Article

Plane Wave Solutions of Moffat & Boal's Non-Symmetric Unified Field Theory

I. S. Mohurley[1], R. K. Jumale[2] & J. K. Jumale[*3]

[1]Dept. of Phys., Shri Dnyanesh Mahavidyalaya, Nawargaon, India. ,*ishwarmohurley@gmail.com*
[2]Kaveri Nagar, Yavatmal Road Darwha,Distt. Yavatmal, (M.S.) India.,
[3]Dept. of Phys., R. S. Bidkar Arts, Comm. & Science College, Hinganghat, India, jyotsnajumale21@gmail.com

Abstract

In the present paper, we have obtained four dimensional plan wave-like solutions of field equations of Moffat and Boal's non-symmetric unified field theory in Peres space-time. If anti-symmetric part of g_{uv} vanishes then the solutions of general relativity proposed by Einstein can be obtained. In this way Einstein general theory of relativity is a special case of Moffat and Boal's non-symmetric unified field theory.

Keywords: plane wave, Moffat, Boal, non-symmetric, unified field theory.

1. Introduction

Einstein's non-symmetric unified field theory is modified by J. W. Moffat and D. H. Boal by considering the field equations as under

$$\partial_a g_{\mu\nu} - g_{\mu\sigma}\Gamma^{\sigma}_{\alpha\nu} - g_{\sigma\nu}\Gamma^{\sigma}_{\mu\alpha} = 0\,, \tag{1.1}$$

$$\Gamma^{\alpha}_{[\mu\alpha]} = 0\,, \tag{1.2}$$

$$R^{*}{}_{(\mu\nu)} = R_{(\mu\nu)} + I_{(\mu\nu)} = 0\,, \tag{1.3}$$

$${}_{[\sigma}R^{*}{}_{[\mu\nu]]} = 0 \tag{1.4}$$

where the contracted curvature tensor is defined by

$$R^{*}_{\mu\nu} = R_{\mu\nu} + I_{\mu\nu}\,, \tag{1.5}$$

[*] Correspondence Author: J. K. Jumale, Department of Physics, R. S. Bidkar College, Hinganghat - 442 301 Wardha, India. E-mail: jyotsnajumale@yahoo.com

$$R_{\mu\nu} = \partial_\alpha \Gamma^\alpha_{\mu\nu} - \frac{1}{2}(\partial_\nu \Gamma^\alpha_{(\mu\alpha)} + \partial\mu \Gamma^\alpha_{(\nu\alpha)} - \Gamma^\alpha_{\mu\sigma}\Gamma^\sigma_{\alpha\nu} + \Gamma^\alpha_{\mu\nu}\Gamma^\sigma_{\alpha\sigma} \tag{1.6}$$

and

$$I_{\mu\nu} = -\frac{1}{2k^2}(g_{\mu\sigma} g^{[\sigma\rho]} g_{\rho\nu} + \frac{1}{2} g_{\mu\nu} g_{\sigma\rho} g^{[\sigma\rho]} + g_{[\varpi\nu]}) . \tag{1.7}$$

Splitting $I_{\mu\nu}$ into its symmetric and skew symmetric part, we have

$$I_{(\mu\nu)} = -\frac{1}{2k^2}(g_{(\mu\rho)} g^{[\rho\sigma]} g_{[\sigma\nu]} + g_{(\nu\rho)} g^{[\rho\sigma]} g_{[\sigma\mu]} + \frac{1}{2} g_{(\mu\nu)} g_{[\sigma\rho]} g^{[\sigma\rho]}) \tag{1.8}$$

and

$$I_{[\mu\nu]} = -\frac{1}{2k^2}(g_{[\mu\rho]} g^{[\rho\sigma]} g_{[\sigma\nu]} + g_{(\mu\rho)} g^{[\rho\sigma]} g_{(\sigma\nu)} + \frac{1}{2} g_{[\mu\nu]} g_{[\rho\sigma]} g^{[\sigma\rho]} + g_{[\mu\nu]}) \tag{1.9}$$

Here, k is a universal constant to be determined later and may be imaginary.

