Article

Dark Energy Model in Bianchi Type-V String Spacetime in Lyra Geometry

Kanika Das* & Gitumani Sarma

Department of Mathematics, Gauhati University, Guwahati-781014, Assam, India

Abstract

In this paper we present a string cosmological model of Bianchi type-V space-time in Lyra geometry with dark energy. Assuming $p = -\rho$ and choosing the average scale factor we have obtained exact solutions for the cosmological model. The study reveals that string dominates the early universe and eventually disappears from the universe for large time, i.e., at the present epoch. Some physical and geometrical properties of the model are discussed.

Keywords: dark energy, Lyra Geometry, cosmic string, Bianchi Type-V, spacetime.

1. Introduction

Recently, considerable work has been done in string cosmology. Einstein's theory of gravity has been the subject of intense study for its success in explaining the observed accelerated expansion of the universe at late times. Bianchi-type cosmological models are important because these are homogeneous and anisotropic. The origin of the universe is one of the greatest cosmological mysteries even today. The exact physical situation at early stage of the formation of our universe is still unknown. The concept of string theory was developed to describe events of the early stage of the evolution of the universe.

The present day observations of the universe indicate the existence of a large scale network of strings in the early universe (Kibble 1976, 1980). In recent years there has been a lot of interest in the study of cosmic strings. Cosmic strings have received considerable attention as they are believed to have served in the structure formation in the early stages of the universe. Kibble (1976) showed that cosmic strings may have been created during phase transitions in the early era and they act as a source of gravitational field (Letelier (1983)).The study of cosmic strings in relativistic framework was intiated by Stachel (1990) and Letelier (1979). Krori et.al (1990, 1994), Raj Bali and Shuchi Dave (2001), Bhattacharjee and Baruah (2001), Rahaman et al.(2003), Reddy (2003) are some of the authors who have studied various aspects of string

*Correspondence Author: Kanika Das, Department of Mathematics, Gauhati University, India. E-mail: daskanika2@gmail.com

cosmologies in general relativistic theory as well as alternative theories of gravitation. Krori et al. (1994) have shown that in the context of general relativity cosmic strings do not occur in Bianchi type V cosmology. Also Adhav et al. (2009) have obtained the same findings as Krori et al. about Bianchi type V cosmology.

Now recent years observations of the luminosity of type Ia supernovae (Riess,1998; Permutter and Bachall, 1999) point towards an accelerated expansion of the universe, which implies that the pressure p and the energy density ρ of the universe should violate the strong energy condition, i.e. $\rho + 3p < 0$. This implies that the universe is dominated by a form of matter with negative pressure which is referred to as dark energy today. There are different candidates for the role of dark energy. The most traditional candidate is a non-vanishing cosmological constant, which can also be thought of as a perfect fluid satisfying the equation of state $p = -\rho$. So far the confirmed information about dark energy is still limited and can be roughly summarized as 'it is a kind of exotic energy form with sufficiently large negative pressure such that drives the universe to undergo a period of accelerating expansion'. The investigation of dark energy is an important mission in the modern cosmology. Much work has been done on this issue and there is still a long way to go. Currently the preferred candidates of dark energy are vacuum energy (or cosmological constant) and dynamical fields.

The geometrization of gravitation by Einstein in his general theory of relativity inspired several authors to geometrize other physical fields. Weyl (1918) proposed a unified theory to geometrize gravitation and electromagnetism. But due to the non-integrability of length transfer this theory was never considered seriously. However, this theory inspired Gehard Lyra to develop what is called Lyra geometry. Lyra (1951) proposed a new modification of Riemannian geometry by introducing a gauge function to remove the non-integrability of the length of a vector under parallel transport. Subsequently Sen (1957), Sen and Dunn (1971) suggested a new scalar tensor theory of gravitation and constructed an analog of the Einstein field equations based on Lyra's geometry which in normal gauge may be written as

$$R_{ij} - \frac{1}{2} g_{ij} R + \frac{3}{2}\phi_i \phi_j - \frac{3}{4} g_{ij}\phi_k \phi^k = -\chi T_{ij}$$

where ϕ_i is the displacement vector, c = 1, $8\pi G = \chi$ and other symbols have their usual meaning in the Reimannian geometry. A brief note on Lyra's geometry is given by Beesham (1988), Singh and Singh (1992).

