B_1 C_1}{BC} + \frac{B_4 C_4}{BC} \\ = \left[\rho - \frac{F_{12}^2}{2\bar{\mu}A^2B^2} \right] A^2, \end{aligned} \tag{17}$$

$$\frac{B_{14}}{B} + \frac{C_{14}}{C} - \frac{A_4}{A} \left(\frac{B_1}{B} + \frac{C_1}{C} \right) - \frac{A_1}{A} \left(\frac{B_4}{B} + \frac{C_4}{C} \right) = 0, \tag{18}$$

where the sub indices 1 and 4 in A,B, C and elsewhere denote ordinary differentiation with respect to x and t respectively.

The velocity field u^i is irrotational. The scalar expansion θ , shear scalar σ^2 , acceleration vector \dot{u}_i and proper volume V^3 are respectively found to have the following expressions:

$$\theta = u^i_{;i} = \frac{1}{A} \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \right), \tag{19}$$

$$\sigma^2 = \frac{1}{2} \sigma_{ij} \sigma^{ij} = \frac{1}{3} \theta^2 - \frac{1}{A^2} \left(\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{C_4 A_4}{CA} \right), \tag{20}$$

$$\dot{u}_i = u_{i;j} u^j = \left(\frac{A_1}{A}, 0, 0, 0 \right) \tag{21}$$

$$V^3 = \sqrt{-g} = A^2 BC, \tag{22}$$

3. Solutions of the Field Equations

As in the case of general-relativistic cosmologies, the introduction of inhomogeneities into the string cosmological equations produces a considerable increase in mathematical difficulty: non-linear partial differential equations must now be solved. In practice, this means that we must proceed either by means of approximations which render the non-linearities tractable, or we must introduce particular symmetries into the metric of the space-time in order to reduce the number of degrees of freedom which the inhomogeneities can exploit. To get a determinate solution, let us assume that expansion (θ) in the model is proportional to the value σ_1^1 of the shear tensor σ_j^i .

This condition leads to

$$A = (BC)^n, \tag{23}$$

where n is a constant. Equations (15) and (16) lead to

$$\frac{F_{12}^2}{\bar{\mu} B^2} = \frac{B_{44}}{B} - \frac{B_{11}}{B} - \frac{C_{44}}{C} + \frac{C_{11}}{C}. \tag{24}$$

and

$$2 \left(\frac{A_4}{A} \right)_4 - 2 \left(\frac{A_1}{A} \right)_1 + \frac{B_{44}}{B} - \frac{B_{11}}{B} + \frac{C_{44}}{C} - \frac{C_{11}}{C} = 0. \tag{25}$$

Using (23) in (18) reduces to

$$\frac{B_{41}}{B} + \frac{C_{41}}{C} - 2n \left(\frac{B_4}{B} + \frac{C_4}{C} \right) \left(\frac{B_1}{B} + \frac{C_1}{C} \right) = 0. \tag{26}$$

To get the deterministic solution, we assume

$$B = f(x) f(t) \text{ and } C = h(x) k(t) \tag{27}$$

and discuss its consequences below in this paper.

In this case Equation (26) reduces to

$$\frac{f_1/f}{h_1/h} = -\frac{(2n-1)(k_4/k) + 2n(g_4/g)}{(2n-1)(g_4/g) + 2n(k_4/k)} = K \text{ (constant)}. \tag{28}$$

which leads to

$$\frac{f_1}{f} = K \frac{h_1}{h}, \tag{29}$$

and

$$\frac{k_4/k}{g_4/g} = \frac{K - 2nK - 2n}{2nK + 2n - 1} = a \text{ (constant)}. \tag{30}$$

From Equations (29) and (30), we obtain

$$f = \alpha h^K, \tag{31}$$

and

$$k = \delta g^a, \tag{32}$$

where α and δ are integrating constants.

From equations (25) and (23), we obtain

$$\begin{aligned} & (2n+1) \frac{B_{44}}{B} - 2n \frac{B^2}{B^2} + (2n+1) \frac{C_{44}}{C} - 2n \frac{C^2}{C^2} \\ & = (2n+1) \frac{B_{11}}{B} + (2n+1) \frac{C_{11}}{C} - 2n \frac{B^2}{B^2} - 2n \frac{C^2}{C^2} = N \text{ (constant)}. \end{aligned} \tag{33}$$

Equations (27) and (33) lead to

$$gg_{44} + rg_4^2 = sg^2, \tag{34}$$

where

$$r = \frac{a(a-1) - 2n(a+1)}{(2n+1)(a+1)}, s = \frac{N}{(2n+1)(a+1)}.$$

Integrating Equation (34), we obtain

$$g = \beta \sinh^{\frac{1}{(1+r)}}(bt + t_0), \tag{35}$$

where $\beta = (c_1)^{\frac{1}{1+r}}$, $b = \sqrt{s(1+r)}$ and t_0, c_1 are constants of integration.