If the anti-symmetric part of $g_{\mu\nu}$ vanishes then $I_{\mu\nu}$ also vanishes and the field equations of Moffat and Boal's unified field theory reduce to those of general relativity proposed by Einstein.

Einstein's unified field theory is based on the non-symmetric field. His non-symmetric theory is nothing but the most natural generalization of his theory of gravitation by incorporating the electromagnetic field into the fundamental tensor $g_{\mu\nu}$.

To our knowledge no attempt has been made in carrying out the work of Takeno(1961) regarding plane wave like solutions in four dimensional space-time V_4 even from mathematical point of view in Moffat and Boal's non-symmetric unified field theory.

In the paper [1], plane wave-like solution of Einstein general theory of relativity obtained by H. Takeno (1961) as follows

$$ds^2 = -dx^2 - dy^2 - dz^2 + dt^2 - 2f(dz - dt)^2 \tag{1.10}$$

$$F_{12} = F_{34} = 0, \; F_{31} = F_{14} = \sigma, \; -F_{23} = F_{24} = -\rho \tag{1.11}$$

where $(x^1, x^2, x^3, x^4 \equiv x, y, z, t)$ and f, ρ and σ are functions of x, y and $Z \equiv z - t$ satisfying

$$\Delta f = -8\pi(\rho^2 + \sigma^2), \qquad (\Delta \equiv (\partial_{11} + \partial_{22})) \tag{1.12}$$

$$\partial_1\rho + \partial_2\sigma = 0, \qquad \partial_2\rho - \partial_1\sigma = 0. \tag{1.13}$$

Here (1.12) and (1.13) are equivalent to

$$K_{ij} = -8\pi E_{ij}, \quad (i, j, \ldots = 1, \cdots, 4) \tag{1.14}$$

and the generalized Maxwell equations

$$F_{ij;k} + F_{jk;i} + F_{ki;j} = 0, \qquad F^{ij}_{;j} = 0 \tag{1.15}$$

respectively.

Furthermore he has solved the field equations of Einstein non-symmetric unified field theory and obtained some plane wave-like solutions in four dimensional space-time for the same line element of Peres given by (1.10):

To solve the field equations of Einstein non-symmetric unified field theory, H Takeno (1961) assumed:

(i) The line element of the Riemannian space-time is given by (1.10)

(ii) The anti-symmetric part of g_{ij} i.e. $g_{[ij]} = f_{ij}$ is given by

$$f_{12} = f_{34} = 0, \quad f_{13} = -f_{14} = \rho, \quad f_{23} = -f_{24} = \sigma. \tag{1.16}$$

From the assumptions we have

$$(g_{ij}) = \begin{bmatrix} -1 & 0 & \rho & -\rho \\ 0 & -1 & \sigma & -\sigma \\ -\rho & -\sigma & -1-2f & 2f \\ \rho & \sigma & 2f & 1-2f \end{bmatrix}, \tag{1.17}$$

$$(g^{ij}) = \begin{bmatrix} -1 & 0 & \rho & \rho \\ 0 & -1 & \sigma & \sigma \\ -\rho & -\sigma & -1+\xi & \xi \\ -\rho & -\sigma & \xi & 1+\xi \end{bmatrix} \tag{1.18}$$

where $\xi = 2f + \rho^2 + \sigma^2$.