Halford (1970) has pointed out that the constant vector displacement field β in Lyra geometry plays the role of cosmological constant in the normal general relativistic treatment. Five dimensional cosmological models in Lyra's manifold are constructed by Rahman et.al (2003), Singh et.al (2004), and Mohanty et.al (2006, 2007 and 2007). Beesham (1988) considered four-dimensional FRW cosmological model in Lyra's geometry with time-dependent displacement field. He has shown by assuming the energy density of the universe to be equal to its critical value, the models have the k = -1 geometry. Singh and Desikan (1997) obtained the exact solutions for four-dimensional FRW cosmological model in Lyra's geometry with constant deceleration parameter. They examined the behaviour of the displacement field β and the energy density ρ for perfect fluid distribution. Mohanty et al. showed the non-existence of five-dimensional perfect fluid cosmological model in Lyra's geometry. Further they obtained the exact solutions of the field equations for empty universe.

The objective of the present paper is to study the Bianchi type V string cosmological model in lyra geometry taking negative pressure, which is a criteria of dark energy and taking variable deceleration parameter. We observe that the universe is accelerating. The physical and geometrical aspects of the solutions are discussed in detail.

This paper is organised as follows: In section 2, the metric and the field equations are presented. In section 3, we deal with the solutions of field equations. Section 4 describes some physical and geometric properties of the model and we have seen that the universe is an accelerating one. The paper ends with a conclusion in section 5.

2. The metric and field equations

The line element for the spatially homogeneous and anisotropic Bianchi-v space time is given by

$$ds^2 = dt^2 - A^2dx^2 - e^{2\alpha x}\left(B^2dy^2 + C^2dz^2\right) \tag{1}$$

where A, B and C are functions of time and α is a constant.

We assume $a = \left(ABC\right)^{\frac{1}{3}}$ as the average scale factor so that the Hubble parameter H is given by

$$H = \frac{\dot{a}}{a} \tag{2}$$

The energy momentum tensor $T^i{}_j$ for a cloud of massive strings and the distribution of perfect fluid is taken as

$$T^i{}_j = (\rho + p)v^i v_j - pg^i{}_j - \lambda x^i x_j \tag{3}$$

where p is isotropic pressure, ρ is proper density for a cloud of strings with particle attached to them, λ is string tension density, $v^i = (0,0,0,1)$ is the four velocity of the particles. And x^i is a unit space-like vector representing the direction of the string. The vector v^i and x^i satisfy the condition

$$v_i v^i = -x_i x^i = -1 \tag{4}$$

The particle density ρ_p is given by

$$\rho = \rho_p + \lambda \tag{5}$$

Einstein's field equation based on Lyra geometry is taken as

$$R_{ij} - \frac{1}{2} g_{ij} R + \frac{3}{2}\phi_i \phi_j - \frac{3}{4} g_{ij}\phi_k \phi^k = -8\pi G T_{ij} \tag{6}$$

The Einstein's field equation (6) for the line element (1) lead to the following equations

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{\alpha^2}{A^2} = -\zeta p - \frac{3}{4}\beta^2 + \lambda \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{\alpha^2}{A^2} = -\zeta p - \frac{3}{4}\beta^2 \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\alpha^2}{A^2} = -\zeta p - \frac{3}{4}\beta^2 \tag{9}$$

$$\frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} + \frac{\dot{A}\dot{B}}{AB} - \frac{3\alpha^2}{A^2} = \zeta\rho + \frac{3}{4}\beta^2 \tag{10}$$

$$\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \tag{11}$$

where $\zeta = 8\pi G$

Taking divergence of (6) with T_{ij} given by (3), the Bianchi identity gives

$$\zeta\dot{\rho} + \frac{3}{2}\beta\dot{\beta} + \left[\zeta(\rho + p) + \frac{3}{2}\beta^2\right]\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0 \tag{12}$$