Thus from (32), we get

$$k = \delta\beta^a \sinh^{\frac{1}{(1+r)}}(bt + t_0). \tag{36}$$

Equations (29) and (33) lead to

$$hh_{11} + \ell h_1^2 = mh^2, \tag{37}$$

where

$$\ell = \frac{K(K-1) - 2n(K+1)}{(2n+1)(K+1)}, m = \frac{N}{(2n+1)(K+1)}.$$

Integrating Equation (37), we obtain

$$h = r_0 \sinh^{\frac{1}{(1+\ell)}}(cx + x_0), \tag{38}$$

where $r_0 = c^{\frac{1}{1+\ell}}$, $c = \sqrt{m(1+\ell)}$ and c, x_0 are constants of integration.

Hence from (31) and (38), we get

$$f = \alpha r_0^K \sinh^{\frac{K}{(1+\ell)}}(cx + x_0). \tag{39}$$

Hence, we obtain

$$B = fg = Q \sinh^{\frac{K}{(\ell+1)}}(cx + x_0) \sinh^{\frac{1}{(r+1)}}(bt + t_0), \tag{40}$$

$$C = hk = R \sinh^{\frac{1}{(\ell+1)}}(cx + x_0) \sinh^{\frac{\alpha}{(r+1)}}(bt + t_0), \tag{41}$$

$$A = (BC)^n = M \sinh^{\frac{n(K+1)}{(\ell+1)}}(cx + x_0) \sinh^{\frac{n(a+1)}{(r+1)}}(bt + t_0), \tag{42}$$

where $Q = \alpha\beta r_0^K, r = r_0\delta\beta^\alpha, M = (QR)^n$.

After using suitable transformation of coordinates metric (1) reduces to

$$ds^2 = M^2 \sinh^{\frac{2n(K+1)}{(\ell+1)}}(cX) \sinh^{\frac{2n(a+1)}{(r+1)}}(bT)(dX^2 - dT^2) + Q^2 \sinh^{\frac{2K}{(\ell+1)}}(cX) \sinh^{\frac{2}{(r+1)}}(bT) dY^2 + R^2 \sinh^{\frac{2}{(\ell+1)}}(cX) \sinh^{\frac{2\alpha}{r+1}}(bT) dZ^2, \tag{43}$$

where $X = x + \frac{x_0}{c}, Y = Qy, Z = Rz$ and $T = t + \frac{t_0}{b}$.

4. Physical and Geometrical Aspects

The energy density (ρ), the string tension density (λ), the particle density (ρ_p), the scalar of expansion (θ), shear tensor (σ), acceleration vector \dot{u}_i and the proper volume for (V^3) for the model (43) are given by

$$\rho = \frac{1}{M^2 \sinh^{\frac{2n(K+1)}{(\ell+1)}} (cX) \sinh^{\frac{2n(a+1)}{(r+1)}} (bT)} \left[\frac{c^2 \{ (K+1)(n(K+1) + \ell) - K^2 \}}{(\ell+1)^2} \coth^2 (cX) + \frac{b^2 \{ n(a+1)^2 + \alpha \}}{(r+1)^2} \coth^2 (bT) - \frac{c^2 (K+1)}{(\ell+1)} + \frac{F_{12}^2}{Q^2 \bar{\mu} \sinh^{\frac{2K}{(\ell+1)}} (cX) \sinh^{\frac{2}{(r+1)}} (bT)} \right], \tag{44}$$

$$\lambda = \frac{1}{M^2 \sinh^{\frac{2n(K+1)}{(\ell+1)}} (cX) \sinh^{\frac{2n(a+1)}{(r+1)}} (bT)} \left[\frac{b^2 \{ a^2 - (a+1)(na + n + r) \}}{(r+1)^2} \coth^2 (bT) - \frac{c^2 \{ n(K+1)^2 - K \}}{(\ell+1)^2} \coth^2 (cX) + \frac{b^2 (a+1)}{(r+1)} + \frac{F_{12}^2}{Q^2 \bar{\mu} \sinh^{\frac{2K}{(\ell+1)}} (cX) \sinh^{\frac{2}{(r+1)}} (bT)} \right], \tag{45}$$