The general solution of weak field equations of Einstein's non-symmetric unified field theory under the assumptions mentioned above is given by ρ and σ satisfying

$$\partial_1\rho + \partial_2\sigma = 0, \qquad \Delta(\partial_2\rho - \partial_1\sigma) = 0 \tag{1.19}$$

and f satisfying

$$-\Delta f = \rho\Delta\rho + \sigma\Delta\sigma + \{(\partial_1\rho)^2 + (\partial_2\sigma)^2 + (\partial_2\rho + \partial_1\sigma)^2 / 2\}. \tag{1.20}$$

And the general solution of strong field equations of Einstein's non-symmetric unified field theory under the same assumptions mentioned above is given by ρ and σ satisfying

$$\partial_1\rho + \partial_2\sigma = 0, \qquad \Delta\rho = 0, \qquad \Delta\sigma = 0 \tag{1.21}$$

and f satisfying

$$-\Delta f = (\partial_1\rho)^2 + (\partial_2\sigma)^2 + (\partial_2\rho + \partial_1\sigma)^2 / 2. \tag{1.22}$$

2. Solution of field equations (1.1) and (1.2) of Moffat and Boal's unified field theory

We have observed that the first two field equations of Moffat and Boal non-symmetric unified field theory are identical to that of Einstein non-symmetric unified field theory and these field equations have already been solved by H Takeno(1961) in the paper refer it to [1]. Therefore solutions of equation (1.1) is given by

$$\Gamma^k_{11} = \Gamma^k_{22} = 0, \quad \Gamma^k_{12} = -\Gamma^k_{21} = \{0, 0, -(\partial_2\rho - \partial_1\sigma)/2, -(\partial_2\rho - \partial_1\sigma)/2\},$$

$$\Gamma^k_{13} = -\Gamma^k_{14} = \{-\partial_1\rho, -(\partial_2\rho + \partial_1\sigma)/2, \partial_1 f, -\alpha_1 + \bar{\rho}, -(\partial_2\rho - \partial_1\sigma)/2\},$$

$$\Gamma^k_{31} = -\Gamma^k_{41} = \{\partial_1\rho, (\partial_2\rho + \partial_1\sigma)/2, \partial_1 f, -\alpha_1 + \bar{\rho}, -(\partial_2\rho - \partial_1\sigma)/2\},$$

$$\Gamma^k_{23} = -\Gamma^k_{24} = \{-(\partial_2\rho + \partial_1\sigma)/2, -\partial_2\sigma, \partial_2 f, -\alpha_2 + \bar{\sigma}, -(\partial_2\rho - \partial_1\sigma)/2\},$$

$$\Gamma^k_{32} = -\Gamma^k_{42} = \{(\partial_2\rho + \partial_1\sigma)/2, \partial_2\sigma, \partial_2 f, -\alpha_2 + \bar{\sigma}, -(\partial_2\rho - \partial_1\sigma)/2\},$$

$$\Gamma^k_{33} = -\Gamma^k_{34} = -\Gamma^k_{43} = \Gamma^k_{44} = \{-\partial_1 f + 2\alpha_1, -\partial_2 f + 2\alpha_2, \bar{f}, \bar{f}\}. \tag{2.1}$$

And the field equation (1.2) is satisfied under the condition

$$\partial_1\rho + \partial_2\sigma = 0. \tag{2.2}$$

3. The components of R_{ij}

To solve the remaining field equations on Moffat and Boal's non-symmetric unified field theory it is essential to calculate components of R_{ij}**,** but they are already have been calculated by H. Takeno(1961) in [1] as under

$$R_{ab} = 0\,, \quad R_{13} = -R_{31} = -R_{14} = R_{41} = -\Delta\rho/2\,, \quad R_{23} = -R_{32} = -R_{24} = R_{42} = -\Delta\sigma/2$$

$$R_{33} = -R_{34} = -R_{43} = R_{44} = Q\,, \quad (a,b=1,2) \tag{3.1}$$

$$-Q = \Delta f + \rho\Delta\rho + \sigma\Delta\sigma + (\partial_1\rho)^2 + (\partial_2\sigma)^2 + (1/2)(\partial_2\rho + \partial_1\sigma)^2\,. \tag{3.2}$$