The conservation law for energy-momentum tensor $T^i_{j;i} = 0$ leads to

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) - \lambda\frac{\dot{A}}{A} = 0 \tag{13}$$

3. Solution of the field equation

Integrating equation (11) and taking the constant of integration equal to one, we get

$$A^2 = BC \tag{14}$$

Using (14), subtracting (8) from (9)

$$\frac{B}{C} = k_1 \exp\left(\int \frac{k}{ABC}\, dt\right) \tag{15}$$

Now there are five independent equations (7) to (11) in seven unknowns A, B, C, ρ, p, λ and β. For the complete determination of the system, we need two extra conditions.

As such, we assume

$$p = -\rho \tag{16}$$

Following Pradhan and Amirhashchi (2011) we also assume that

$$a = \left(t^n e^t\right)^{\frac{1}{m}} \tag{17}$$

where m and n are positive constants.

Now the spatial volume V of the model read as

$$V = (a(t))^3 = \left(t^n e^t\right)^{\frac{3}{m}} \tag{18}$$

Using equations (14), (17) and (18) we obtain

$$A(t) = \left(t^n e^t\right)^{\frac{1}{m}} \tag{19}$$

Also using equation (19) in (14) and (15), we get

$$B(t) = \left(t^n e^t\right)^{\frac{1}{m}} \left\{k_1 \exp\left(\int \frac{k}{ABC} dt\right)\right\}^{\frac{1}{2}} \tag{20}$$

$$C(t) = \left(t^n e^t\right)^{\frac{1}{m}} \left\{k_1 \exp\left(\int \frac{k}{ABC} dt\right)\right\}^{\frac{-1}{2}} \tag{21}$$

4. Some Physical and Geometrical Properties

The time-displacement field β, pressure p, density ρ, the string tensor density λ, particle density ρ_p, deceleration parameter q are given by

$$\beta^2 = \frac{2}{3}\left[\begin{array}{l} \frac{2n}{m}t^{-2} + \frac{1}{m^2} - \frac{3nk}{m}t^{-1}a^{-3} - \frac{3k}{m}a^{-3} - \frac{k^2}{4}a^{-6} \\ + \frac{k}{2m}\left(\frac{n}{t}+1\right)a^{-3} - \frac{k}{m}\left(\frac{n}{t}+1\right)a^{-1} - 2\alpha^2 a^{-2} - \frac{k^2}{4}a^{-4} \end{array}\right] \tag{22}$$

$$\rho = \frac{1}{\zeta}\left[\begin{array}{l} \frac{n(3n-m)}{m^2}t^{-2} + \frac{6n}{m^2}t^{-1} + \frac{5}{2m^2} + \frac{3k}{m}\left(\frac{n}{t}+1\right)a^{-3} \\ -\frac{k}{2m}\left(\frac{n}{t}+1\right)a^{-1} - \frac{k^2}{8}\left(a^{-4} - a^{-6}\right) - 2\alpha^2 a^{-2} + \left(\frac{3nk}{2m}t^{-1} + \frac{3k}{2m}\right)a^{-3} \end{array}\right] \tag{23}$$

$$p = -\frac{1}{\zeta}\left[\begin{array}{l} \frac{n(3n-m)}{m^2}t^{-2} + \frac{6n}{m^2}t^{-1} + \frac{5}{2m^2} + \frac{3k}{m}\left(\frac{n}{t}+1\right)a^{-3} \\ -\frac{k}{2m}\left(\frac{n}{t}+1\right)a^{-1} - \frac{k^2}{8}\left(a^{-4} - a^{-6}\right) - 2\alpha^2 a^{-2} + \left(\frac{3nk}{2m}t^{-1} + \frac{3k}{2m}\right)a^{-3} \end{array}\right] \tag{24}$$

$$\lambda = \frac{n^2}{m^2}t^{-2} + \frac{2n}{m^2}t^{-1} + \left\{\frac{3nk}{m}t^{-1} + \frac{3k}{m} + \frac{k}{2m}\left(\frac{n}{t}+1\right)\right\}a^{-3} + \frac{k^2}{4}a^{-4}(a^{-2}-1) \tag{25}$$

