Report

A Note on the Hall Effect and Graphene

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Abstract

We briefly comment on the parallel between Quantum Theory and the behaviour of graphene and thus provide an explanation for the Hall Effect and related effects.

In this note we comment on the Hall Effect from the following novel point of view namely the strong parallel between two dimensional Quantum Mechanics and the analysis of graphene [1, 2]. The Hall Effect itself was observed in the nineteenth century, in the case of a current (or a moving electron) I_x along the x axis, a magnetic field B_z along the z axis, leading to the so called Hall Voltage along the y axis. This is given by

$$V_H = \left(\frac{1}{n|e|}\right) \cdot \frac{I_x}{d} B_z \tag{0.1}$$

The expression $\left(\frac{1}{n|e|}\right)$ is called the Hall Resistence and V_H is the Hall Voltage while d is the thickness of the conductor. We would like to point out that the Hall Effect has a parallel in relativistic electromagnetic theory.

In two dimensions and one dimension electrons will display strange neutrino like properties as had been pointed out by the author, starting the mid nineties [3, 4, 5]. The two component equation that is obeyed [6] is

$$\left(\sigma^\mu \partial_\mu - \frac{mc}{\hbar}\right) \psi = 0 \tag{0.2}$$

where σ^μ denote the 2×2 Pauli matrices. In case the mass vanishes (0.2) gives the neutrino equation. This has relevance to graphene that was discovered nearly a decade later. For the electron quasi particles in graphene we have

$$\nu_F \vec{\sigma} \cdot \vec{\nabla} \psi(r) = E\psi(r) \tag{0.3}$$

$\nu_F \sim 10^6 m/s$ is the Fermi velocity replacing c, the velocity of light and $\psi(r)$ being a two component wave function, E denoting the energy.

Indeed going a step further, the author has argued that graphene (or more generally other two dimensional structures) could be a test bed for high energy physics itself, in the sense of the role played by a wind tunnel, given Reynold's numbers, for the actual problem

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[1, 2]. Surprisingly we can then resolve puzzles like the minimum conductivity observed in graphene as also get an explanation for the fractional Quantum Hall effect, something that has eluded us.
We would now like to observe that the relativity that can be seen in graphene with the Fermi velocity ν_F replacing the velocity of light gives a "Lorentz" transform. Furthermore in the electromagnetic case, this leads to the Lorentz force equal to $\vec{v} \times \vec{B}$ where $\vec{v}$ is the velocity of the moving or conduction electron and $\vec{B}$ is the magnetic field. This Lorentz force can be immediately identified with the Hall effect emf.
More specifically the Lorentz force is given by

$$\text{Force } = \frac{d\vec{p}}{dt} = \frac{e}{c}\vec{u} \times \vec{B} \tag{0.4}$$

So the energy is given by

$$\text{Energy } = \frac{1}{c}R\frac{I_x B_z}{d} \tag{0.5}$$

where a factor R has been introduced to indicate a resistance, in case the electron is not a free moving particle. Comparing (0.1) and (0.5) we can immediately see that the Hall Effect is nothing but a manifestation of the Lorentz force in relativistic electrodynamics.
It must also be pointed out, that as shown in references [1, 2], it is the non-commutative nature of the graphene space which leads to mysteries like the minimum conductance in graphene or the Fractional Quantum Hall Effect.

References

[1] Sidharth, B.G. (2014). *New Advances in Physics* Vol.8 (2), 2014.

[2] Sidharth, B.G. (2014). *Graphene and High Energy Physics* to appear in *Int.J.Mod.Phys.E* 2014.

[3] Sidharth, B.G. (1995). *A Note on Two Dimensional Fermions* in *BSC-CAMCS-TR-95-04-01.*

[4] Sidharth, B.G. (1999). *Low Dimensional Electrons* in *Solid State Physics*, Eds., R. Mukhopadhyay et al., 41, (Universities Press, Hyderabad, 1999), p.331.

[5] Sidharth, B.G. *arXiv. quant-ph. 9506002.*

[6] Sidharth, B.G. (2001). *The Chaotic Universe: From the Planck to the Hubble Scale* (Nova Science Publishers, Inc., New York, 2001).