$$\rho_p = \rho - \lambda = \frac{1}{M^2 \sinh^{\frac{2n(K+1)}{(\ell+1)}} (cX) \sinh^{\frac{2n(a+1)}{(r+1)}} (bT)} \times \left[\frac{b^2 \{ (a+1)(2na + 2n + r) + a(1-a) \}}{(r+1)^2} \coth^2 (bT) + \frac{c^2 (K+1) \{ (2nK + 2n + \ell) - K \}}{(\ell+1)^2} \coth^2 (cX) - \frac{c^2 (K+1)}{(\ell+1)} - \frac{b^2 (a+1)}{(r+1)} \right] \tag{46}$$

where

$$F_{12}^2 = \bar{\mu} Q^2 \sinh^{\frac{2K}{(\ell+1)}} (cX) \sinh^{\frac{2}{(r+1)}} (bT) \left[\frac{b^2 (1-a)}{(r+1)} + \frac{c^2 (1-K)}{(\ell+1)} \right]$$

$$\left. -\frac{b^2[r+a(a-r-1)]}{(r+1)^2} \coth^2(bT) - \frac{c^2[\ell+K(K-\ell-1)]}{(\ell+1)^2} \coth^2(cX) \right], \quad (47)$$

$$\theta = \frac{b(a+1)(n+1)\coth(bT)}{(r+1)M \sinh^{(\ell+1)}(cX) \sinh^{(r+1)}(bT)}, \quad (48)$$

$$\sigma^2 = \frac{b^2\{(a+1)^2(n^2-n+1)-3a\} \coth^2(bT)}{3(r+1)^2 M^2 \sinh^{(\ell+1)}(cX) \sinh^{(r+1)}(bT)}, \quad (49)$$

$$\dot{u}_i = \left(\frac{cn(K+1)}{(\ell+1)} \coth(cX), 0, 0, 0 \right). \quad (50)$$

$$V^3 = (QR)^{(2n+1)} \sinh^{(\ell+1)}(cX) \sinh^{(r+1)}(bT). \quad (51)$$

From Equations (48) and (49), we obtain

$$\frac{\sigma^2}{\theta^2} = \frac{(a+1)^2(n^2-n+1)-3a}{3(n+1)^2(a+1)^2} = (\text{constant}) \quad (52)$$

The deceleration parameter (q) is given by

$$q = -1 + \frac{3(r+1)M \sinh^{(\ell+1)}(cX) \sinh^{(r+1)}(bT)}{b(a+1)(n+1) \coth(bT)} \times \left[\frac{2b}{\sinh(bT)} + \frac{nb(a+1)}{(r+1)} \coth(bT) \right]. \quad (53)$$

5. Conclusions

We have investigated the behaviour of string in the cylindrically symmetric inhomogeneous cosmological model with electromagnetic field. If we choose the suitable values of constants K and M, we find that energy conditions $\rho \geq 0$, $\rho_p \geq 0$ are satisfied. The string tension (λ) and energy density (ρ) increases as F_{12} increases.

The model (43) starts with a big bang at $T = 0$. The expansion in the model decreases as time increases. The expansion in the model stops at $T = \infty$. Since $\frac{\sigma}{\theta} \neq 0$, hence the model does not approach isotropy in general. However, if $(a+1)^2(n^2-n+1)-3a = 0$, then $\frac{\sigma}{\theta} = 0$ which leads to the isotropy of the universe. We also observe that ρ, λ, ρ_p tend to ∞ when $X \rightarrow 0, T \rightarrow 0$. The

energy density (ρ) and string tension density (λ) increases as electromagnetic field component (F_{12}) increases. The proper volume V^3 increases exponentially as time increases. Thus, the model represents the inflationary scenario. The model (43) has a point type singularity at $T = 0$ (MacCallum [47]).

We observe that $q < 0$ if

$$\frac{3(r+1) M \sinh^{n(K+1)}(cX) \sinh^{n(a+1)}(bT)}{b(a+1)(n+1) \coth(bT)} \times \left[\frac{2b}{\sinh(bT)} + \frac{nb(a+1)}{(r+1)} \coth(bT) \right] < 0.$$

The deceleration parameter q approaches the value (-1) as in the case of de-Sitter universe if $2(r+1) + n(a+1) \cosh(bT) = 0$.

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