An important quantity, q, the deceleration parameter is given by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 + \frac{mn}{(n+t)^2} \tag{26}$$

Equation (26) shows that deceleration parameter q depends on the constant m and n.

From (22) and (25) we get

$$\lambda = S - \frac{3}{2}\beta^2 \tag{27}$$

where

$$S = \left\{\left(\frac{2nm+n^2}{m^2}\right)t^{-2} + \frac{2n}{m^2}t^{-1} + \frac{k}{2m}\left(\frac{n}{t}+1\right)a^{-3} - \frac{k^2}{2}a^{-4} - 2\alpha^2 a^{-2} - \frac{k}{m}\left(\frac{n}{t}+1\right)a^{-1}\right\} + \frac{1}{m^2}$$

Equation (27) shows that there is a relation between the string density and displacement field.

The average Hubble parameter H and expansion scalar θ is found to be

$$H = \frac{1}{m}\left(\frac{n}{t}+1\right) \tag{28}$$

$$\theta = 3H = \frac{3}{m}\left(\frac{n}{t}+1\right) \tag{29}$$

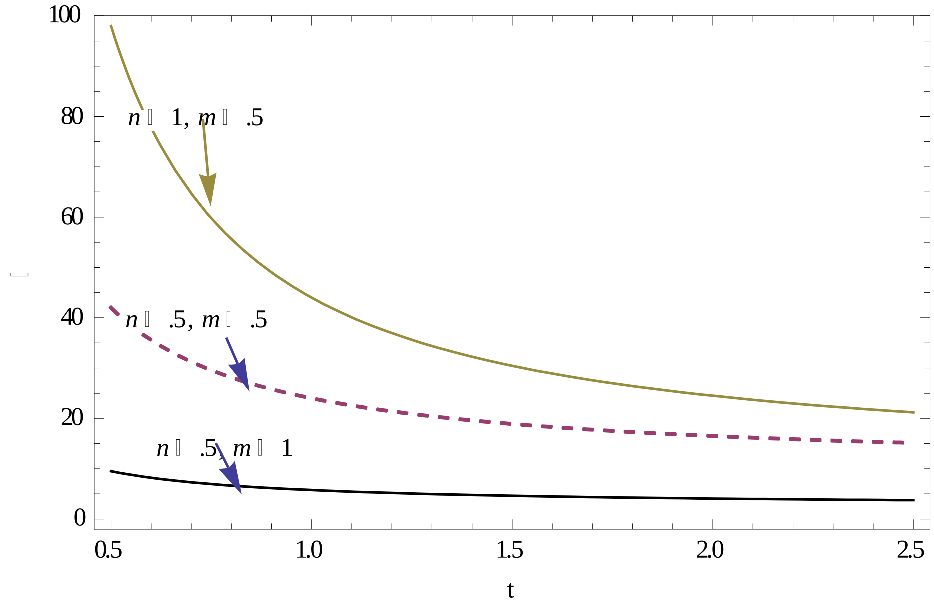

Fig.1 shows the variation of ρ vs. time. It is observed that density decreases as the time increases (taking $k = .009$, $\alpha = .002$)