The non-vanishing independent components of symmetric $R_{(ij)}$ and anti-symmetric $R_{[ij]}$ are respectively

$$R_{(33)} = -R_{(34)} = -R_{(43)} = R_{(44)} = Q\,,$$

$$R_{[13]} = -R_{[14]} = -\Delta\rho/2\,, \qquad R_{[23]} = -R_{[24]} = -\Delta\sigma/2\,. \tag{3.3}$$

4. Solutions of (1.3) and (1.4) of Moffat and Boal's unified field theory

To solve these field equations it is necessary to calculated the components of $I_{\mu\nu}$, therefore from equation (1.8) the symmetric components of $I_{\mu\nu}$ are calculated as under

$$I_{(33)} = I_{(44)} = -I_{(34)} = -I_{(43)} = \frac{-(\sigma^2+\rho^2)}{k^2}\,. \tag{4.1}$$

and from (1.9) the anti-symmetric components of $I_{\mu\nu}$ are calculated as under

$$I_{[13]} = -I_{[14]} = -\rho/k^2\,, \qquad I_{[23]} = -I_{[24]} = -\sigma/k^2\,. \tag{4.2}$$

Using (3.1) and (4.1) the symmetric components of $R^*{}_{\mu\nu}$ are obtained as

$$R^*{}_{(13)} = R^*{}_{(14)} = 0\,, \qquad R^*{}_{(23)} = R^*{}_{(24)} = 0\,,$$

$$R^*{}_{33} = -R^*{}_{(34)} = R^*{}_{44} = Q - \frac{1}{k^2}(\sigma^2 + \rho^2)\,. \tag{4.4}$$

Similarly the anti-symmetric components of $R^*{}_{\mu\nu}$ are obtained as

$$R^*{}_{[13]} = -R^*{}_{[14]} = -\Delta^*\rho\,, \qquad R^*{}_{[23]} = -R^*{}_{[24]} = -\Delta^*\sigma \tag{4.5}$$

where $\Delta^* = (\frac{\Delta}{2} + \frac{1}{k^2})\,.$ (4.6)

From (4.4) we have a solution of the field equation (1.3) of Moffat and Boal as

$$Q = \frac{1}{k^2}(\sigma^2 + \rho^2)\,. \tag{4.7}$$

And with the help of (4.5) the field equation (1.4) of Moffat and Boal is satisfies under the condition

$$\Delta^*(\partial_2\rho - \partial_1\sigma) = 0\,. \tag{4.8}$$

5. Conclusion

The $g_{\mu\nu}$ given by (1.17) is a plane wave-like solution of Moffat and Boal's field equation if ρ, σ satisfy (2.2), (4.7) and (4.8). It has been observed that if anti-symmetric part of $g_{\mu\nu}$ vanishes then the plane wave-like solutions of field equations of general relativity proposed by Einstein in the space-time of Peres obtained by Takeno(1961) can be brought out.

Acknowledgement: The authors are thankful to Professor K. D. Thengane of India for his constant inspiration.

References

[1] H. Takeno(1961), Some plane wave-like solutions of non-symmetric unified field theory. Tensor New Series, Japan.

[2] H. Takeno(1961),The Mathematical theory of plane gravitational wave in general relativity". *Scientific report of the Research Institute to theoretical physics. Hiroshima, University, Takchara, Hiroshima-ken, Japan*-1961.

[3] A. Peres (1959), Some gravitational waves, Phys. Rev. Let., 3, 571-572.

[4] J. W. Moffat & D. H. Boal (1975), Solutions of the non-symmetric unified field theory, Physical Review D, 11,1375.

[5] D. H. Boal and J. W. Moffat (1975), Physicaal consequences of a solution of the non-symmetric unified field theory, Physical Review D, 11, 2026.

[6] A. Einstein (1955), The meaning of Relativity (Princeton Uni. Press, Princeton), Appendix II, PP.133-166