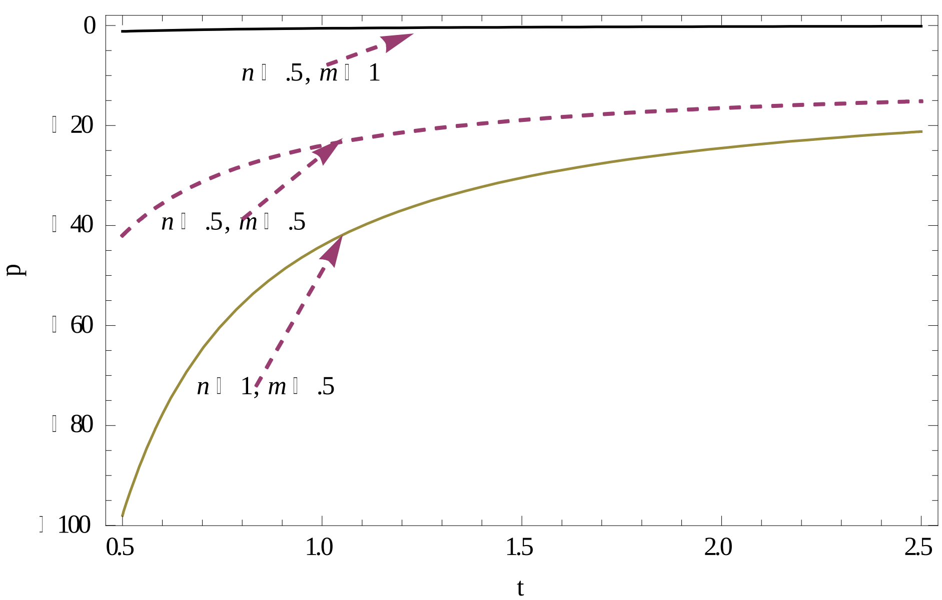

Fig.2 shows the variation of p vs. time. It is observed that pressure increases as the time increases. Also pressure is negative.

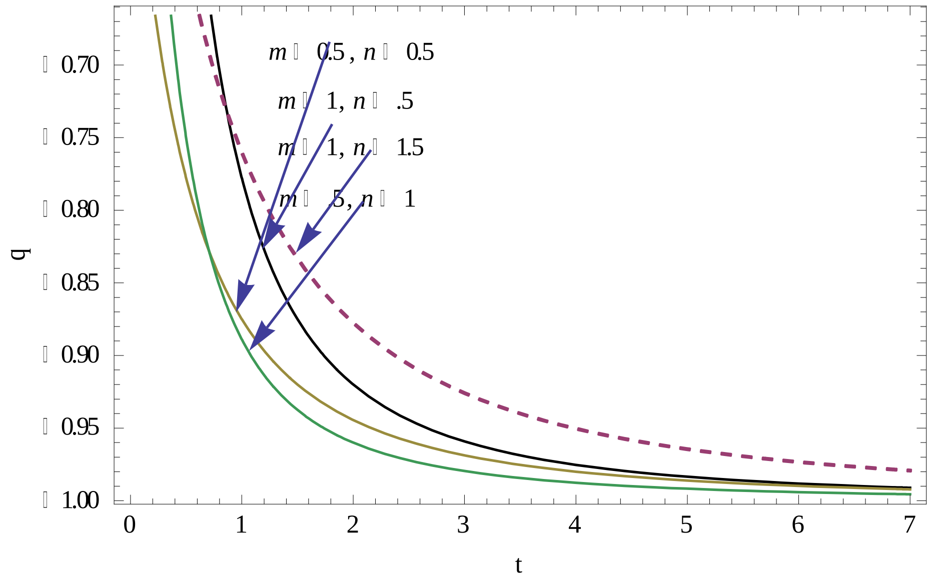

Fig.3 shows the variation of DP (q) vs. time. It is observed that deceleration parameter depends on two free parameters m and n. And it decreases with time. For all values of m and n the universe is accelerating as the time increasing. The dynamics of DP yields two different phase of the universe.

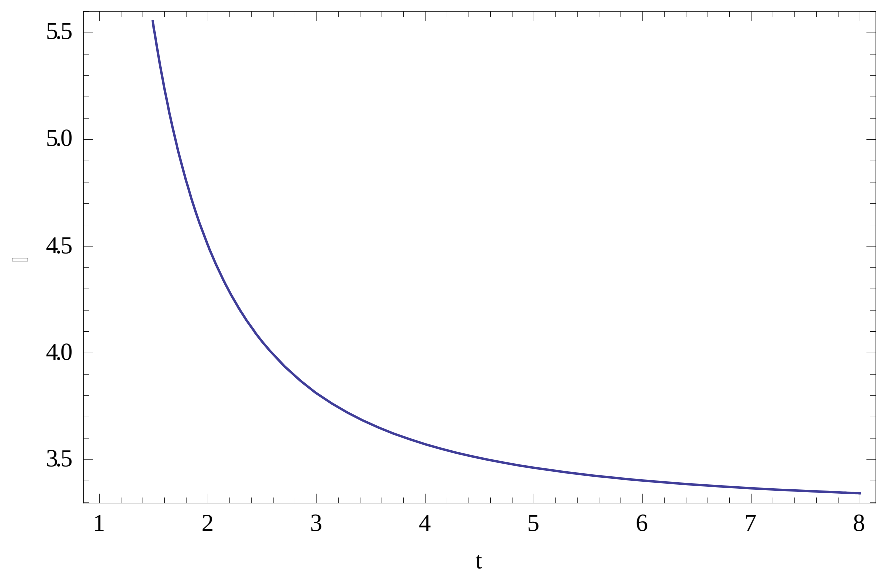

Fig.4 shows the variation of β vs. time, using parameter $n = 1.5$, $m = .5$, $k = -2$, $\alpha = 6$.

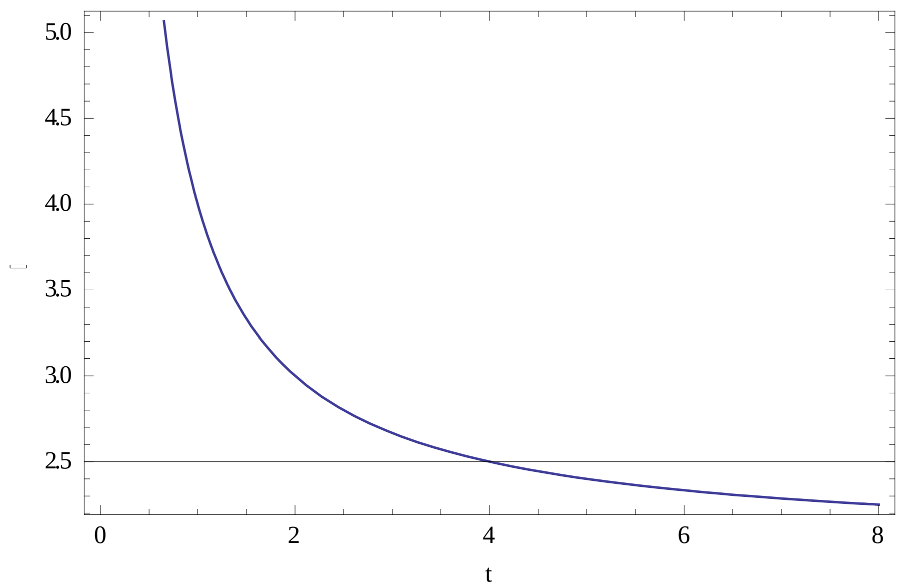

Fig.6 shows the variation of λ vs. time. It is observed that the string tension density starts off with extremely large values and tends to zero. Thus the string dominates the early universe and decreases as the time increases.

5. Discussions

In the present work we have derived some exact solutions for Bianchi type V string cosmological model in Lyra geometry and with dark energy. From fig.3 it is observed that the dynamics of DP (q) depends on two free parameters m and n. Also from equation (26) it is clear that deceleration parameter is time dependent. We have examined that at $t = 0$, the spatial volume vanishes and parameter H and θ diverge. Hence the model starts with big bang singularity at $t = 0$. Here the expansion scalar is finite at $t = 0$ and decreases as the time increases. From equation (28) we have observed that density depends on gauge function and also on time. And density and gauge function (time displacement field) both decrease as the time increases. That means at the very early stage of the universe density and time displacement field are very large.

The pressure is negative and increases as time increases (Fig.2). The pressure p and the energy density ρ of the universe should violate the strong energy condition $(\rho + 3p > 0)$. From fig-5 we have shown that string tension density is very large in early universe and decreases as time increases. Thus the string dominates the early universe and eventually disappears from the universe at later times. This paper is based on an exact solution of Einstein's field equations for anisotropic Bianchi type V space time with dark energy equation of state. The model is accelerating as q, the deceleration parameter is negative. Therefore, this dark energy model in

Bianchi type V string space time is consistent with the recent observations of Type-Ia Supernovae. Finally, we conclude that our model is accelerating and represent not only the early stages of evolution but also the present stage of the universe.

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References

Kibble, T. W. B., J. Phys. A **9**, 1387 (1976)
Kibble, T. W. B., Phys. Rep. **67**, 183 (1980)
Letelier, P. S., Phys. Rev. *D* **28**, 2414 (1983)
Stachel, J., Phys. Rev. D **28**, 2171 (1990)
Letelier, P. S., Phys. Rev. D **20**, 1294 (1979)
Krori, K.D., Choudhury, T. Mahanta, C. R., Gen. Relativity gravit. **22**, 123 (1990)
Krori, K.D., Choudhury, T. Mahanta, C. R., Gen. Relativity gravit. **26**, 265 (1994)
Balsi, R., Dave, S., Pramana, J. Phys. **56**, 4, 513 (2001)
Bhattacharjee, R., Baruah, K. K.,Indian J. Pure Appl. Math. **32**, 47 (2001)
Rahaman, F., Chakravorty, S., das, S., Hossain, M., Bera, J., Pramana, J. Phys. **60**, 453(2003)
Reddy, D. R. K.,Astrophys. Space Sci. **286**, 359 (2003)
Adhav, K. S., Nimkar, A. S.,Dawande, M. V., Ugale, M. A., Rom. Journ. Phys. Vol. **54**, 207(2009)
Riess, A. et al., Astron. J. **116**, 1009 (1998)
Perlmutter, S. J et al., Astroph. J. **517**, (1999) 565-586
Bachall, N. A., Ostriker, J. P., Perlmutter, S and Steinhardt, P. J., Science **284**, 1481(1999)
Weyl, H., Sber Preuss. Akad . Wiss. **465** (1918)
Lyra, G. , Math. Z. **54**, 52 (1951)
Sen, D. K., Z.f¨ur Physik **149,** 311 (1957)
Sen, D. K., Dunn, K. A., Math. Phys. **12**, 578 (1971)
Beesham, A., Aust . J. Phys. **41**, 833 (1988)
Singh, T and Singh, G. P., Int. J. Theo.Phys.**31**, 1433 (1992)
Halford, W. D., Aust. J. Phys.**23**, 863 (1970)
Singh, G. P., Despandey, R . V. and Singh T., Pramana J. Phys. **63**, 937 (2004)
Mohanty, G., Mahanta, K. L. and Sahoo R. R.**,** Astrophys space sci . **306** 269 (2006)
Mohanty, G., Mahanta K . L., and Bishi, B. K.,Astrophys . space. Sci, 310, 273 (2007)
Mohanty,G., Mahanta, K. L. Astrophys .space sci , **312** , 301 (2007)
Singh, G. P and Desikan, K., Pramana J. Phys. **49**, 205 (1997)
Pradhan, A., and Amirhshachi, H.,Modern Physics Letters A, **26**, 2261 (2011)